FUNDAMENTALS OF **LEFM** AND APPLICATIONS TO **FATIGUE CRACK GROWTH**

FUNDAMENTALS OF **LEFM** & APPLICATIONS TO **FATIGUE CRACK GROWTH**

- Overview
- LEFM Concepts
- Crack Tip Plasticity
- Fracture Toughness
- Fatigue Crack Growth
- Mean Stress Effects on FCG
- Cyclic Plastic Zone, LEFM Limitations, EPFM
- Crack Closure
- Small Cracks

OVERVIEW

FUNDAMENTALS OF LEFM & APPLICATIONS TO FATIGUE CRACK GROWTH, FCG (**OVERVIEW**)

- The presence of a crack can significantly reduce the life of a component or structure.

- To make life estimations for fatigue crack growth and damage tolerant design, the following information are often needed:
  - The stress intensity factor, K.
  - The fracture toughness, K_c.
  - The applicable fatigue crack growth rate expression.
  - The initial crack size, a_i (a_o).
  - The final or critical crack size, a_f (a_c).

FUNDAMENTALS OF LEFM & APPLICATIONS TO FCG, **OVERVIEW**

- Damage tolerant and fatigue crack growth analysis design assume a discontinuity, flaw, or crack exists.

- Cracks can form
  – due to fatigue,
  – as a consequence of manufacturing processes (i.e. deep machining marks or voids in welds),
  – and metallurgical discontinuities (i.e. inclusions).

FUNDAMENTALS OF LEFM & APPLICATIONS TO FCG, **OVERVIEW**

$$N_{total} = N_{nucleation} + N_{growth}$$

$N_{nucleation}$	N_{growth}
S-N or **ε-N**	**LEFM**

$N_{nucleation}$ may be 0 or almost the entire life

N_{growth} may be very small or almost the entire life

FUNDAMENTALS OF LEFM & APPLICATIONS TO FCG, **OVERVIEW**

- Use of fracture mechanics in fatigue crack growth and damage tolerant design requires knowledge of "pre-existing" cracks, either assumed or found using nondestructive flaw detection techniques (reviewed in Chapter 2).

- Fracture mechanics has been used heavily in the aerospace, nuclear, and ship industries with a recent extension to the ground vehicle industry.

FUNDAMENTALS OF LEFM & APPLICATIONS TO FCG, **OVERVIEW**

- This chapter
  - Provides an introduction to the important aspects of linear elastic fracture mechanics (LEFM),
  - Shows how LEFM is used to describe and predict fatigue crack growth and final fracture.
  - Does not contain the mathematics used to develop the theory (See textbooks on fracture mechanics 1-4), but does provide the back-ground fracture mechanics concepts needed for fatigue design.

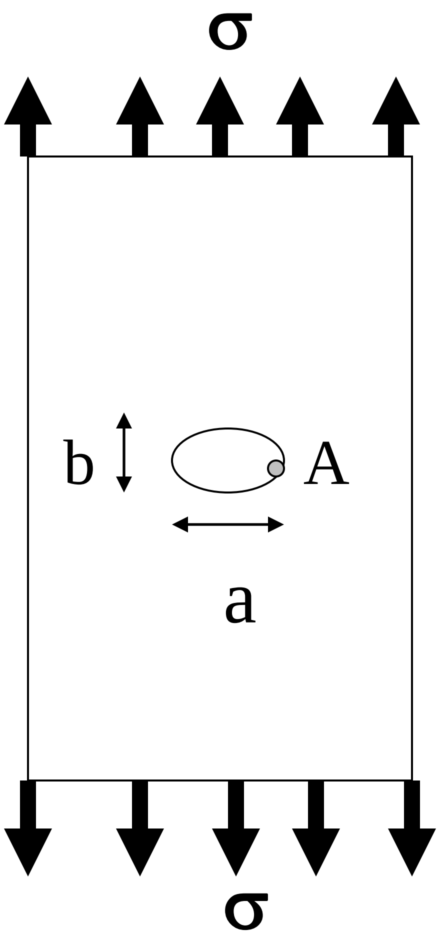
LEFM & APPLICATIONS TO FCG, **OVERVIEW**

Stress Concentration Effect of Flaws and the need for Fracture Mechanics.

- Consider a plate with an elliptical hole.
- Based on Mechanics of Materials approach for stress concentrations:

$$\sigma_A = K_t\, \sigma = (1 + 2\, a/b)\, \sigma$$

- For circular hole, $a = b$, $K_t = 3$
- As $a \gg b$ (i.e. crack), σ_A goes to infinity (even for small σ).
- Therefore a different approach is needed (Fracture Mechanics)

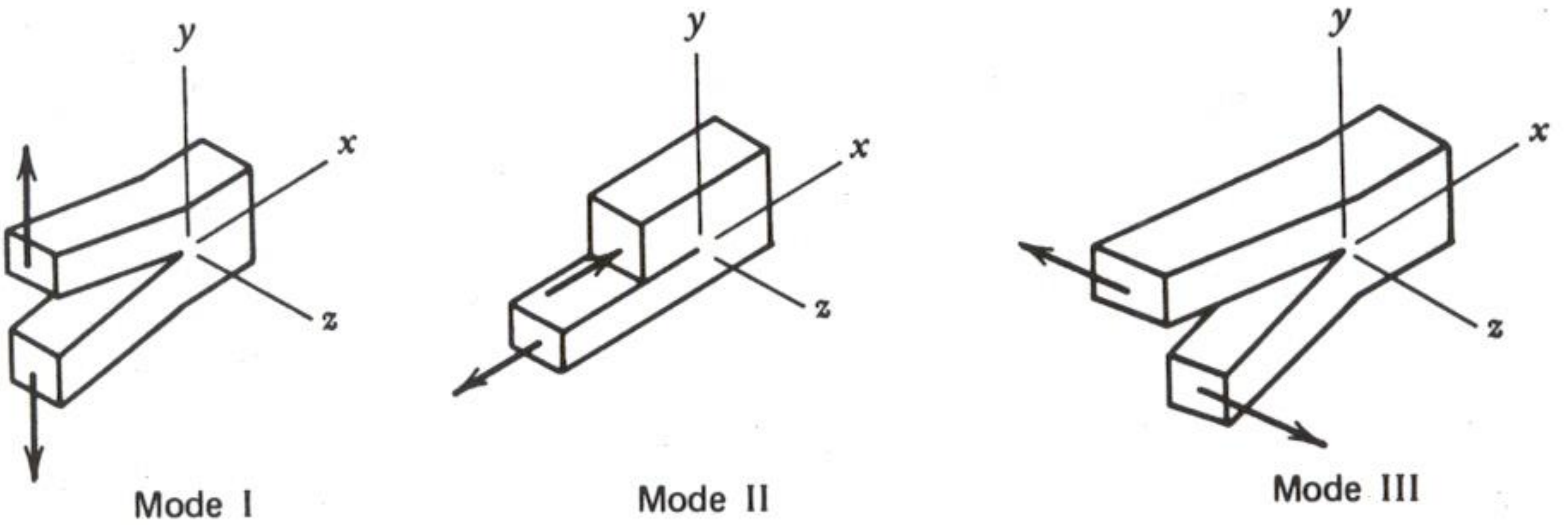


LEFM & APPLICATIONS TO FCG (**LEFM CONCEPTS**)

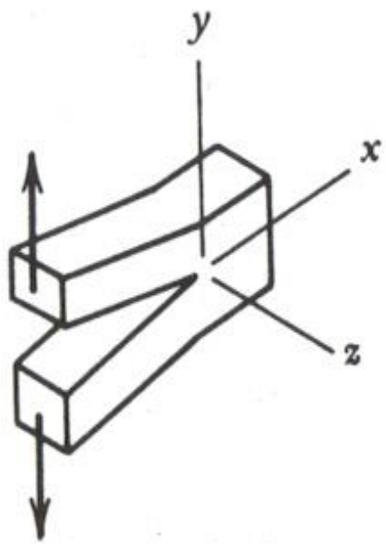
- **Fracture mechanics** is used to evaluate the strength of a structure or component in the presence of a crack or flaw.
- **LEFM** is used for material conditions which are predominantly linear elastic during the fatigue process.
- For crack growth or fracture conditions that violate this basic assumption, elastic-plastic fracture mechanics (**EPFM**) approaches are used to describe the fatigue and fracture process.

LEFM & APPLICATIONS TO FCG
(LEFM CONCEPTS)

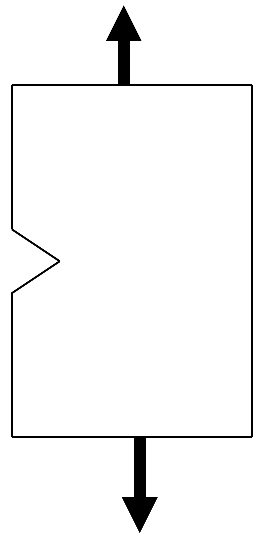
Three modes in which a crack can extend:

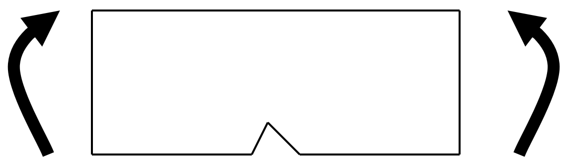


MODE I



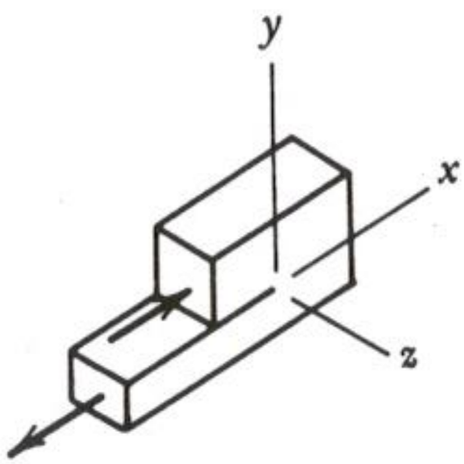
Mode I



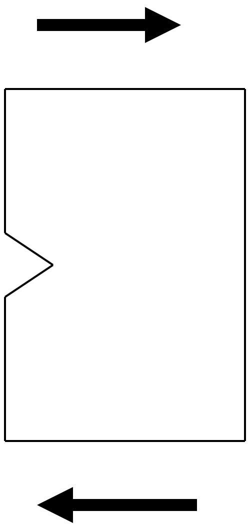


- Mode I is the crack opening mode
- It is the most common mode, particularly in fatigue, because cracks tend to grow on the plane of maximum tensile stress.

MODE II

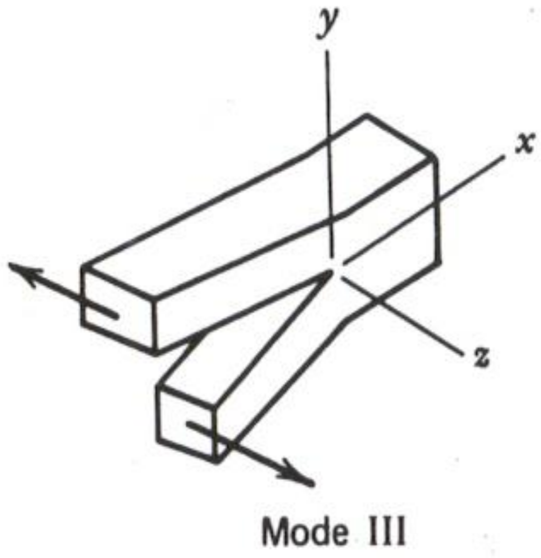


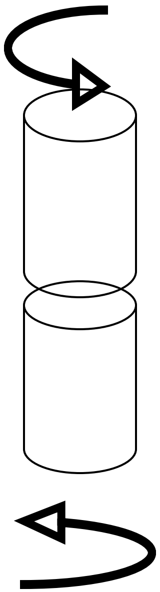
Mode II



Mode II is the in-plane shearing or sliding mode

MODE III

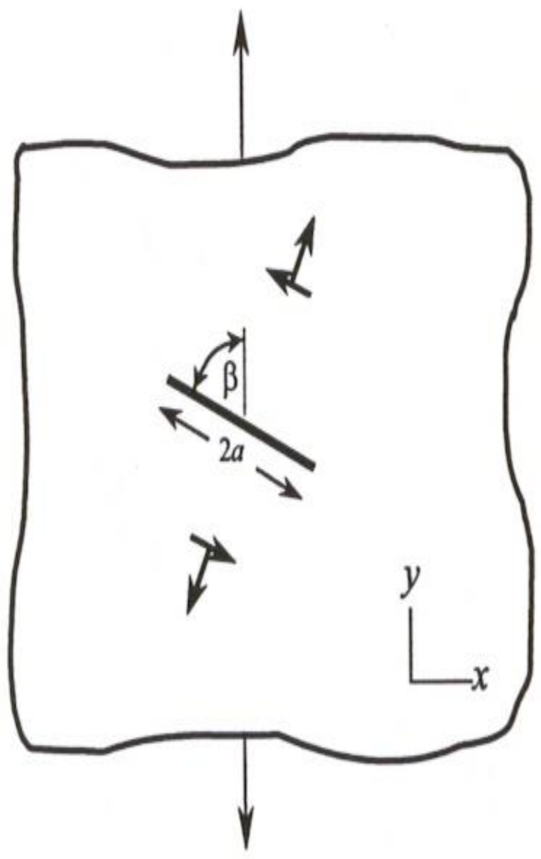




- Mode III is the tearing or anti-plane shear mode.
- It is associated with a pure shear condition, typical of a round notched bar loaded in torsion.

LEFM & APPLICATIONS TO FCG
(LEFM CONCEPTS)

- Combinations of these crack extension modes can also occur.
  - An example of mixed mode I-II crack extension is a crack on an inclined plane.
  - If β = 90, pure Mode I



LEFM & APPLICATIONS TO FCG
(LEFM CONCEPTS)

- **Mode I** crack extension will only be covered because:
  - It most commonly occurs, and
  - Other mode cracks (II and III) in combination with mode I cracks often turn into mode I cracks.

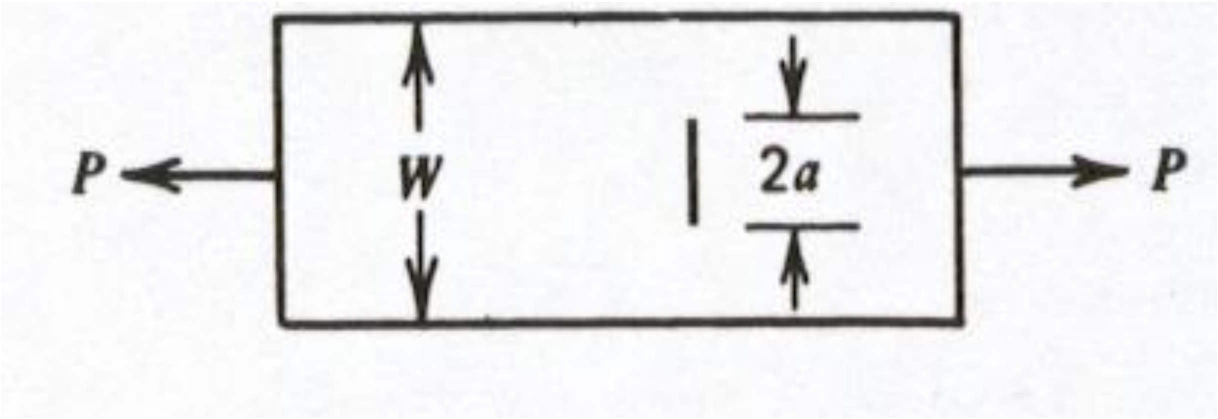
- K used without a mode subscript I, II, or III normally refers to mode I.

LEFM CONCEPTS
Stress Intensity Factor K

- Based on energy balance of a cracked body for ideally brittle behavior (i.e. glass) **Griffith** showed that:

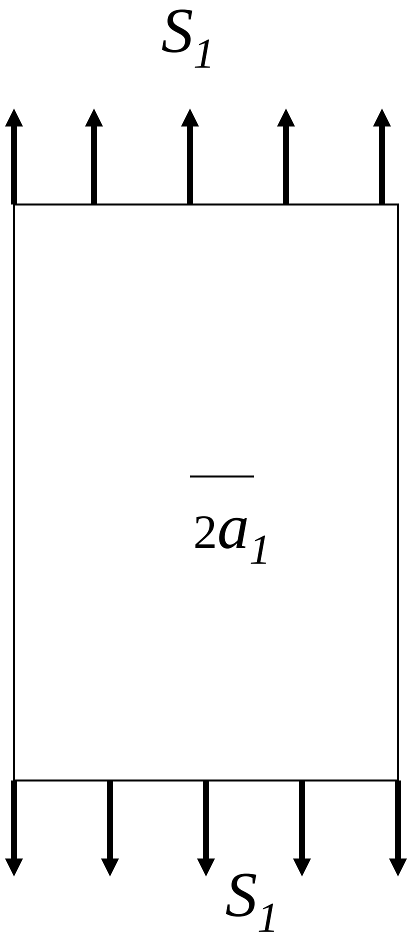
$$S\sqrt{a} = \text{constant} \qquad \text{where } S = P/A$$

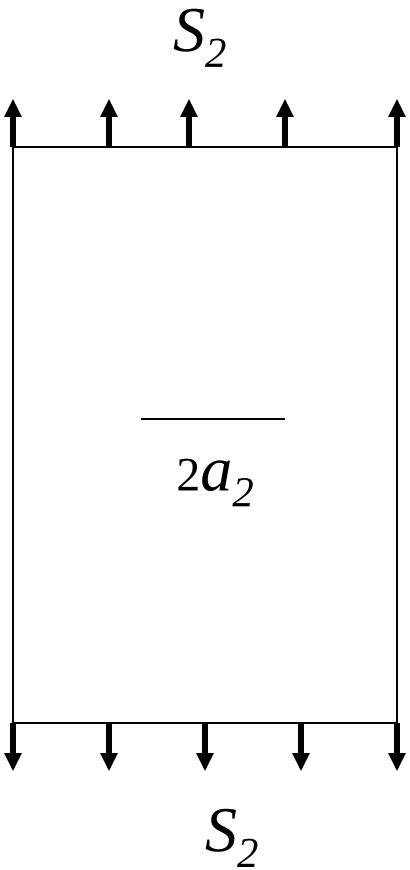
- This product is related to the energy release rate, G.



LEFM CONCEPTS
Stress Intensity Factor K

$$S_1\sqrt{a_1} = S_2\sqrt{a_2}$$





LEFM CONCEPTS
Stress Intensity Factor K

- **Irwin** applied Griffith's theory to metals with small plastic deformation at the crack tip and used the **stress intensity factor K** to quantify the crack tip driving force.

- Using Griffith's energy approach, Irwin showed:

For plane stress:

$$G = \frac{K^2}{E}$$

For plane strain:

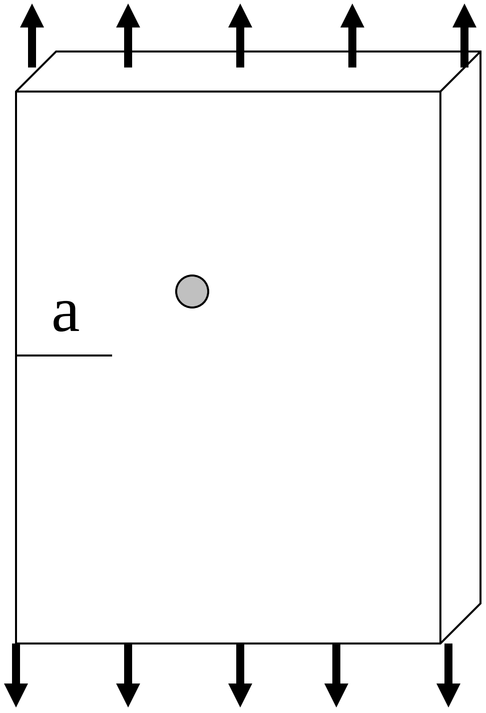
$$G = \frac{K^2}{E}(1-\nu^2)$$

LEFM CONCEPTS
Stress Intensity Factor K

- Consider a through thickness sharp crack in a linear elastic isotropic body subjected to Mode I loading.

- Consider an arbitrary point in the vicinity of the crack tip.

- Stresses at this point can be obtained from linear elasticity theory.

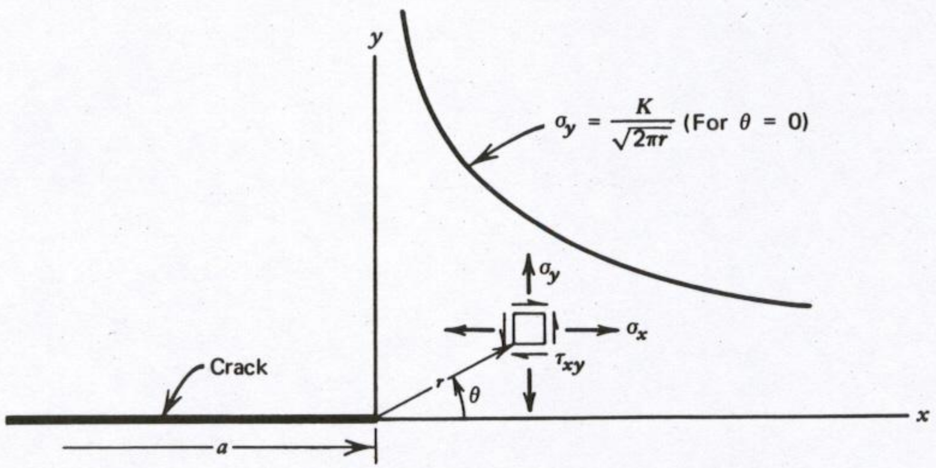


LEFM CONCEPTS
Stress Intensity Factor K

- Consider the **arbitrary point** in the vicinity of the crack tip with coordinates r and θ.

- Using the mathematical theory of linear elasticity and the stress function in complex form, the **stress field** at any point near the crack tip can be described, as given in Fig. 6.2.

- Note that the normal and shear stresses in the z direction are zero for plane stress, while the normal stress in the z direction is not zero for plane strain.



$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin 3\frac{\theta}{2}\right]$$

$$\sigma_x = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin 3\frac{\theta}{2}\right]$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos 3\frac{\theta}{2}$$

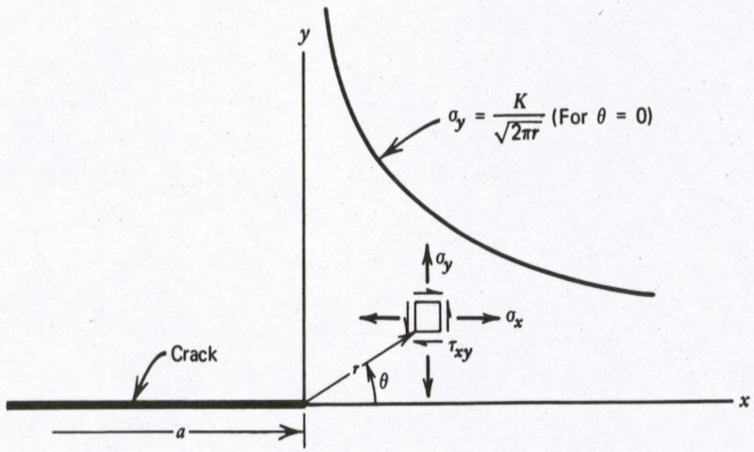
$$\sigma_z = \tau_{xz} = \tau_{yz} = 0 \quad \text{for plane stress}$$

$$\left.\begin{aligned} &\sigma_z = \mu\,[\sigma_x + \sigma_y] \\ &\tau_{xz} = \tau_{yz} = 0 \end{aligned}\right\} \text{for plane strain}$$

Figure 6.2 Elastic stresses near the crack tip (*r/a* << 1).

LEFM CONCEPTS
Stress Intensity Factor K

- Stresses in the vicinity of the crack tip are dependent on r, θ, and K.
- The ***magnitudes*** of these stresses at a given point are entirely dependent on K.
  - K is called a stress field parameter, or stress intensity factor.
  - K is not to be confused with the elastic stress concentration factor K_t.
  - The stress intensity factor, K, is the fundamental parameter of LEFM.
  - K depends on the:
    - Loading mode
    - Crack shape and component, specimen, or structure configuration.



$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin 3\frac{\theta}{2}\right]$$

$$\sigma_x = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin 3\frac{\theta}{2}\right]$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos 3\frac{\theta}{2}$$

$\sigma_z = \tau_{xz} = \tau_{yz} = 0$ for plane stress

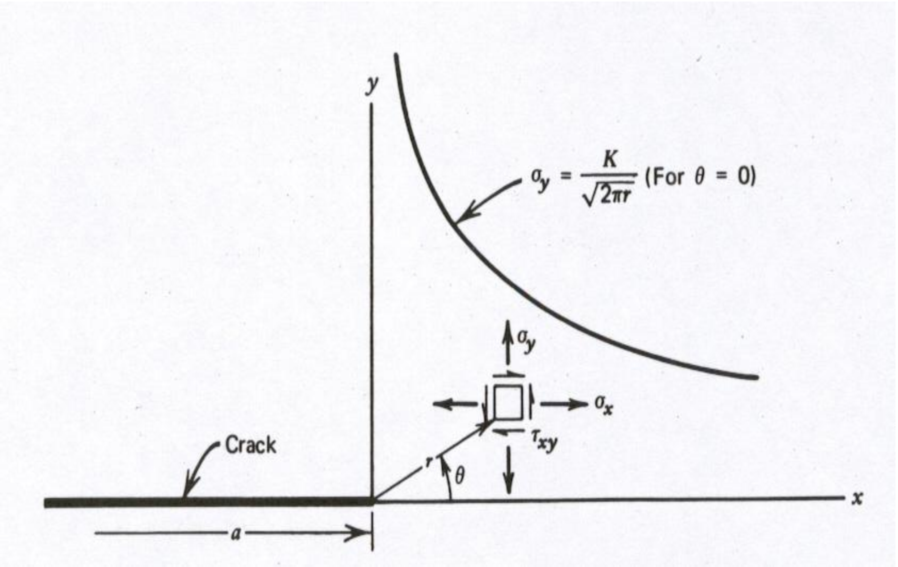
$\left.\begin{array}{l}\sigma_z = \mu\,[\sigma_x + \sigma_y] \\ \tau_{xz} = \tau_{yz} = 0\end{array}\right\}$ for plane strain

Figure 6.2 Elastic stresses near the crack tip ($r/a << 1$).

LEFM CONCEPTS
Stress Intensity Factor K

The elastic stress distribution in the
y direction for **θ = 0** is shown.

- As r approaches zero, the stress at the crack tip approaches infinity, thus a singularity exists at $r = 0$.
- Since infinite stresses cannot exist, the elastic solution must be modified to account for some crack tip plasticity.
- If, the plastic zone radius, r_y, at the crack tip is small relative to local geometry, little or no modification to the stress intensity factor, K, is needed.
- Thus an important restriction to the use of LEFM is that the plastic zone size at the crack tip must be small.
- Crack tip plasticity and LEFM limitations are discussed in section 6.2.



LEFM CONCEPTS
Stress Intensity Factor K

- Values of K for various loadings and configurations can be calculated using:
  - The theory of elasticity involving
    - **Analytical** calculations
    - **Computational** calculations (i.e. FEA)
  - **Experimental** methods (i.e. photo-elasticity)

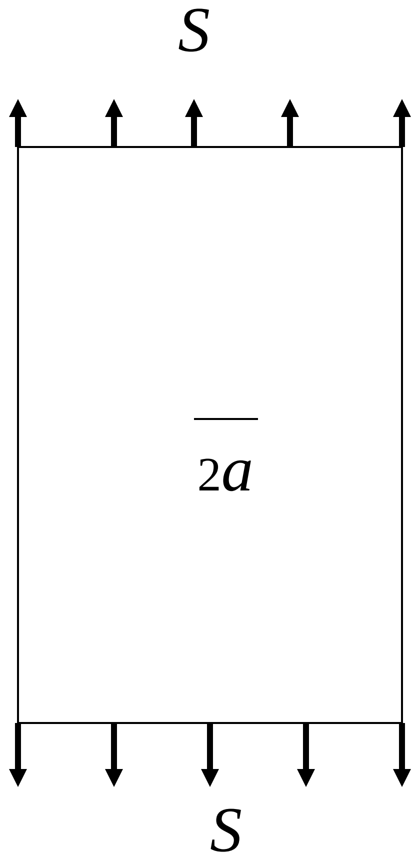
- When the crack is small compared to other dimensions of the component, the crack is viewed as being contained within an infinite body. The reference value of K is for a 2-D center crack of length 2a in an infinite sheet subjected to a uniform tensile stress S.

LEFM CONCEPTS
Stress Intensity Factor K

- For the **infinite sheet**, K is:

$$K = S\sqrt{\pi a} \approx 1.77\, S\sqrt{a}$$

  - Units of K are MPa√m and ksi√in
    1 MPa√m = 0.91 ksi√in
    1 ksi√in ≈ 1.1 MPa√m)
  - S is nominal stress, assuming the crack did not exist.
  - For central cracks, the crack length is 2a (or 2c) and for edge cracks the crack length used is just a (or c).



LEFM CONCEPTS
Stress Intensity Factor K

- For other crack geometries, configurations, and 1oadings:

$$K = S\sqrt{\pi a}\,\alpha \quad \text{or} \quad S\sqrt{\pi a}\, f\left(\frac{a}{w}\right) \quad \text{or} \quad S\sqrt{a}\,Y$$

Where:

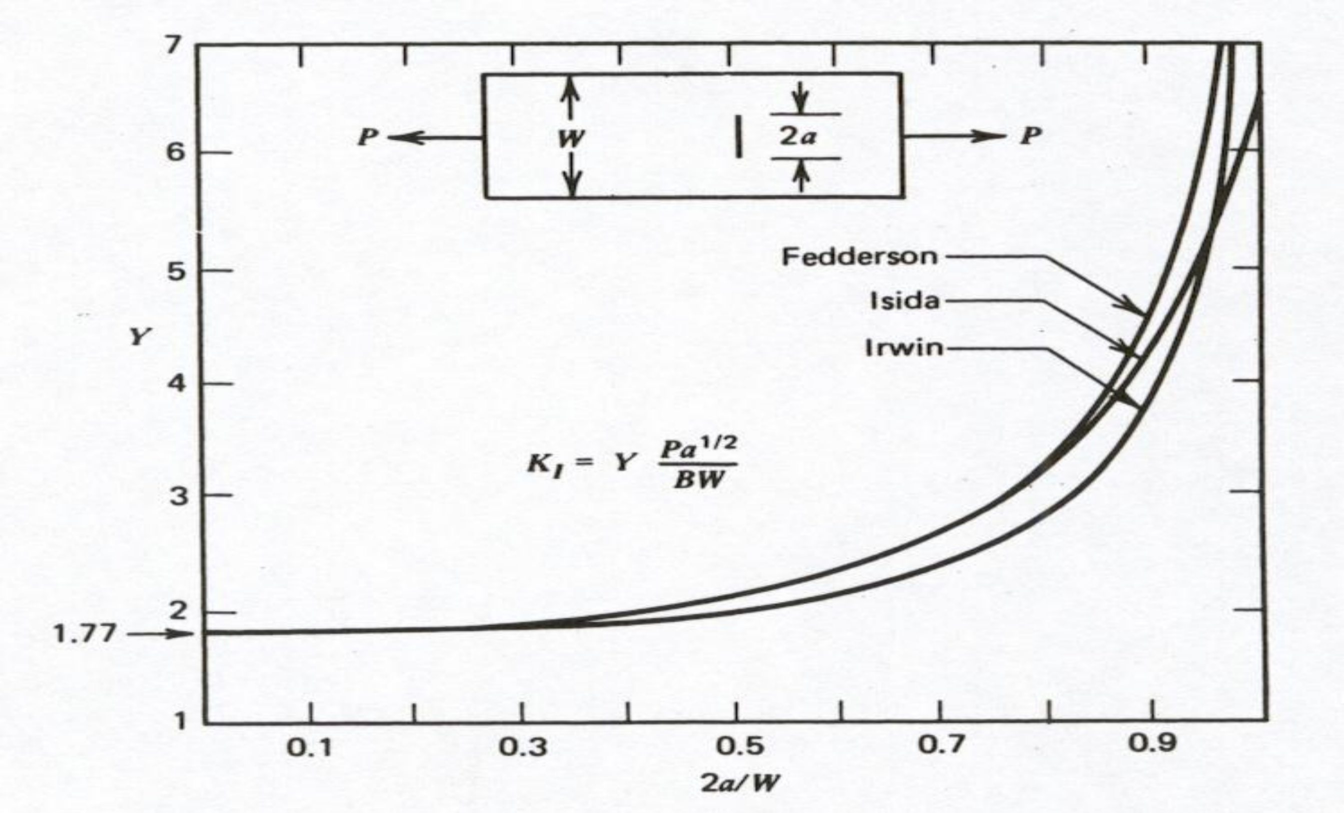
α, $f(a/w)$, and Y are dimensionless geometry parameters,

w is a width dimension.

LEFM CONCEPTS
Stress Intensity Factor K

- Opening mode I stress intensity expressions for several common configurations of thickness B are given in the form of:
  - Dimensionless curves in Fig. 6.3.
  - Mathematical expressions in Table 6.1.

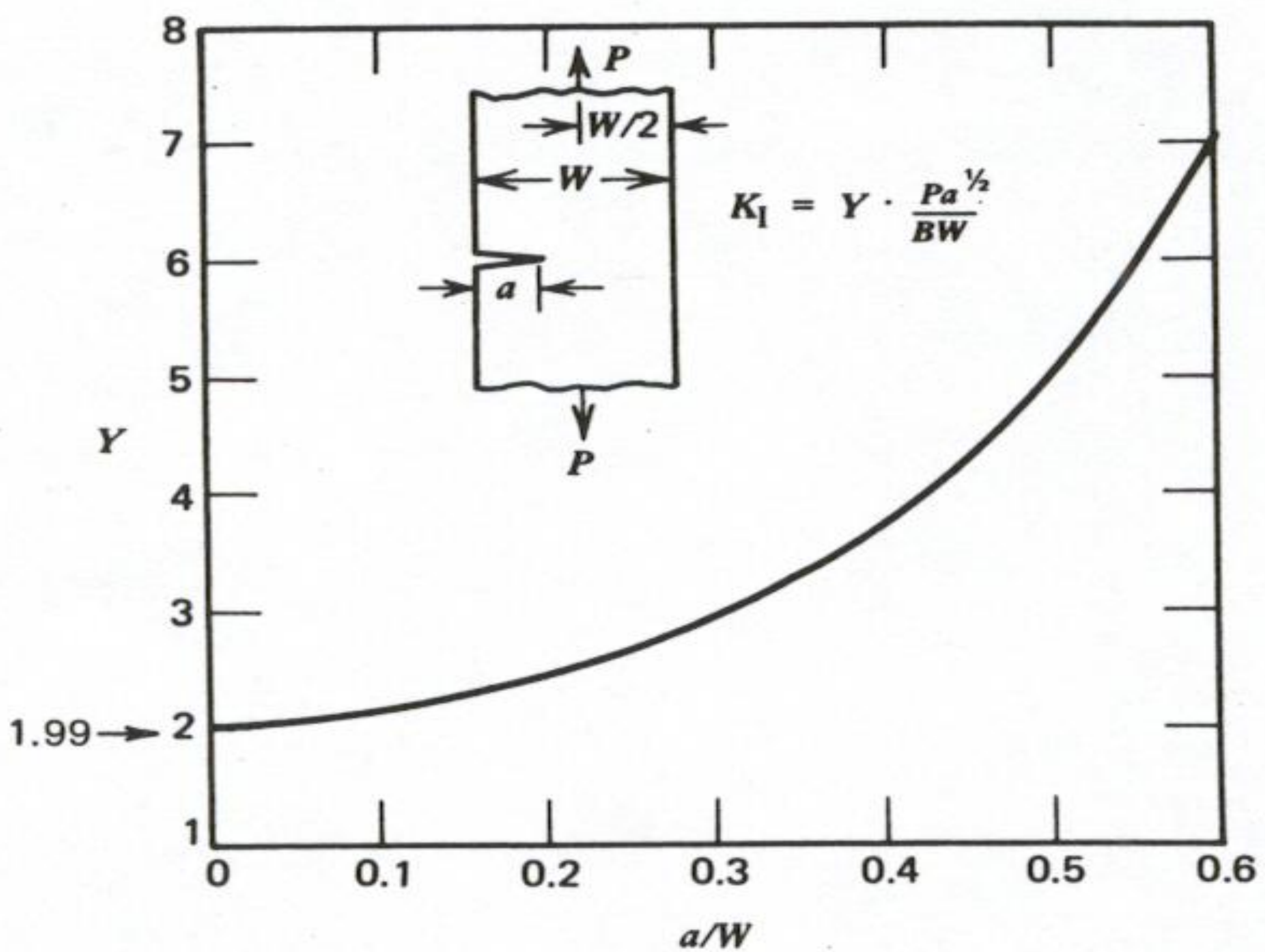
Stress Intensity Factor K
Center Cracked Plate in Tension



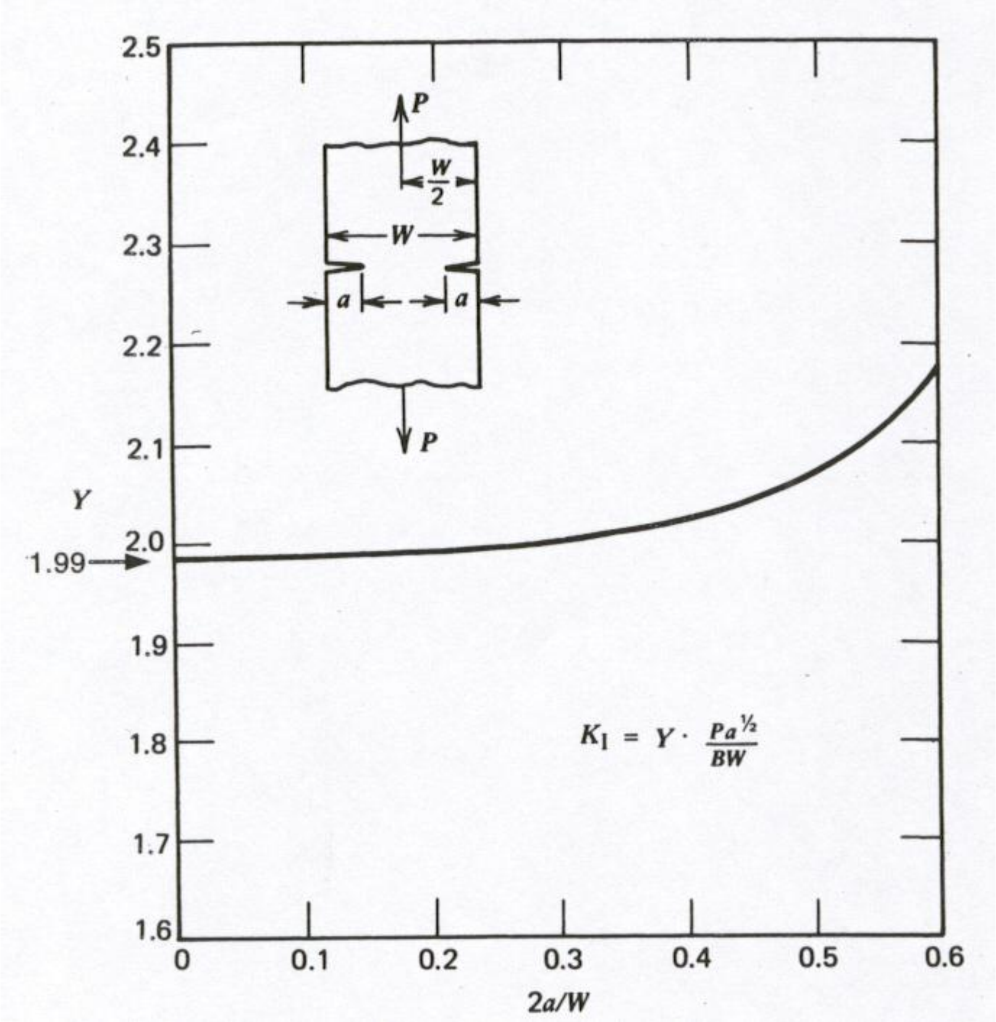
- As a/w→0, K approaches an infinite body solution.
- The term $\sqrt{\sec(\pi a / w)}$ shown for a center cracked plate in tension is used for finite width solutions.

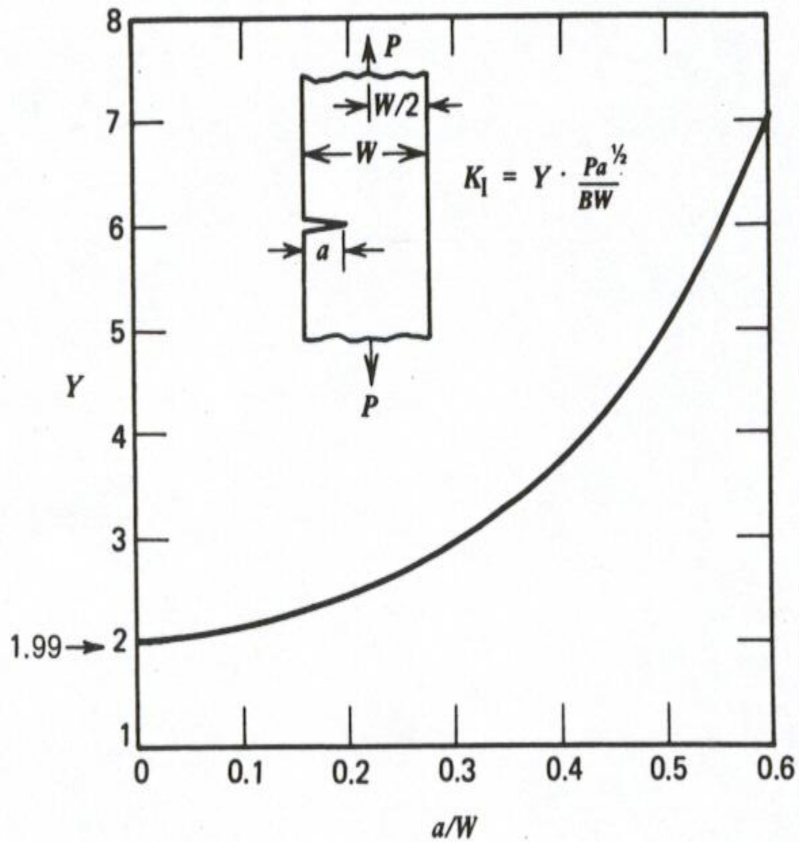
Stress Intensity Factor K

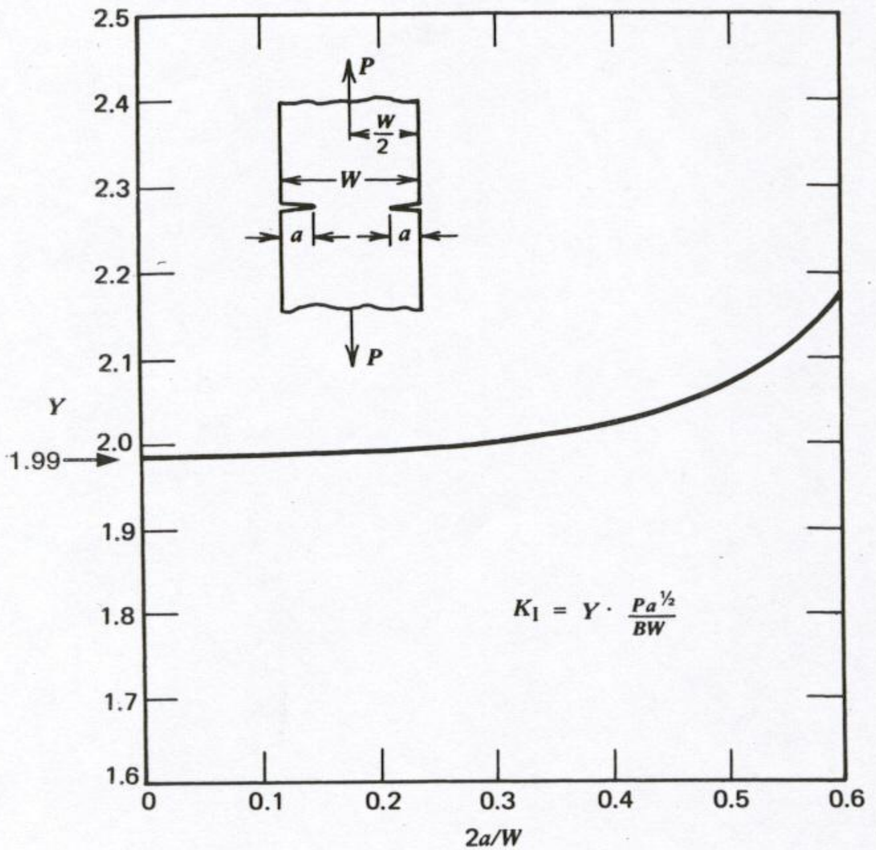
Single Edge Cracked Plate in Tension



Stress Intensity Factor K
Double Edge Cracked Plate in Tension







- Note that Y, and therefore, K increase much faster with a/w for the single edge crack plate, as compared with the double edge crack plate.
- For the single or double edge crack in a semi-infinite plate (a/w $\rightarrow$ 0)

$$K = 1.12S\sqrt{\pi a} \approx 2S\sqrt{a}$$

where 1.12 is the free edge correction.

Stress Intensity Factor K
Single Edge Cracked Plate in **Bending**

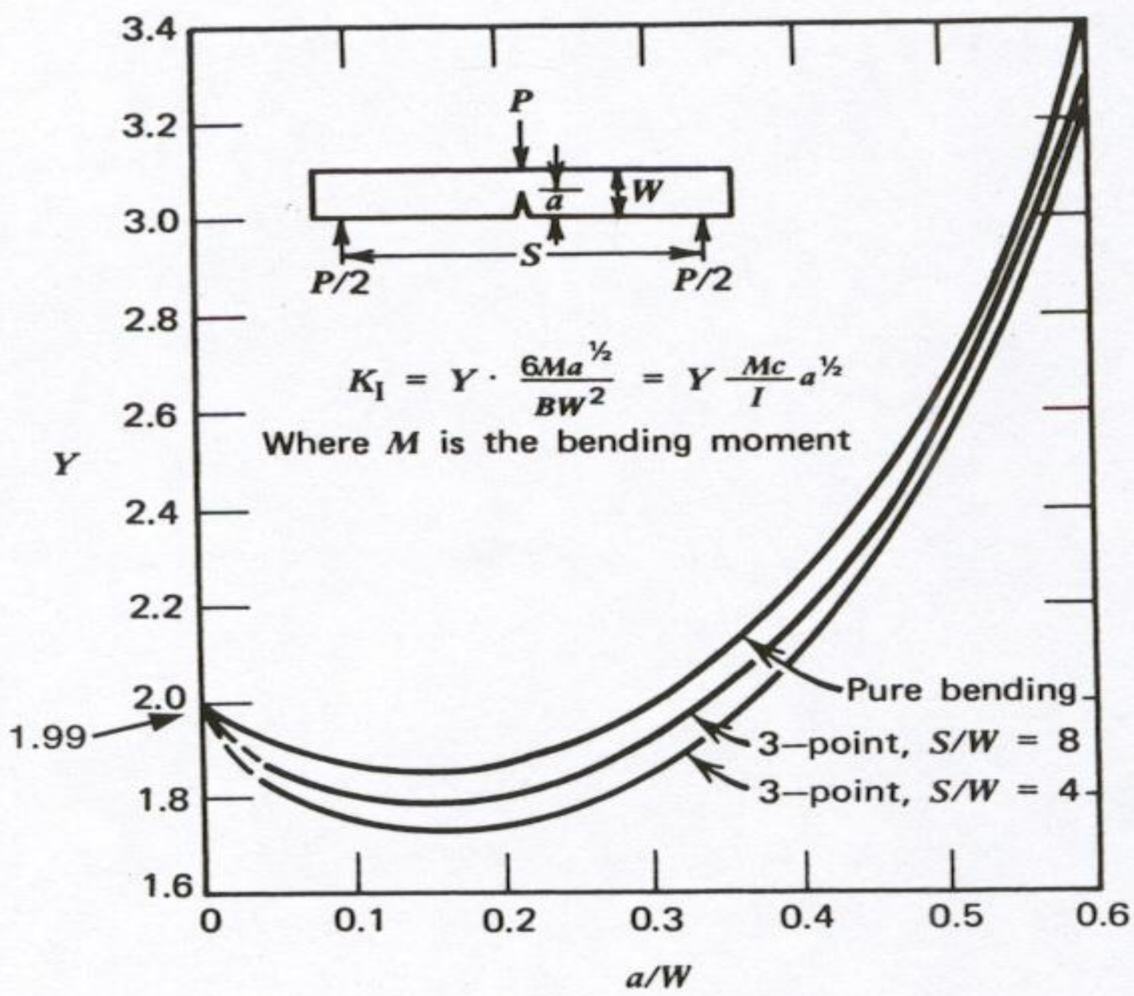


TABLE 6.1 *K* Expressions for Fig. 6.3

a. Center-cracked plate in tension (for $0 < 2a/w < 0.95$)

Fedderson: $K_I = S\sqrt{\pi a}\left[\sec\left(\frac{\pi a}{w}\right)\right]^{1/2}$

Irwin: $K_I = S\sqrt{\pi a}\left[\frac{w}{\pi a}\tan\left(\frac{\pi a}{w}\right)\right]^{1/2}$

where $S = P/Bw$

b. Single-edge crack in tension (for $0 < a/w < 0.95$)

$$K_I = S\sqrt{a}\left[1.99 - 0.41\left(\frac{a}{w}\right) + 18.7\left(\frac{a}{w}\right)^2 - 38.48\left(\frac{a}{w}\right)^3 + 53.85\left(\frac{a}{w}\right)^4\right]$$

where $1.12\sqrt{\pi} = 1.99$ $\quad S = P/Bw$

c. Double-edge crack in tension (for $0 < 2a/w < 0.95$)

$$K_I = S\sqrt{a}\left[1.98 + 0.36\left(\frac{2a}{w}\right) - 2.12\left(\frac{2a}{w}\right)^2 + 3.42\left(\frac{2a}{w}\right)^3\right]$$

where $S = P/Bw$

d. Single-edge crack in pure bending of a beam (for $0 < a/w < 1$)

$$K_I = S\sqrt{a}\left[1.99 - 2.47\left(\frac{a}{w}\right) + 12.97\left(\frac{a}{w}\right)^2 - 23.17\left(\frac{a}{w}\right)^3 + 24.8\left(\frac{a}{w}\right)^4\right]$$

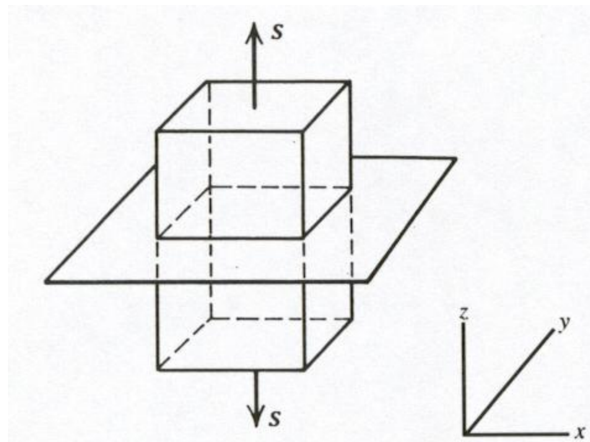
where $S = Mc/I = 6M/Bw^2$

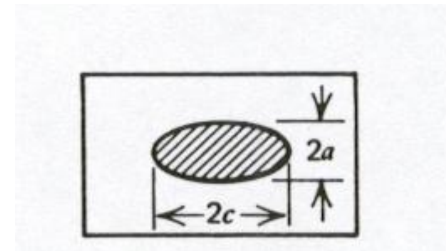
LEFM CONCEPTS
Stress Intensity Factor K

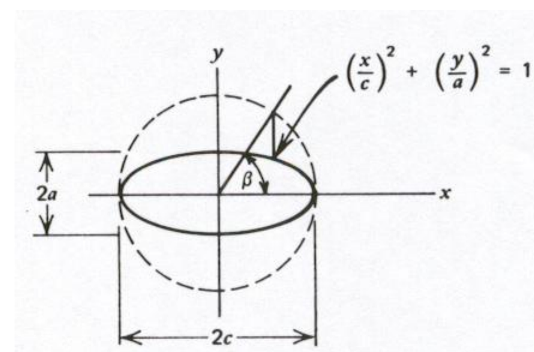
- Additional stress intensity factor expressions for all three modes, K_I, K_{II}, and K_{III} can be found in handbooks of stress intensity factors.
  - H. Tada, P. C. Paris, and G. R. Irwin, *The Stress Analysis of Cracks Hand-book*, 2nd edition, Paris Productions Inc., St. Louis, Mo, 1985.
  - G. C. Sih, *Handbook of Stress Intensity Factors*, Institute of Fracture and Solid Mechanics, Lehigh University, Bethleham, PA, 1973.
  - D.P. Rooke and D. J. Cartwright, *Compendium of Stress Intensity Factors*, Her Majesty's Stationery Office, London, 1976.
- *t* and *B* are often used interchangeably for thickness.

LEFM CONCEPTS
Stress Intensity Factor K

- **Elliptical-shaped cracks** approximate many cracks found in engineering components & structures.







- The general reference specimen is the embedded elliptical crack in an infinite body subjected to uniform tension S perpendicular to the crack plane:

$$K = \frac{S\sqrt{\pi a}}{\Phi}\left[\sin^2\beta + \left(\frac{a}{c}\right)^2\cos^2\beta\right]^{1/4}$$

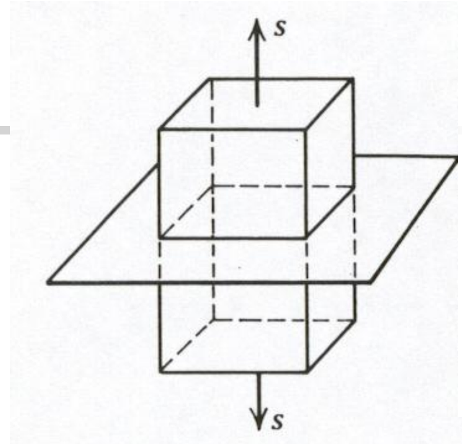
a/c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Φ	1.0	1.016	1.051	1.097	1.151	1.211	1.277	1.345	1.418	1.493	1.571

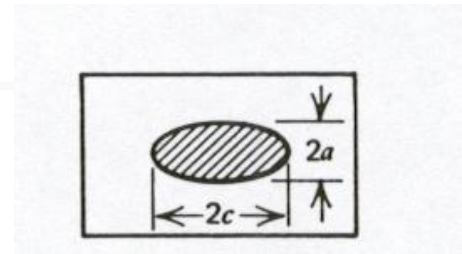
- β is the angle shown, $2a$ is the minor axis, and $2c$ is the major axis.

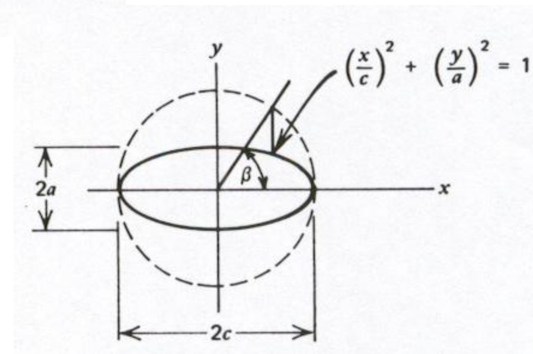
- Values of Φ are given in Fig. 6.4*h*. K varies along the elliptical crack tip.

$$\Phi = \int_0^{\pi/2} \left[1 - \left(1 - \frac{a^2}{c^2} \right) \sin^2 \phi \right]^{1/2} d\phi$$

- K varies along the elliptical crack tip.
  - The maximum value of K exists at the minor axis and the minimum is at the major axis (β = 90).
  - Therefore the embedded elliptical crack subjected to uniform tension tends to grow to a circle



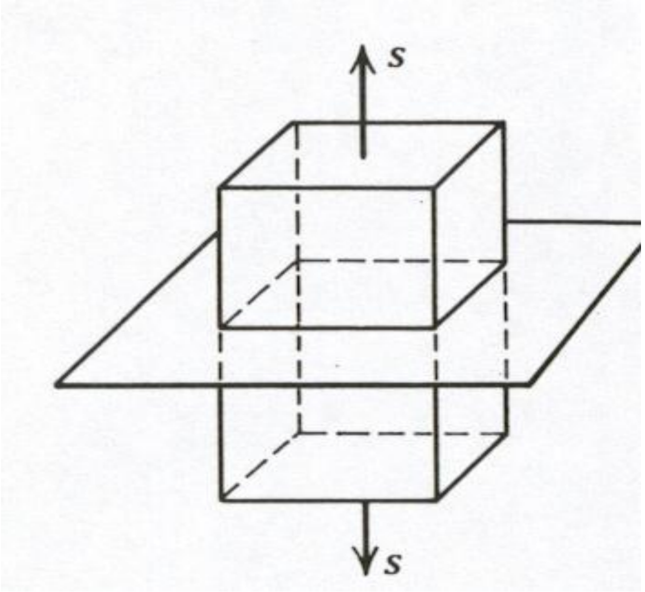




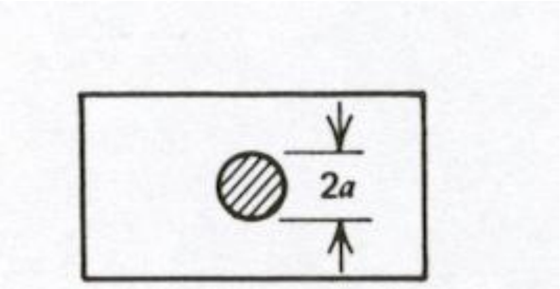
a/c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Φ	1.0	1.016	1.051	1.097	1.151	1.211	1.277	1.345	1.418	1.493	1.571

Stress Intensity Factor K

- For the **circular embedded crack** in an infinite solid, with a/c = 1 and $\Phi = \pi/2$:



$$K = S\sqrt{\pi a}\left(\frac{2}{\pi}\right) = 2S\sqrt{\frac{a}{\pi}} \approx 1.13S\sqrt{a}$$

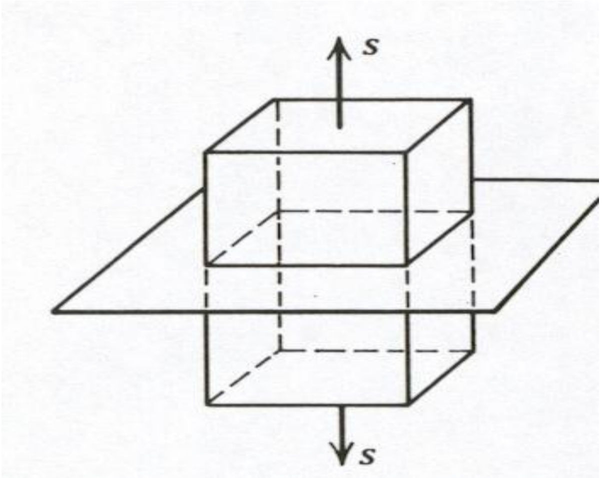


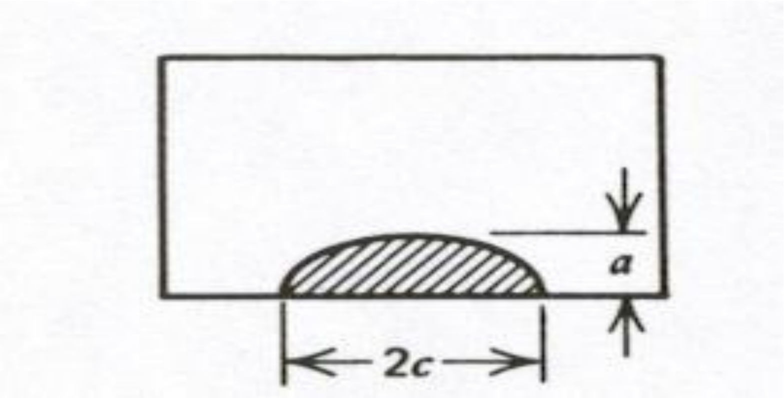
LEFM CONCEPTS
Stress Intensity Factor K

- A general expression for mode I **semi-elliptical surface crack** in a finite thickness plate is

$$K = \frac{S\sqrt{\pi a}}{\Phi} M_f M_b \left[\sin^2 \beta + \left(\frac{a}{c} \right)^2 \cos^2 \beta \right]^{1/4}$$

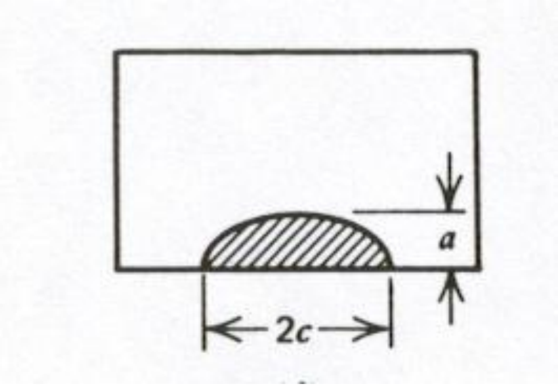
where M_f is a front face correction factor and M_b is a back face correction factor. M_f and M_b are functions of β.





LEFM CONCEPTS
Stress Intensity Factor K

- For a **semi-elliptical surface crack** in a plate of thickness *t*, K **at the deepest point** is:

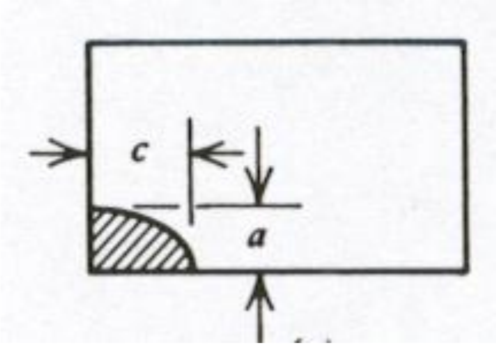


$$K \cong \frac{1.12\, S \sqrt{\pi a}}{\Phi} \sqrt{\sec(\pi a / 2t)}$$

  – Where 1.12 is the free edge correction

  – $\sqrt{\sec(\pi a/2t)}$ is the finite thickness correction factor.

- For the **quarter-circular corner crack** (*a/c* = 1) in an infinite solid with two free edges, K is:



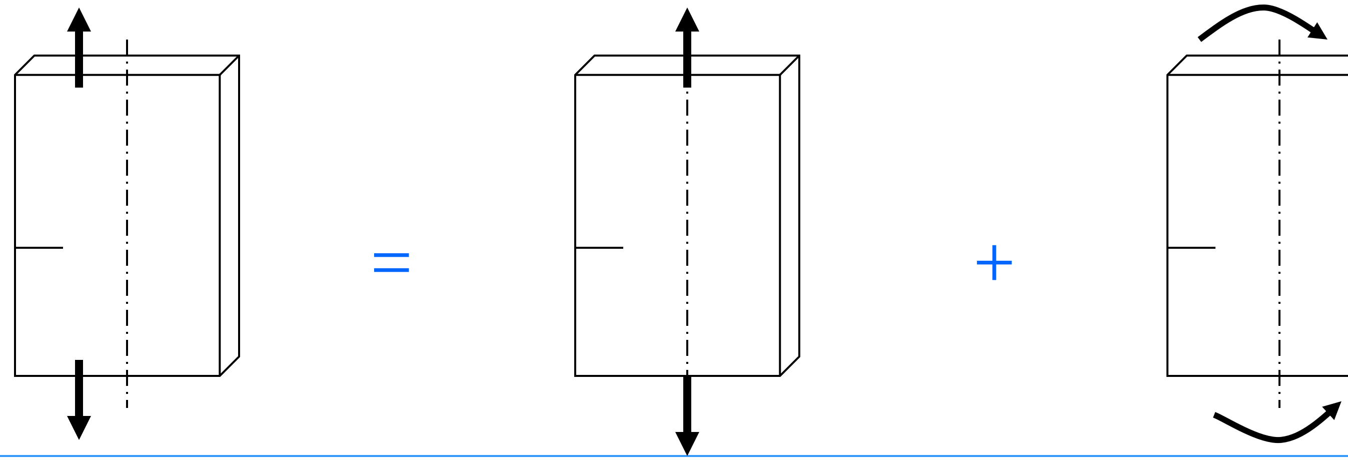
$$K \cong \frac{(1.12)^2\, S \sqrt{\pi a}}{\Phi} \approx 1.41\, S \sqrt{a}$$

LEFM CONCEPTS
Stress Intensity Factor K

- **Superposition** for Combined Mode I Loading.
  - The principle of superposition can be used to determine stress intensity factor solutions for combined loadings.
  - For example, for an eccentrically loaded member that experiences both axial and a bending loading, the resultant stress intensity solution is

$$K_I \neq (S_{axial} + S_{bending})\sqrt{\pi a}\,\alpha \qquad K_I = K_{I(axial)} + K_{I(bending)}$$

- Noted that:
  - as the geometry factor α, for axial and bending are different.
  - Also, K values of different modes, i.e. I, II, and III, cannot be added together.



EXAMPLE PROBLEM

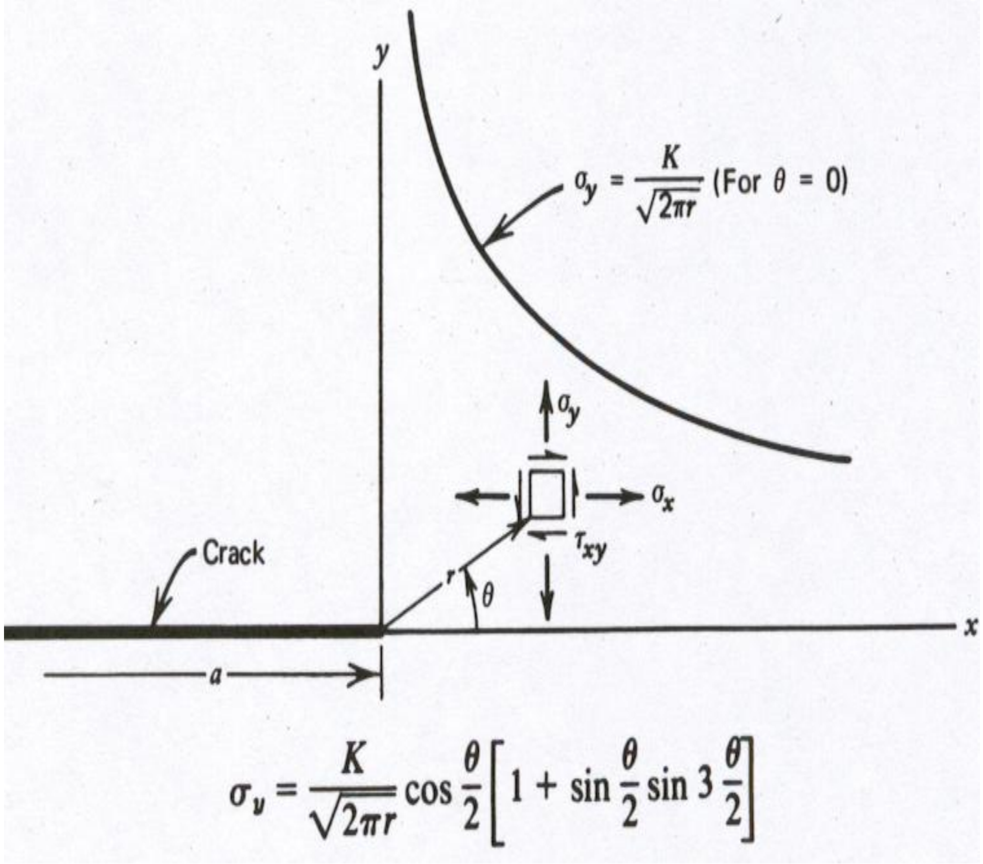
CRACK TIP PLASTICITY

CRACK TIP PLASTICITY

- A distance r_y ahead of the crack tip ($\theta = 0$) can be determined that identifies the plastic zone boundary:
  - By using the stress field equation for σ_y and
  - Substituting the yield strength, S_y, for σ_y, and r_y for r.

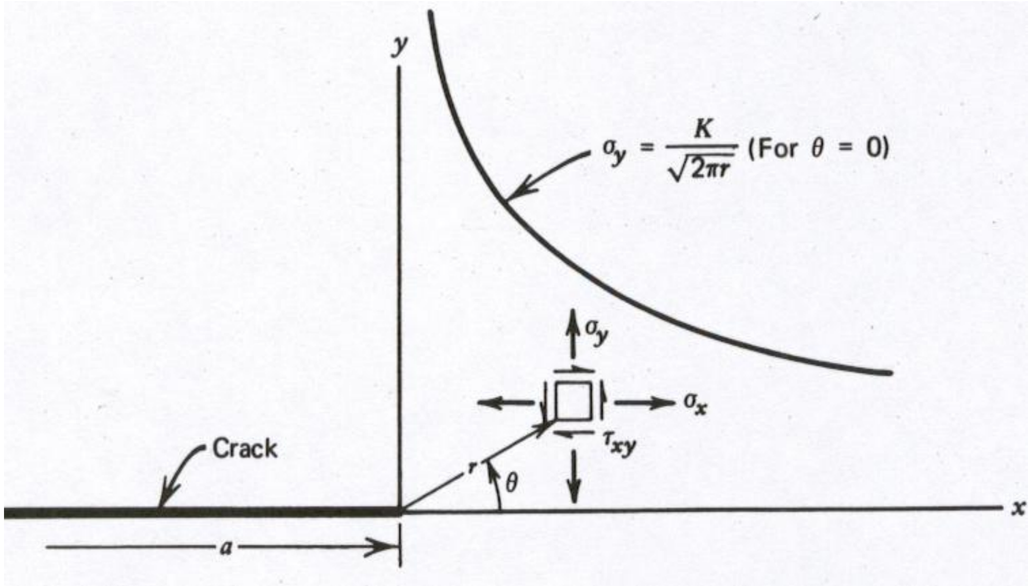
$$S_y = K / (\sqrt{2\pi r_y})$$

$$r_y = 1/2\pi\ (K/S_y)^2$$



CRACK TIP PLASTICITY

- Plastic relaxation and redistribution of the stress field occurs in the plastic zone.
  - The stress distribution for σ_y shown must shift to the right to accommodate the plastic deformation and satisfy equilibrium conditions.
  - As a result, the actual plastic zone size is approximately *twice* the calculated value.

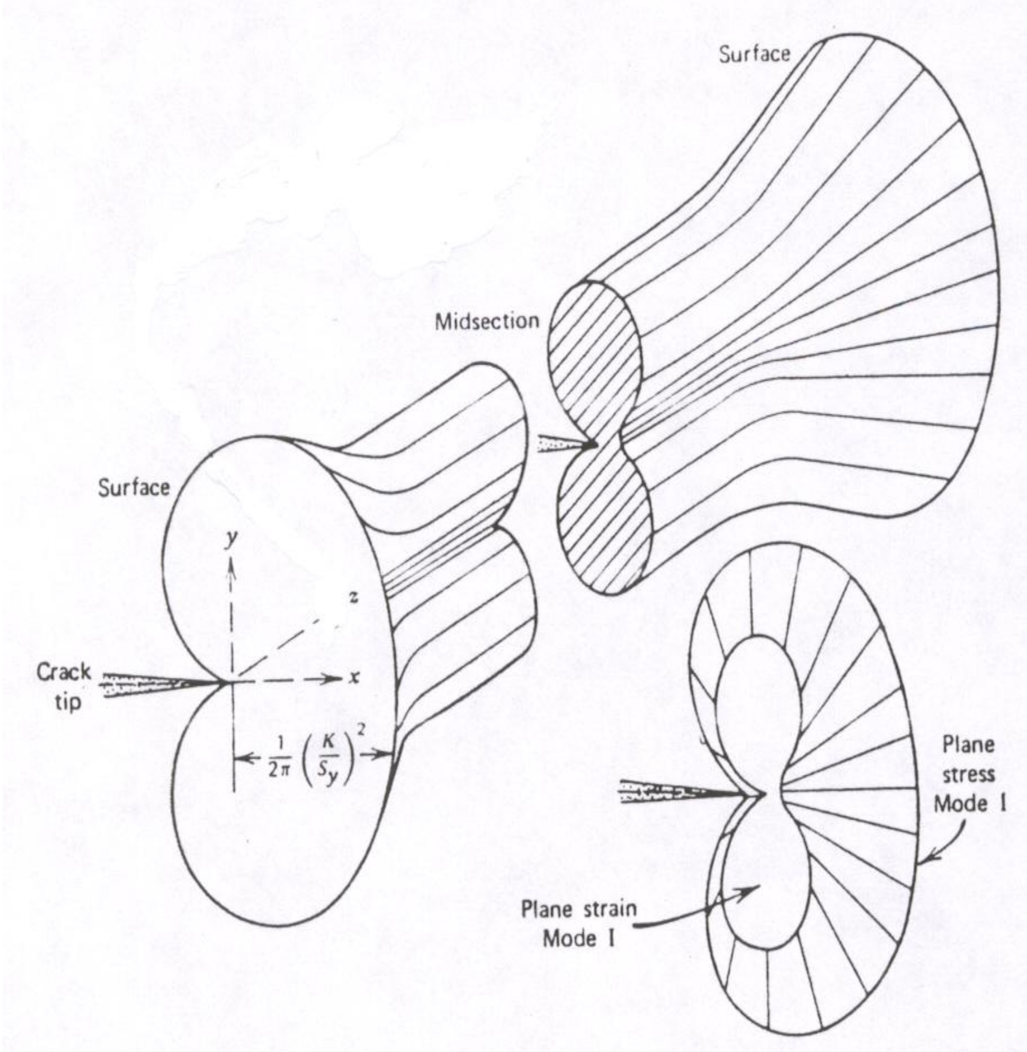


CRACK TIP PLASTICITY

- Using the stress field equations in Fig. 6.2 and the von Mises or maximum shear stress yield criteria, **plastic zone shape** can be determined.

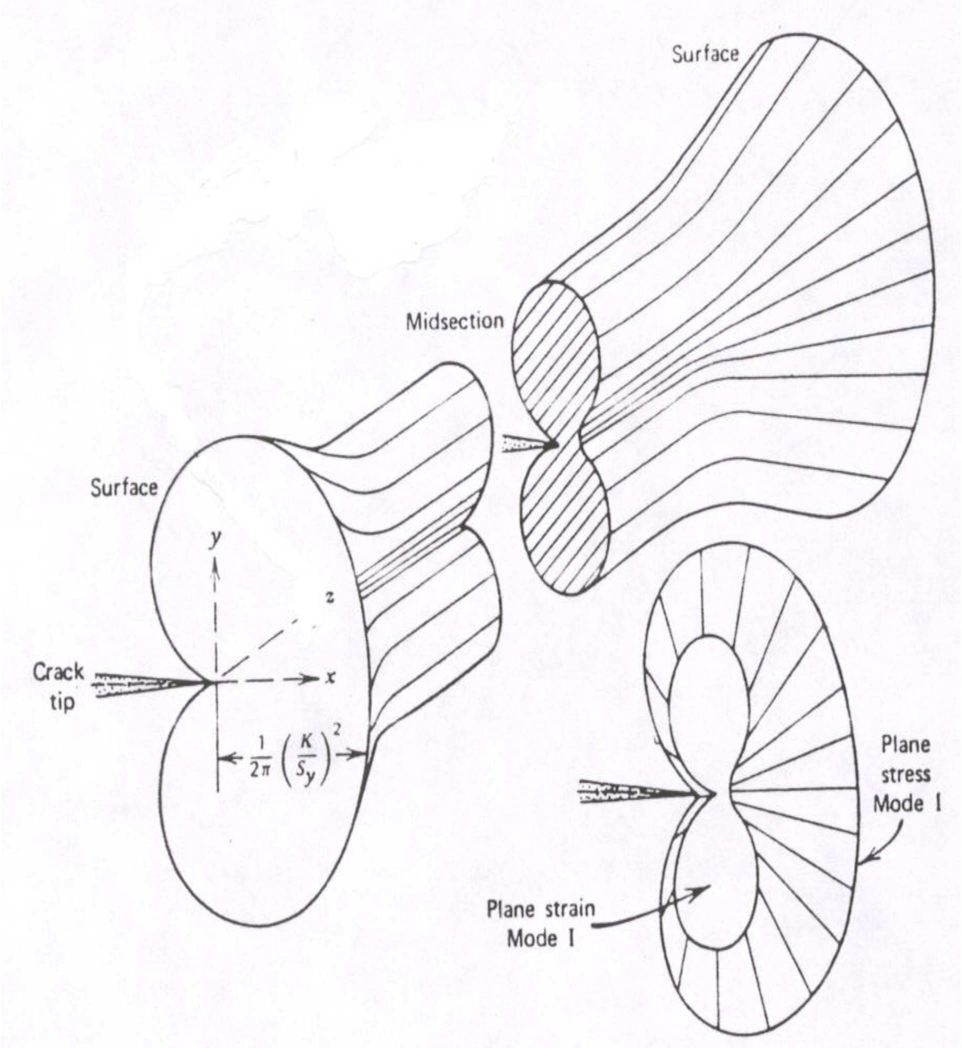
- The resultant plastic zone shape for mode I using the von Mises criterion is shown.

- For plane stress conditions (where $\sigma_z = 0$) a much larger plastic zone exists compared to plane strain condition, where the tensile stress component, σ_z, restricts plastic flow.



CRACK TIP PLASTICITY

- Plane strain plastic zone size is usually taken as one-third the plane stress value.
- For a through crack in a thick plate,
  - The free surfaces with zero normal and shear stresses are in a plane stress condition.
  - The interior region of the plate near the crack tip is closer to plane strain conditions as a result of elastic constraint.
  - Thus the plastic zone size along the crack tip varies similarly to that shown schematically.



CRACK TIP PLASTICITY

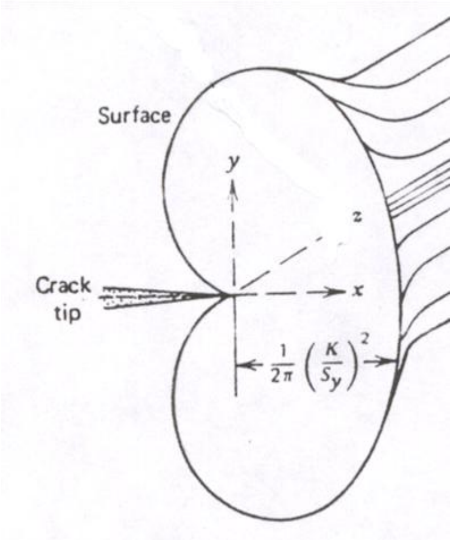
- Under **monotonic loading**, the plastic zone size, $2r_y$, at the crack tip, in the plane of the crack, is:

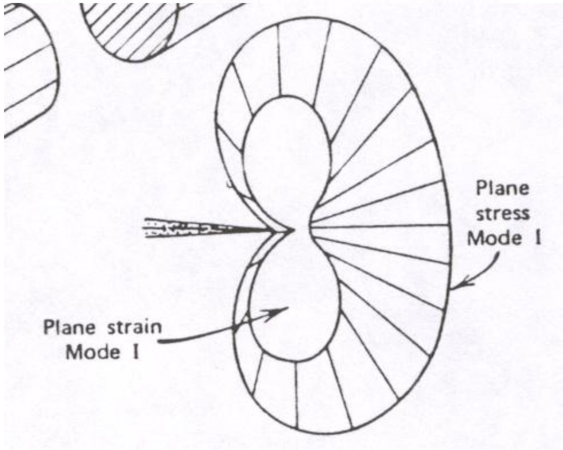
$$2r_y = 2\left[\frac{1}{2\pi}\left(\frac{K}{S_y}\right)^2\right] = \frac{1}{\pi}\left(\frac{K}{S_y}\right)^2 \qquad \text{For plane stress}$$

$$2r_y \cong \frac{1}{3\pi}\left(\frac{K}{S_y}\right)^2 \qquad \text{For plane strain}$$

- The criteria for deciding between plane stress and plane strain are discussed in Section 6.3.

- The value r_y is often called the plastic zone radius.



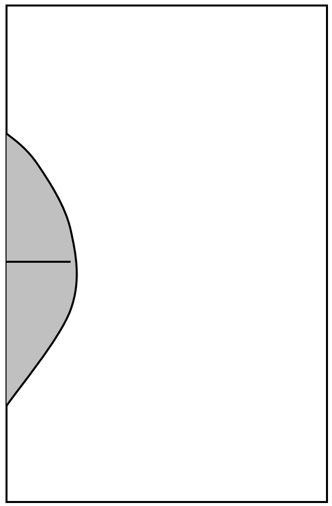


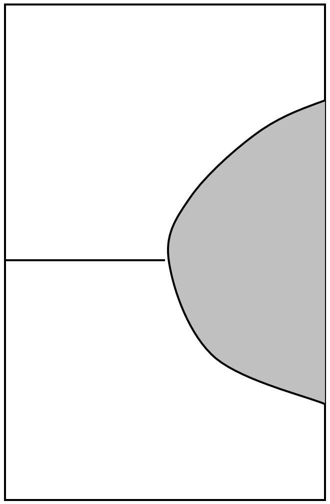
CRACK TIP PLASTICITY

- An important restriction to the use of LEFM is that plastic zone size at the crack tip must be small relative to the crack length as well as the geometrical dimensions of the specimen or part.
  - A definite limiting condition for LEFM is that net (nominal) stresses in the crack plane must be less than $0.8S_y$ (80% of yield strength).
  - Under monotonic loading $r_y \leq (1/8)a$.
  - Other restrictions include $r_y \leq 1/8$ of t and (w-a) where t is the thickness and (w-a) is the uncracked ligament along the plane of the crack.

- Otherwise, a plasticity correction is required for the stress intensity factor, K, or elastic-plastic fracture mechanics may be needed (Section 6.9).

CRACK TIP PLASTICITY





– Under monotonic loading $r_y \leq (1/8)a$.

– Other restrictions include $r_y \leq 1/8$ of t and (w-a) where t is the thickness and (w-a) is the uncracked ligament along the plane of the crack.

FRACTURE TOUGHNESS

FRACTURE TOUGHNESS - K_c, K_{Ic}

- **Critical values of K** refer to the condition when a crack extends in a rapid (unstable) manner.

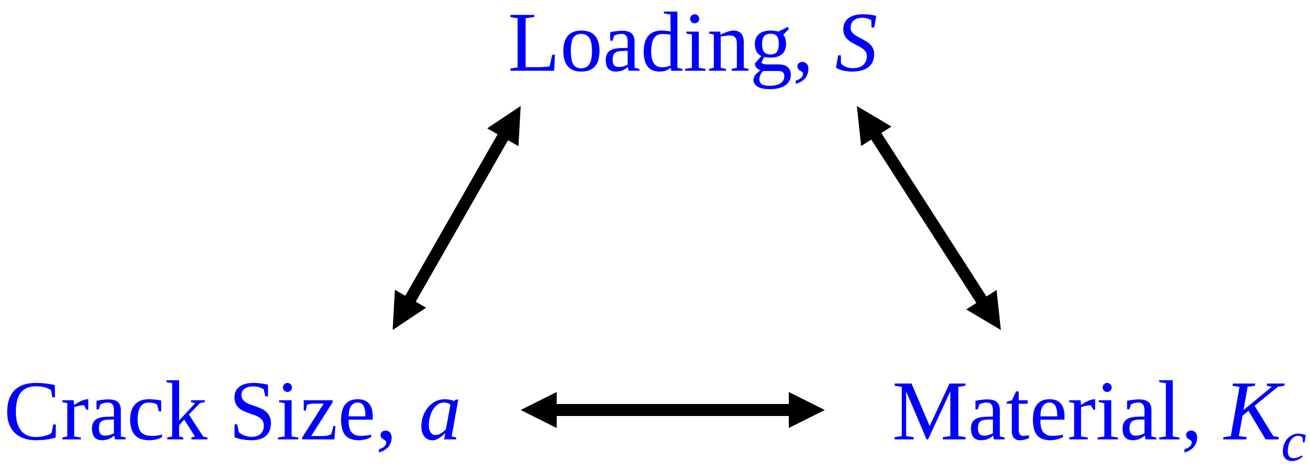
$$K_c = S_c \sqrt{\pi a_c}\, f\left(\frac{a_c}{w}\right)$$

S_c is the applied nominal stress at crack instability..
a_c is the crack length at instability.
K_c is called **fracture toughness** and depends on the material, temperature, strain rate, environment, and thickness.

- This equation provides a quantitative design parameter to prevent fracture involving ***applied stress, material selection, and crack size***.

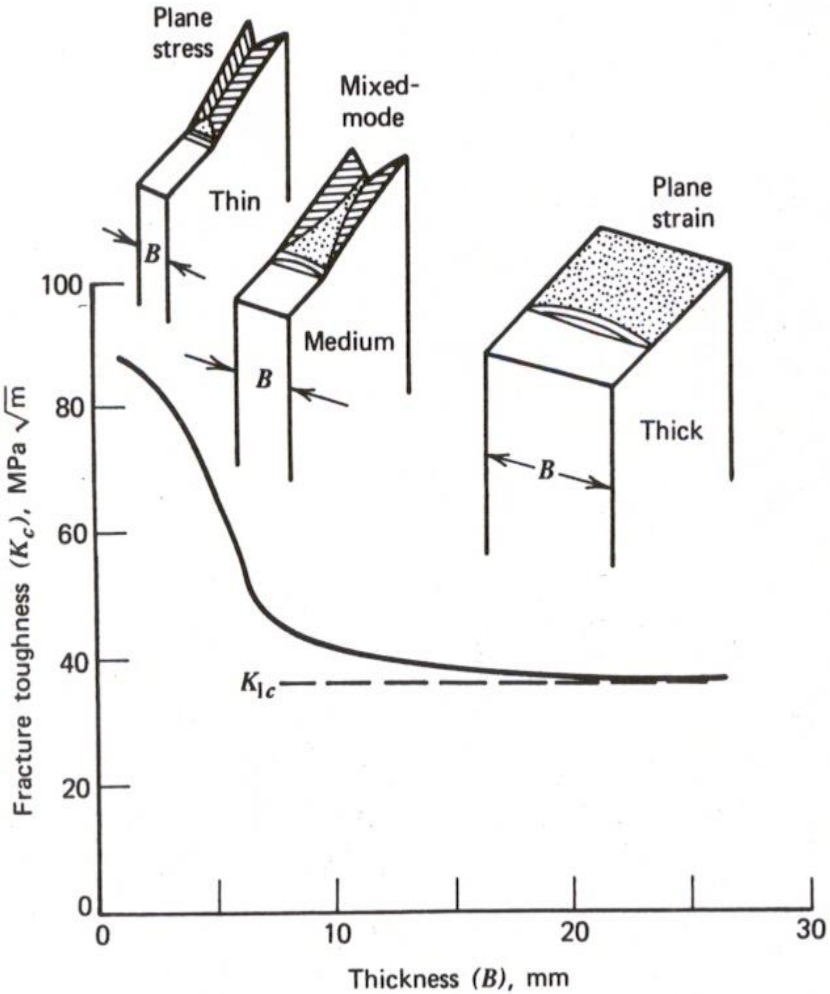
DESIGN BASED ON FRACTURE MECHANICS

$$K_c = S_c \sqrt{\pi a_c}\, f\left(\frac{a_c}{w}\right)$$



FRACTURE TOUGHNESS - K_c, K_{Ic}

- The general relationship between fracture toughness, K_c, and thickness is shown.
  - Thin parts have a high value of K_c accompanied by appreciable “shear lips" or slant fracture.
  - As the thickness is increased, the percentage of "shear lips" or slant fracture decreases, as does K_c. This type of fracture appearance is called mixed-mode implying both slant and flat fracture.
  - For thick parts, the entire fracture surface is flat and K_c approaches an asymptotic minimum value, called the ***"*plane strain fracture toughness" K_{Ic}.**
  - Plastic zone sizes at fracture are much larger in thin parts as compared to thick parts.



FRACTURE TOUGHNESS - K_c, K_{Ic}

- **Plane strain fracture toughness K_{Ic}** is considered a true material property because it is independent of thickness.

- In order for a plane strain fracture toughness value to be considered valid, it is required that:

$$a \;\; and \;\; t \geq 2.5\left(\frac{K_{Ic}}{S_y}\right)^2$$

- Approximate thickness required for steels and aluminums to obtain valid K_{Ic} values are given in Table 6.2.

- Low strength, ductile materials are subject to plane strain fracture only if they are very thick.

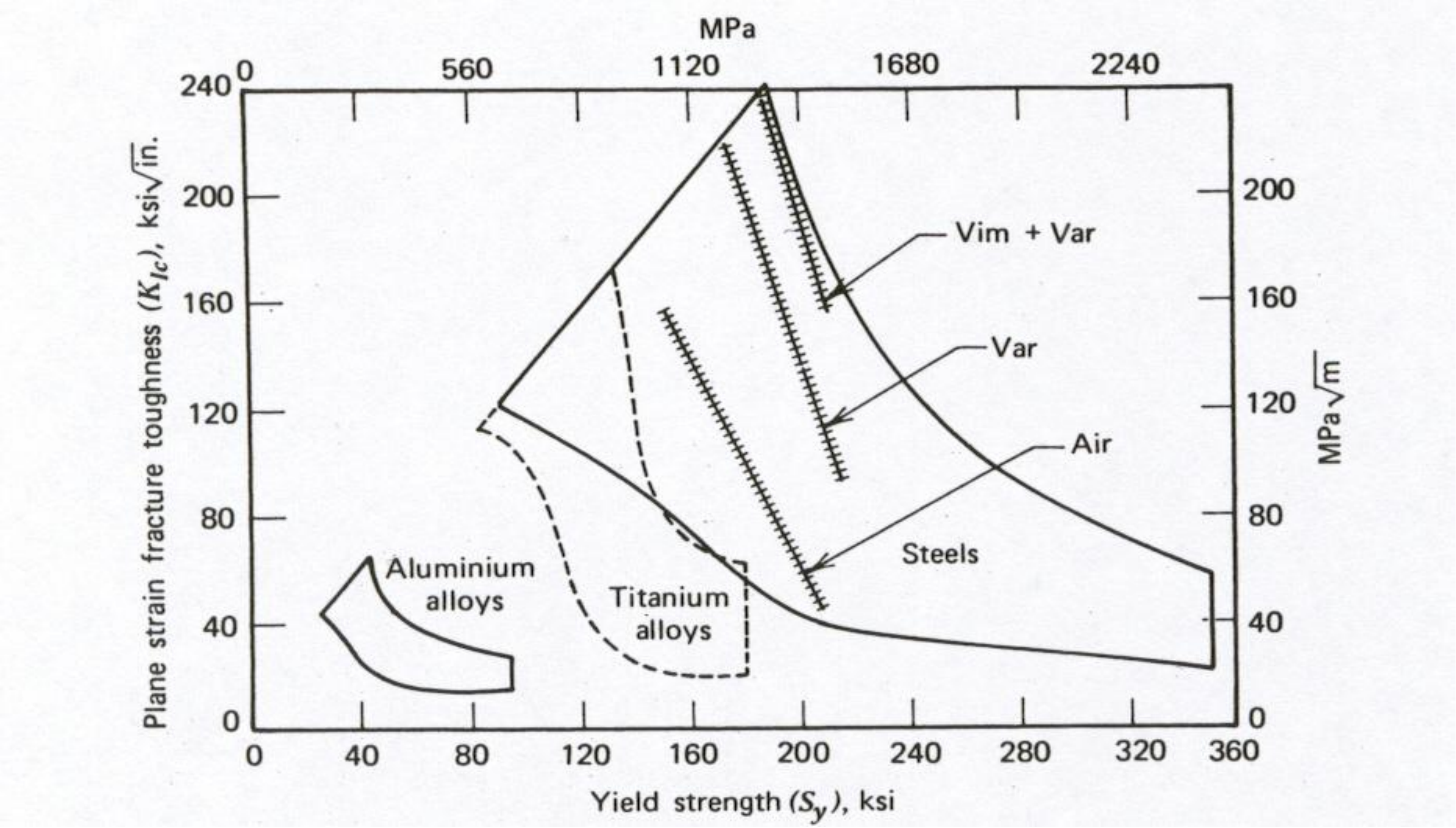
FRACTURE TOUGHNESS - K_c, K_{Ic}

TABLE 6.2 Approximate **Thickness Required for Valid K_{IC} Tests**

Steel S_y, MPa (ksi)	Aluminum S_y, MPa (ksi)	**Thickness**, mm (in.)
690 (100)	275 (40)	>76 (3)
1030 (150)	345 (50)	76 (3)
1380 (200)	448 (65)	45 (1-3/4)
1720 (250)	550 (80)	19 (3/4)
2070 (300)	620 (90)	6 (1/4)

FRACTURE TOUGHNESS - K_c, K_{Ic}

- A higher yield or ultimate strength generally produces a decrease in K_{Ic}, and thus a **greater** susceptibility for catastrophic fracture!



Some representative K_{Ic} values are given in Table A.3.

Material	Process Description	S_y MPa	S_y (ksi)	K_{Ic} MPa$\sqrt{m}$	K_{Ic} (ksi$\sqrt{in}$)
Steel					
11V41[c]	666°C temper	670	(97)	113	(104)
11V41[c]	As-forged	524	(76)	67	(62)
4340	425°C temper	1360–1455	(197–211)	79–89	(72–81)
4340	350°C temper	1380	(200)	66–68	(60–62)
4340	260°C temper	1495–1640	(217–238)	50–63	(45–57)
4330V	275°C temper	1400	(203)	85–92	(77–84)
300M	315°C temper	1625	(236)	56–57	(51–52)
300M	245°C temper	1655	(240)	37–38	(34–35)
D6AC	540°C temper	1495	(217)	102	(93)
HP 9-4-20	550°C temper	1280–1310	(186–190)	132–154	(120–140)
HP 9-4-30	540°C temper	1320–1420	(192–206)	90–115	(82–105)
10Ni (vim)	510°C temper	1770	(257)	54–56	(49–51)
18Ni (200)	Marage	1450	(210)	110	(100)
18Ni (250)	Marage	1785	(259)	88–97	(80–88)
18Ni (300)	Marage	1905	(277)	50–64	(45–58)
Ph13–8Mo	Annealed	1380–1420	(200–206)	113–141	(103–128)
Aluminum					
2014-T651		435–470	(63–68)	23–27	(21–25)
2020-T651		525–540	(76–78)	22–27	(20–25)
2024-T351		370–385	(54–56)	31–44	(28–40)
2024-T851		455	(66)	23–28	(21–25)
2124-T851		440–460	(64–67)	27–36	(25–33)
2219-T851		345–360	(50–52)	36–41	(33–37)
7050-T73651		460–510	(67–74)	33–41	(30–37)
7075-T651		515–560	(75–81)	27–31	(25–28)
7075-T7351		400–455	(58–66)	31–35	(28–32)
7079 T651		525–540	(76–78)	29–33	(26–30)
7178-T651		560	(81)	26–30	(24–27)
A356–T6[d]	Cast	217–229	(31–33)	17–18	(15.5–16.5)
Titanium					
Ti–6Al–4V	Mill annealed	875	(127)	123	(112)
Ti–6Al–4V	Recrystallized Annealed	815–835	(118–121)	85–107	(77–97)
Ti–6Al–6V–2Sn	Mill annealed	1165	(169)	41–51	(37–47)
Ti–6Al–6V–2Sn	Mill annealed	990–1070	(144–155)	48–67	(44–61)

FRACTURE TOUGHNESS - K_c, K_{Ic}

- Low impurity materials provide better fracture toughness.

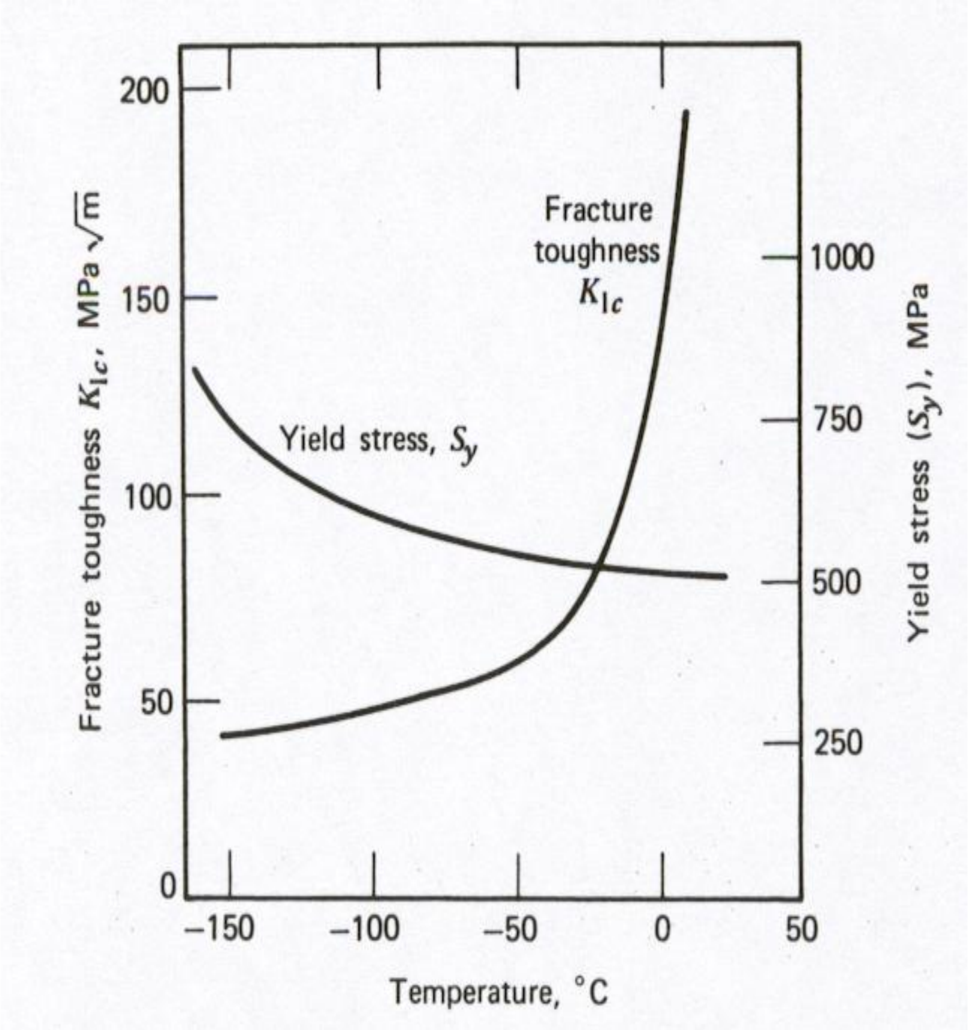
- $\mathbf{K_{Ic}}$ can be very sensitive to metallurgical conditions such as grain orientation, chemistry, and microstructure.

- Fracture toughness $\mathbf{K_c}$ of metals is also dependent on temperature, strain rate, and corrosive environment.

FRACTURE TOUGHNESS - K_c, K_{Ic}

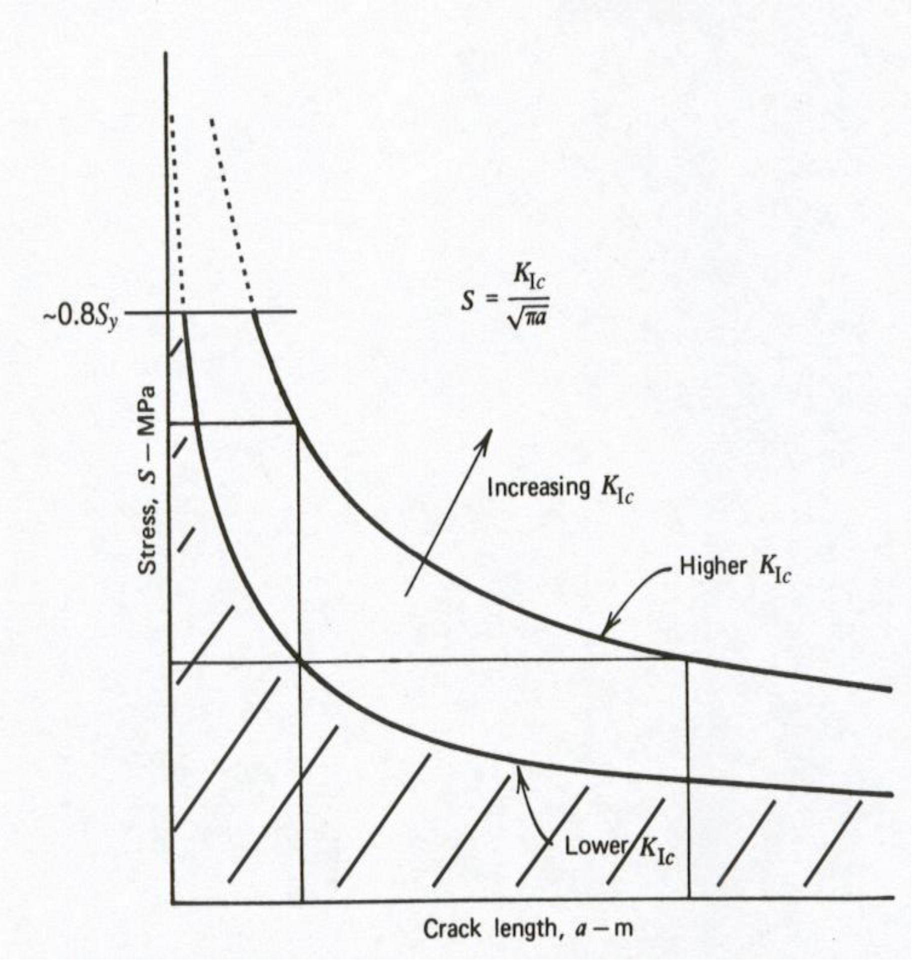
- Typical results for a low alloy nuclear pressure vessel steel.
  - As the **temperature** decreases, K_c usually decreases, while the yield strength increases.
  - Thus, even though unnotched or uncracked tensile strength increases with decreasing temperature, the flaw or crack resistance can be drastically reduced.

- Increased **strain rate** tends to lower fracture toughness, and hence, greater crack sensitivity similar to that of decreasing the temperature.



FRACTURE TOUGHNESS - K_c, K_{Ic}

- General schematic of how changes in fracture toughness influence the relationship between allowable nominal stress and allowable crack size.
  – The allowable stress in the presence of a given crack size is directly proportional to the fracture toughness.
  – the allowable crack size for a given stress is proportional to the square of the fracture toughness.
  – Thus increasing K_{Ic} has a much larger influence on allowable crack size than on allowable stress.



- $0.8S_y$ is the upper bound limit of allowable stress in LEFM approach.

FRACTURE TOUGHNESS - K_c, K_{Ic}

- **ASTM Standard E399** contains a detailed description of the specimen geometry, experimental procedure, and data collection and & techniques used to determine valid K_{Ic} values.

- When a specimen or component has a thickness less than that required for plane strain conditions, it will experience either mixed-mode or plane stress conditions depending on thickness.

- Under plane strain conditions, once a critical stress is reached, unstable crack growth occurs.

- Under plane stress conditions where the plastic zone size is greater, the crack may first extend by slow stable crack growth prior to unstable fracture.

- **ASTM Standard E561** [24] provides a recommended practice for plane stress fracture toughness testing.

FRACTURE TOUGHNESS - K_c, K_{Ic}

- In design situations, the stress state may be plane stress, where K_c for the particular thickness is required but is often not available.
  - K_{Ic} is often used over K_c because of availability as well as K_{Ic} being a more conservative value.
  - However, use of K_{Ic} rather than K_c may be inefficient and costly in some situations.
  - Use of K_{Ic} or K_c is dependent on the application and the safety critical aspects of the component or structure.

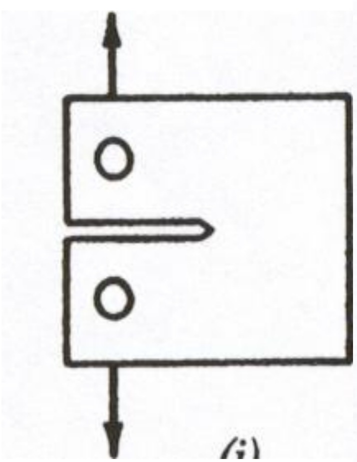
EXAMPLE PROBLEM

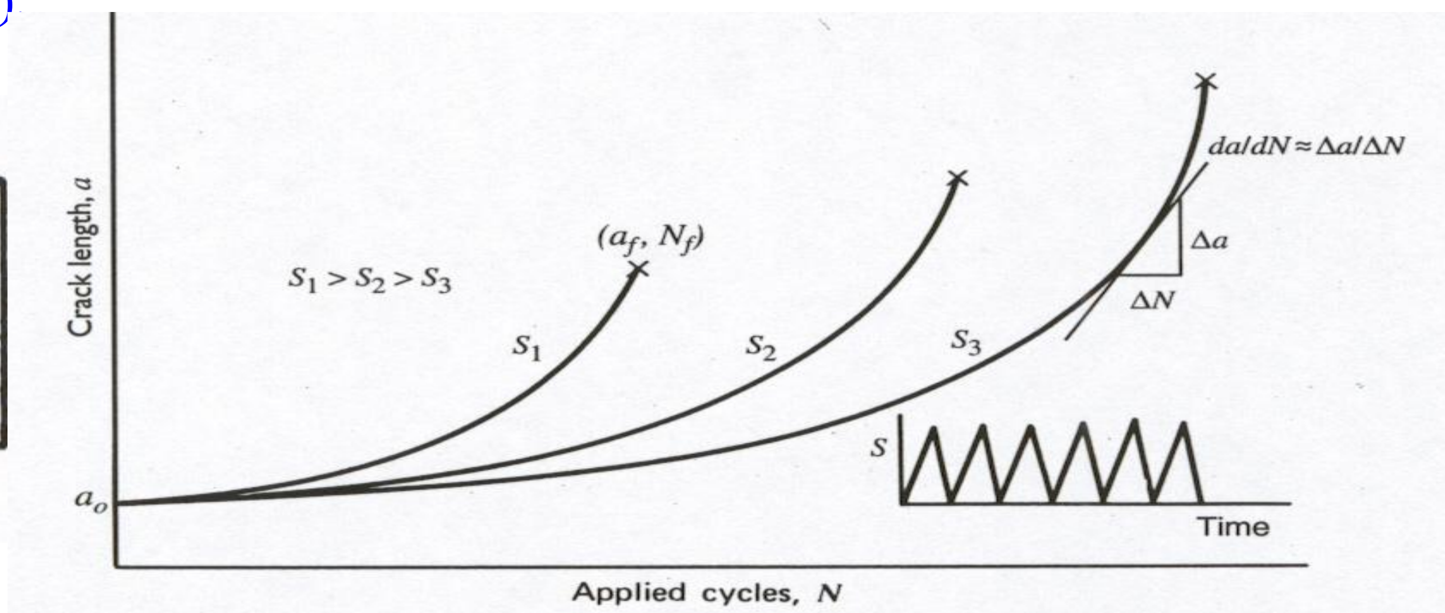
FATIGUE CRACK GROWTH
da/dN - ΔK

- In many structural components subjected to cyclic loading, sub-critical crack growth often occurs due to fatigue until a critical crack size is reached causing fracture.

FATIGUE CRACK GROWTH, da/dN-ΔK

- Three ***a-N* curves** for three identical test specimens (with the same initial crack length, a_o) subjected to different repeated stress levels are shown.
- At higher stresses the **crack growth rates** (slopes of the curves) are higher and the fatigue life is shorter.
- The crack lengths at fracture are shorter at the higher stress levels.
- Therefore, for the given initial crack size, the life to fracture depends on the magnitude of the applied stress and the final fracture resistance of the material (which dictates final crack length)



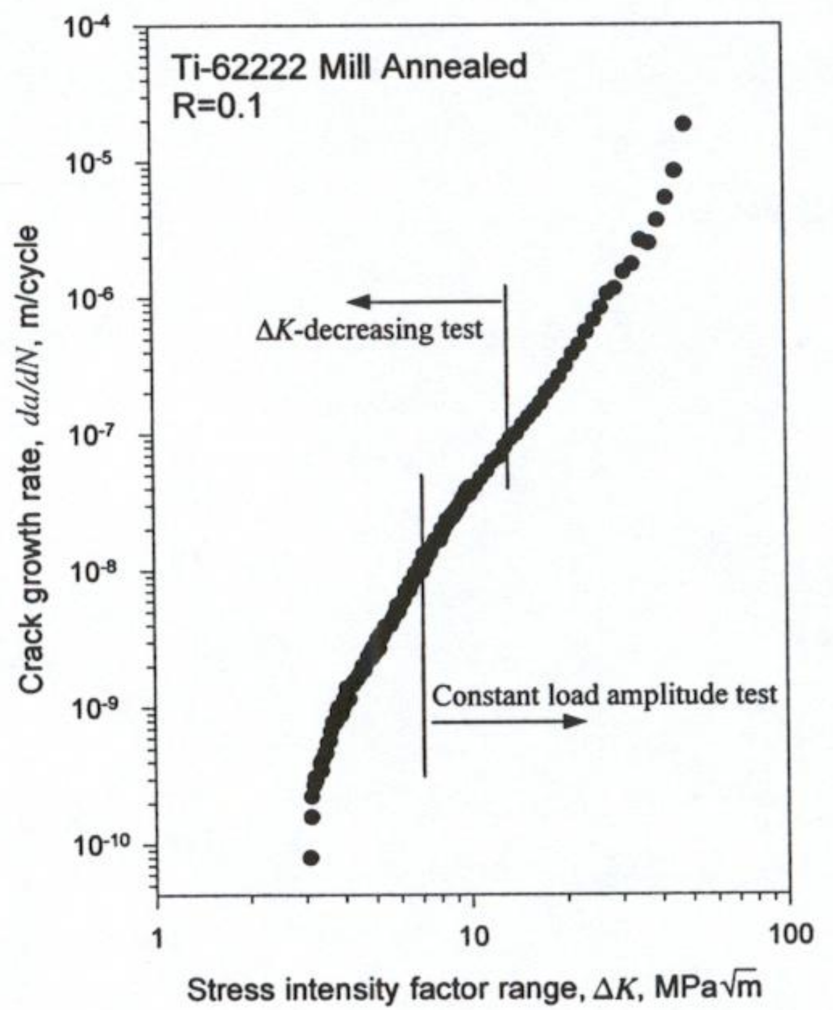


FATIGUE CRACK GROWTH, da/dN-ΔK

- By applying LEFM concepts to *a*-*N* data we can obtain crack growth rate, *da/dN*, versus the applied stress intensity factor range, *ΔK*.

- The fatigue crack growth rate, *da/dN*, is simply the slope of the *a* vs. *N* curve at a given crack length or given number of cycles as identified by da/dN (*Δa/ΔN*).

- The corresponding applied stress intensity factor range, *ΔK*, is calculated knowing the crack length, *a*, applied stress range, *ΔS*, and the stress intensity factor solution, *K*, for the part in question.



FATIGUE CRACK GROWTH, da/dN-ΔK

- ΔK is defined as:

$$\Delta K_I = \Delta K = K_{max} - K_{min} = S_{max}\sqrt{\pi a}\,\alpha - S_{min}\sqrt{\pi a}\,\alpha = (S_{max} - S_{min})\sqrt{\pi a}\,\alpha = \Delta S\sqrt{\pi a}\,\alpha$$

Since the stress intensity factor is undefined in compression, K_{min} is taken as zero if S_{min} is compression.

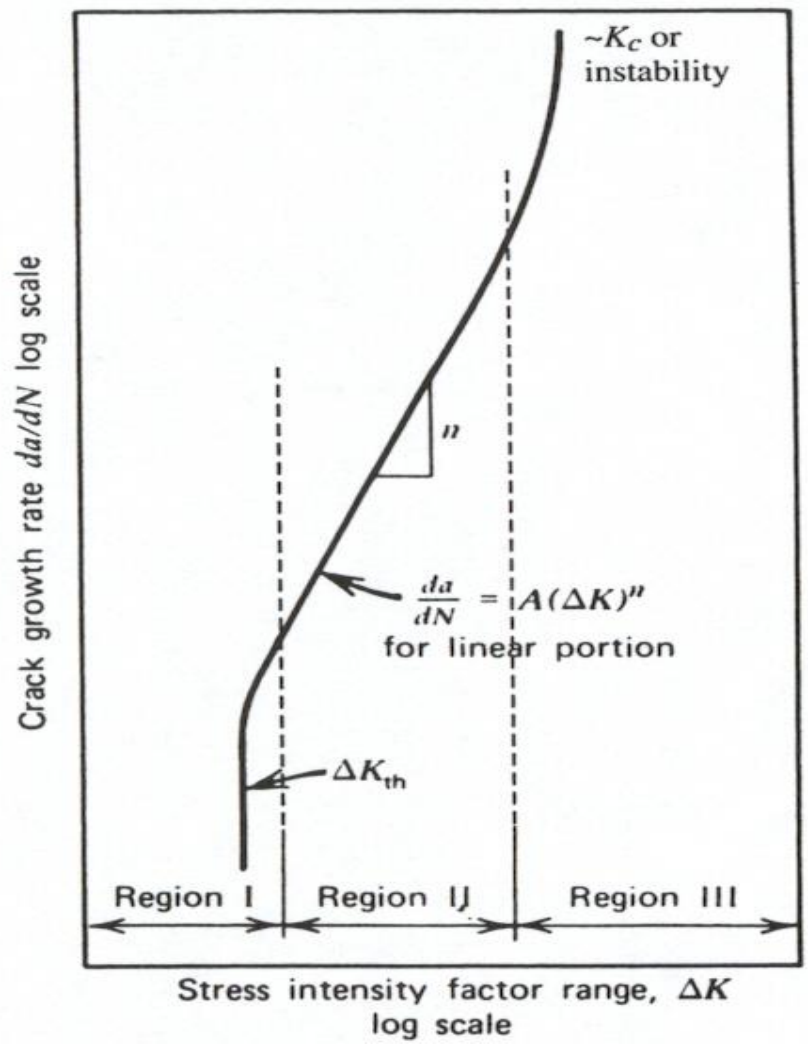
- As ΔK primarily depends on ΔS, *a*, and geometry (for example α), many models of the following form have been proposed and developed

$$\frac{da}{dN} = f(\Delta S, a, \alpha) = f(\Delta K)$$

FATIGUE CRACK GROWTH, da/dN-ΔK

- Log-log plot of *da/dN* versus ΔK has a sigmoidal shape that can be divided into **three regions**.

- **Region I** is the near threshold region and indicates a threshold value $\mathbf{\Delta K_{th}}$, below which there is no crack growth.
  - This threshold occurs at crack growth rates on the order of $1\text{x}10^{-10}$ m/cycle (~4 x 10^{-9} in./cycle) or less.
  - Microstructure, mean stress, and environment mainly control region I crack growth.

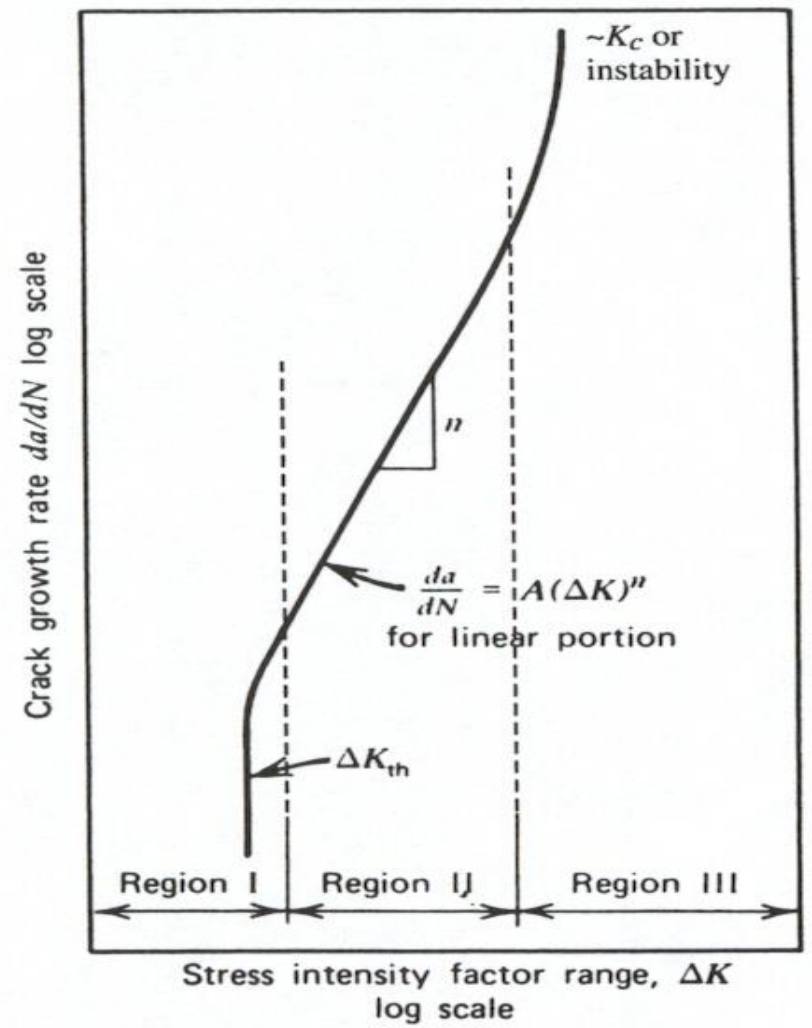


FATIGUE CRACK GROWTH, da/dN-ΔK

- **Region II** shows essentially a linear relationship between log *da/dN* and log ΔK, first suggested by **Paris**

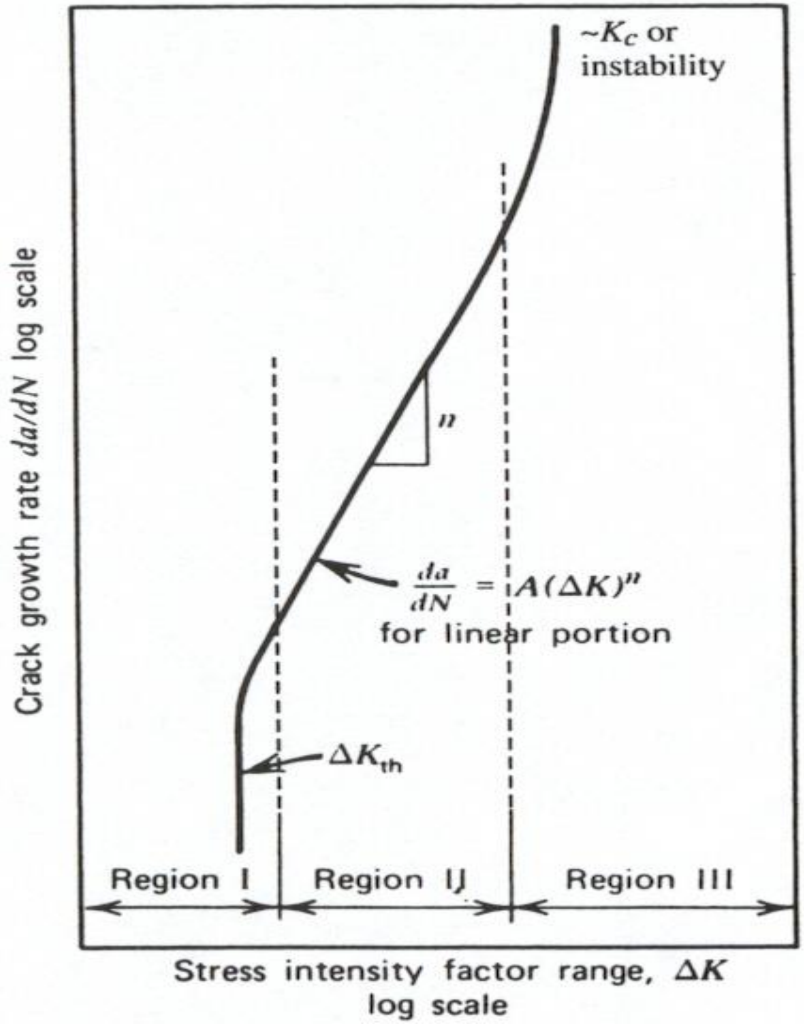
$$\frac{da}{dN} = A(\Delta K)^n$$

  - n is the slope of the curve
  - A is the coefficient found by extending the straight line to ΔK = 1 MPa√m (or 1 ksi√in.).
  - Region II (Paris region) corresponds to stable macroscopic crack growth. Microstructure and mean stress have less influence on fatigue crack growth behavior in region II than in region I.



FATIGUE CRACK GROWTH, da/dN-ΔK

- In **region III**
  - the fatigue crack growth rates are very high as they approach instability
  - little fatigue crack growth life is involved.
  - this region is controlled primarily by **fracture toughness K_c or K_{Ic}.**



FATIGUE CRACK GROWTH, da/dN-ΔK

- For a given material the fatigue crack growth behavior shown in Fig. 6.12 is essentially the same for different specimens or components.

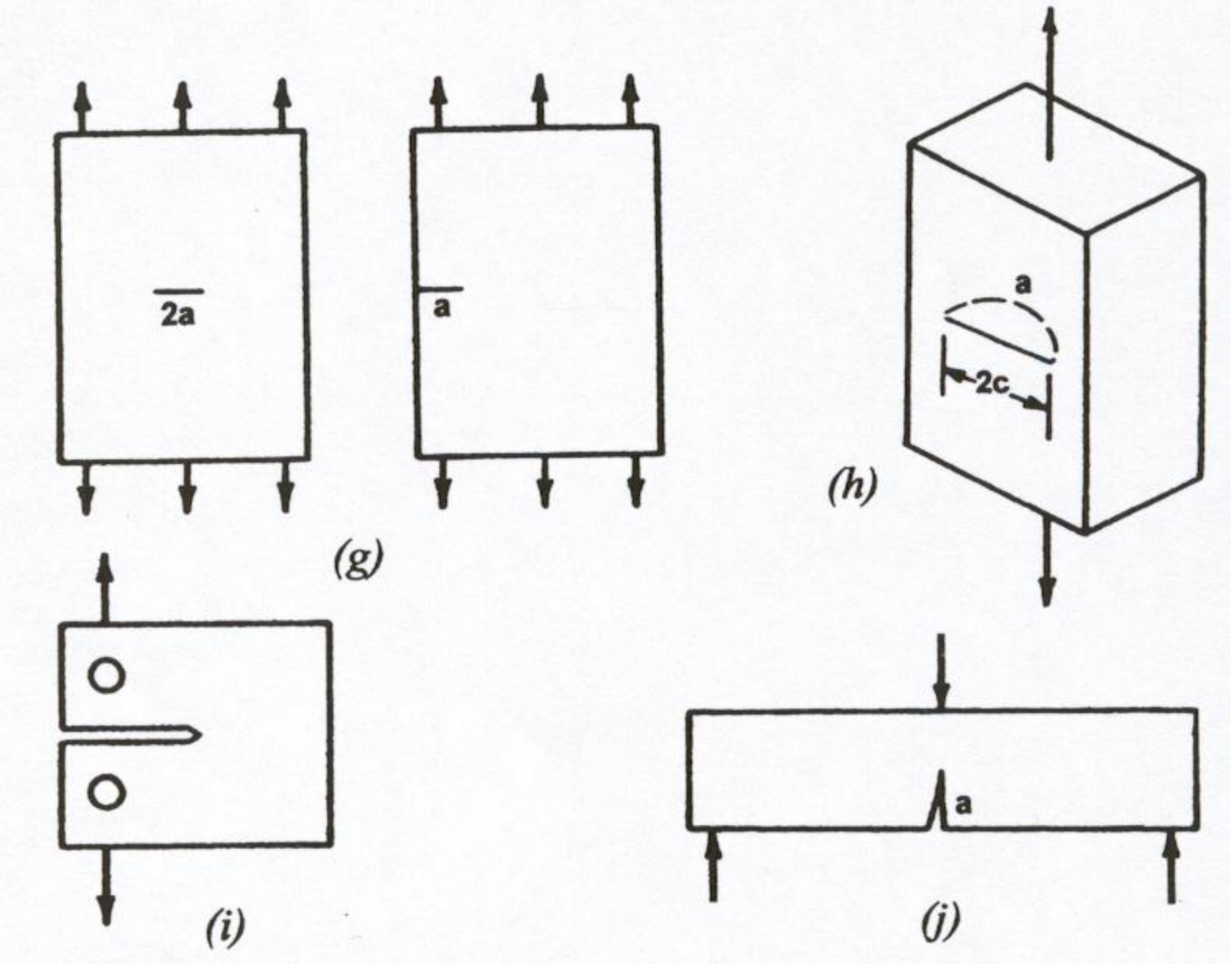
- This allows fatigue crack growth rate versus ΔK data obtained under constant amplitude conditions with simple specimen configurations to be used in design situations.

$$\Delta S \text{ and } a \longrightarrow \Delta K \longrightarrow da/dN \longrightarrow N_f$$

- In many cases, integration of Paris by extrapolating to both regions I and III may be satisfactory as it often gives conservative fatigue crack growth life values.

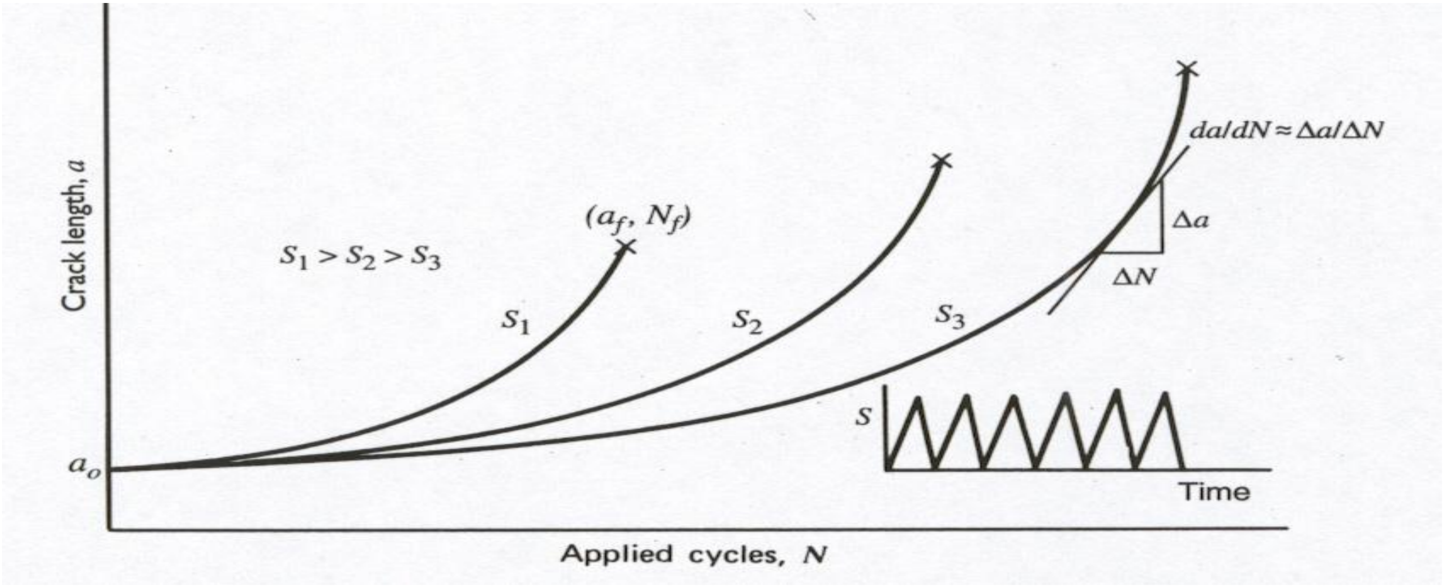
FATIGUE CRACK GROWTH, da/dN-ΔK

- Standard test methods are available for performing constant amplitude fatigue crack growth tests (**ASTM Standard E647**).
  - Common test specimens are shown, although other types may be used.
  - A typical fatigue crack growth experiment is performed under constant amplitude cyclic loading with R = constant
  - Generally a fatigue precrack is formed at a sharp machined notch at a relatively low ΔK level to provide a sharp crack tip.
  - Extension of the growing crack is documented in terms of crack length and number of cycles until failure occurs.
  - Crack growth is usually measured with optical, compliance, ultrasonic, eddy current, electrical potential, or acoustic emission techniques.


2a
a
(g)
a
2c
(h)
(i)
a
(j)

FATIGUE CRACK GROWTH, da/dN-ΔK

- **Data reduction techniques**, as recommended in ASTM Standard E647, include a secant or incremental polynomial methods to smoothen a-N curve.
- These data are then reduced to generate da/dN vs. ΔK data.
- Fatigue crack growth rates that correspond to near threshold conditions, $\sim 10^{-10}$ m/cycle, are very slow (i.e. it takes about 14 hours of continuous testing at 50 Hz frequency to grow a crack 0.25 mm at this crack growth rate).
- Details of the recommended load shedding procedure for threshold tests are summarized in ASTM Standard E647.



da/dN-ΔK for R = 0

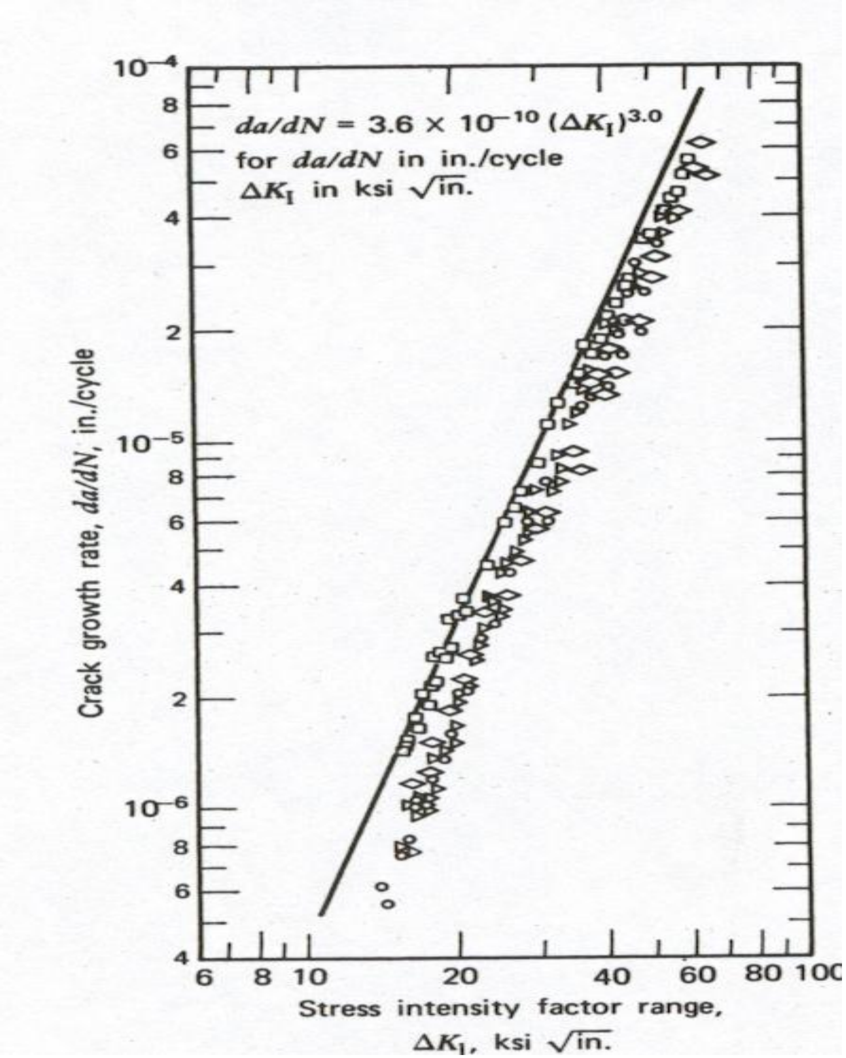
- Conventional S-N or ε-N fatigue behaviors are usually with reference to the fully reversed stress or strain conditions (R = -1).

- Fatigue crack growth data, however, are usually referenced to the pulsating tension condition with **R = 0** or approximately zero, since during compression loading the crack is closed and hence no stress intensity factor, K, can exist.

- Many mathematical equations depicting fatigue crack growth rates above threshold levels have been formulated. The Paris equation **(for region II)** is the most popular equation for R = 0 loading.

da/dN-ΔK for R = 0

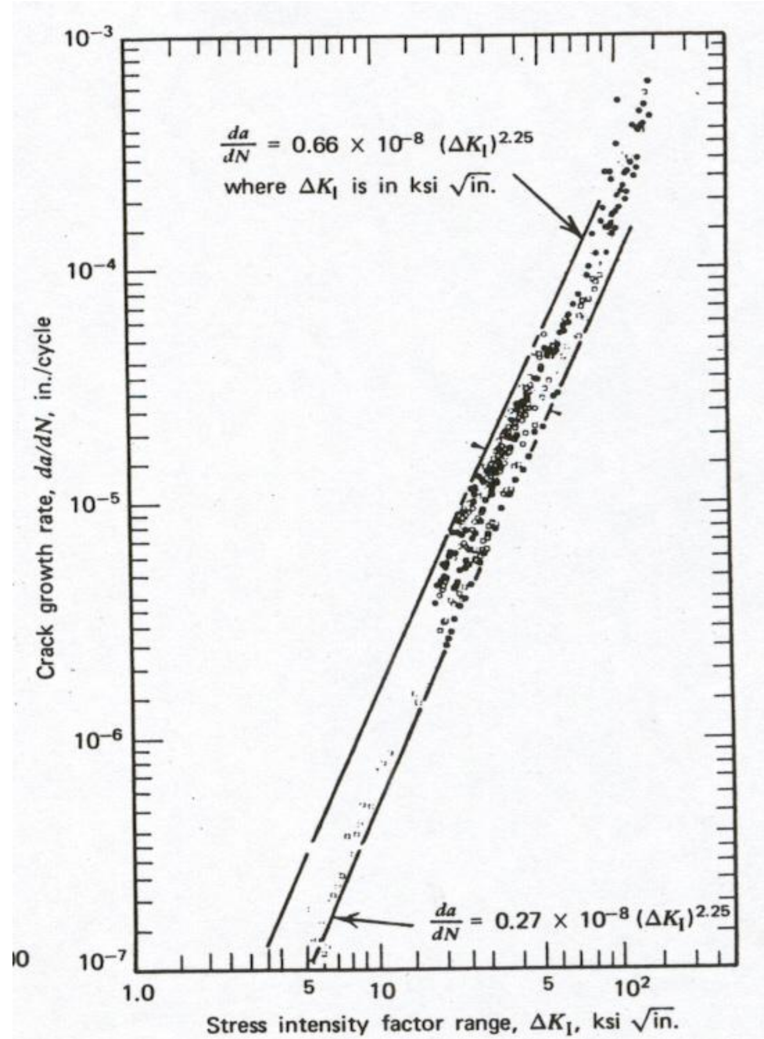
- Barsom has evaluated Paris equation (for region II) for a wide variety of steels varying in yield strength from 250 to 2070 MPa (36 to 300 ksi).
  - He shows that the scatter band for a given ΔK, with many **ferritic-pearlitic steels**, varies by a factor of about 2 (shown in Fig. 6.14a).



da/dN-ΔK for R = 0

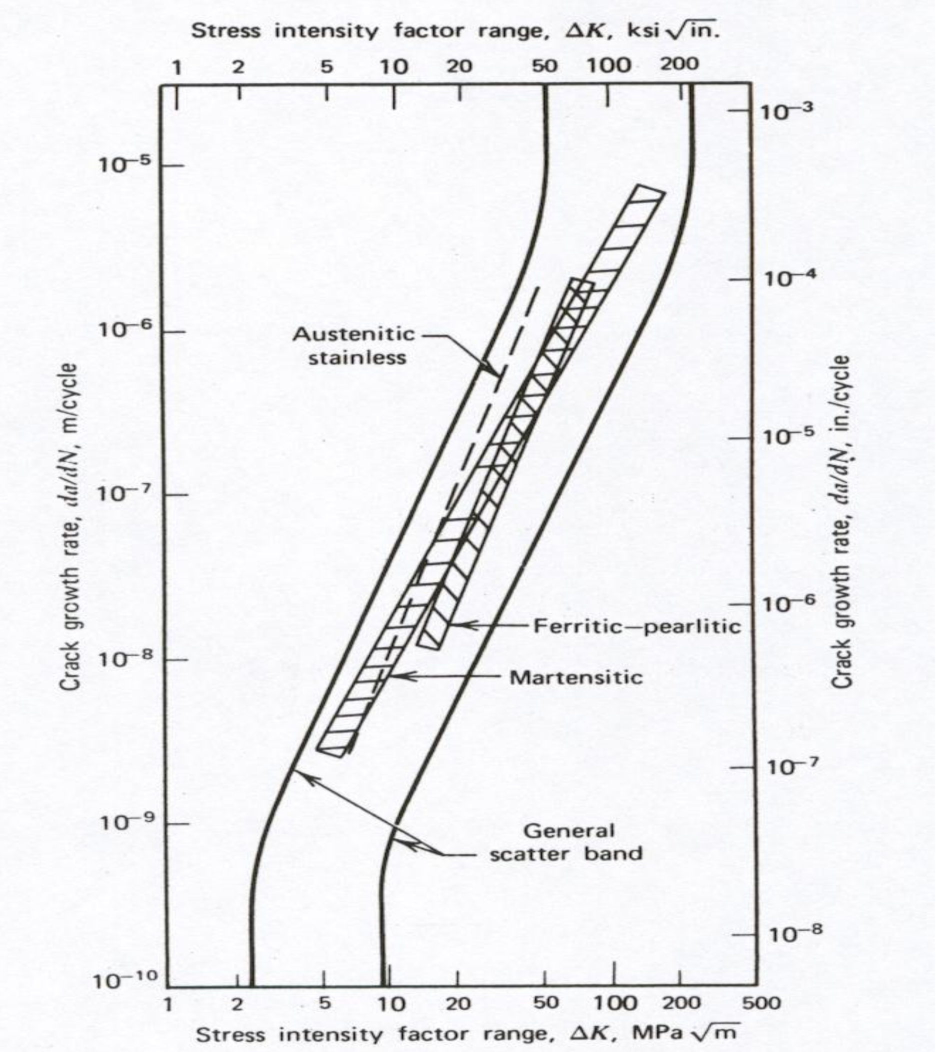
- He also found a similar scatter band width for martensitic steels.

- He suggested that conservative values of the upper boundaries of these scatter bands could be used in design situations if actual data could not be obtained.



da/dN-ΔK for R = 0

- An approximate schematic sigmoidal shaped scatter band for steels with Barsom's scatter bands superimposed is shown, which also includes austenitic stainless steels.



da/dN-ΔK for R = 0

- Typical values of n and A for some metals are given in Table 6.3.
- In general, the crack growth rate exponent n is higher for materials that behave in a more brittle manner.

TABLE 6.3 Approximate Region II Fatigue Crack Growth Rate Properties for the Paris Equation (Eq. 6.19) for Various Metals

Material	Slope, *n*	Intercept, *A* (m/cycle)	Intercept, *A* (in./cycle)
Ferritic-pearlitic steels*	3.0	6.9×10^{-12}	3.6×10^{-10}
Martensitic steels*	2.25	1.35×10^{-10}	6.6×10^{-9}
Austenitic stainless steels*	3.25	5.6×10^{-12}	3.0×10^{-10}
7075-T6 wrought aluminum [22]	3.7	2.7×10^{-11}	1.5×10^{-9}
A356-T6 cast aluminum [29]	11.2	1.5×10^{-20}	7.8×10^{-19}
Ti-6-4 mill annealed titanium [30]	3.2	1.0×10^{-11}	5.2×10^{-10}
Ti-62222 mill annealed titanium [31]	3.2	2.3×10^{-11}	1.2×10^{-9}
AZ91E-T6 cast magnesium [32]	3.9	1.8×10^{-10}	9.4×10^{-9}

* Conservative values suggested by Barsom [28].

da/dN-ΔK for R = 0

- Frequency, wave shape, and thickness effects on constant amplitude fatigue crack growth rates are secondary compared to environmental effects such as corrosion and temperature.

- The **thickness influence** can be greatest in region III because of the inverse relationship between fracture toughness and thickness.
  - A high fracture toughness is desirable because of the longer final crack size at fracture.
  - This allows easier and less frequent inspection and, therefore, safer components or structures.

Threshold stress intensity factor ranges, ΔK_{th}, for selected engineering alloys.

TABLE A.4 Fatigue Crack Growth Threshold, ΔK_{th}, for Selected Engineering Alloys

Material	S_u, MPa (ksi)	$R = K_{min}/K_{max}$	ΔK_{th} MPa$\sqrt{m}$ (ksi$\sqrt{in}$)
Mild steel[a]	430 (62)	0.13	6.6 (6.0)
		0.35	5.2 (4.7)
		0.49	4.3 (3.9)
		0.64	3.2 (2.9)
		0.75	3.8 (3.5)
A533B[b]	—	0.1	8.0 (7.3)
		0.3	5.7 (5.2)
		0.5	4.8 (4.4)
		0.7	3.1 (2.8)
		0.8	3.1 (2.8)
A508[b]	606 (88)	0.1	6.7 (6.1)
		0.5	5.6 (5.1)
		0.7	3.1 (2.8)
18/8 stainless[a]	665 (97)	0	6.0 (5.5)
		0.33	5.9 (5.4)
		0.62	4.6 (4.2)
		0.74	4.1 (3.7)
D6AC[c]	1970 (286)	0.03	3.4 (3.1)
7050-T7[c]	497 (72)	0.04	2.5 (2.3)
2219-T8[d]	—	0.1	2.7 (2.5)
		0.5	1.4 (1.3)
		0.8	1.3 (1.2)
Titanium[a]	540 (78)	0.6	2.2 (2.0)
Ti–6A1–4V[e]	1035 (150)	0.15	6.6 (6.0)
		0.33	4.4 (4.0)
Copper[a]	215 (31)	0	2.5 (2.3)
		0.33	1.8 (1.6)
		0.56	1.5 (1.4)
		0.69	1.4 (1.3)
		0.80	1.3 (1.2)
60/40 brass[b]	325 (47)	0	3.5 (3.2)
		0.33	3.1 (2.8)
		0.51	2.6 (2.4)
		0.72	2.6 (2.4)
Nickel[c]	430 (62)	0	7.9 (7.2)
		0.33	6.5 (5.9)
		0.57	5.2 (4.7)
		0.71	3.6 (3.3)

da/dN-ΔK for R = 0

– These values are usually less than 10 MPa$\sqrt{m}$ for steels and less than 4 MPa$\sqrt{m}$ for aluminum alloys.

– Values of ΔK_{th} are substantially less than K_{Ic} values

– The threshold stress intensity factor, ΔK_{th}, has often been considered analogous to the fatigue limit, S_f, since an applied stress intensity factor range below ΔK_{th} does not cause fatigue crack growth.

– Use of ΔK_{th} in design may be appropriate for conditions involving high frequency or high cycle applications such as turbine blades, or for metals with very high da/dN-ΔK slopes.

da/dN-ΔK for R = 0

- Figure 6.16 shows the use of ΔK_{th} as a design parameter for no crack growth using a single-edge cracked infinitely wide plate subjected to R = 0 loading as shown.

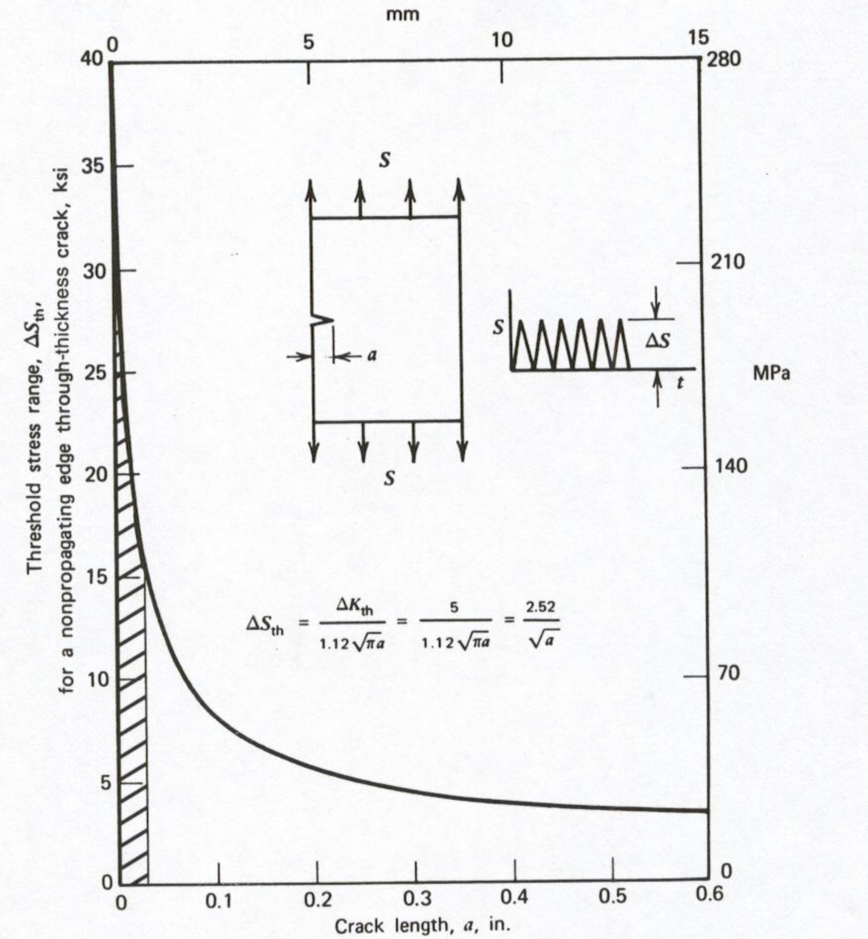


Figure 6.16 Threshold stress range for nonpropagating edge through-thickness cracks in wide plates, $R = 0$, $\Delta K_{th} = 5.5$ MPa$\sqrt{\text{m}}$ (5 ksi$\sqrt{\text{in}}$). The hashed region, where $a \leq 1$ mm ($a \leq 0.04$ in.), indicates LEFM breakdown due to small crack size effects.

Crack Growth Life Integration

$$\Delta K = \Delta S \sqrt{\pi a}\,\alpha$$

$$\frac{da}{dN} = A(\Delta K)^n = A(\Delta S \sqrt{\pi a}\,\alpha)^n = A(\Delta S)^n (\pi a)^{n/2} \alpha^n$$

$$N_f = \int_0^{N_f} dN = \int_{a_i}^{a_f} \frac{da}{A(\Delta S)^n (\pi a)^{n/2} \alpha^n} = \frac{1}{A(\Delta S)^n (\pi)^{n/2}} \int_{a_i}^{a_f} \frac{da}{\alpha^n a^{n/2}}$$

$$= \frac{1}{A(\Delta S)^n (\pi)^{n/2} \alpha^n} \int_{a_i}^{a_f} \frac{da}{a^{n/2}}$$

The last part of this equation is not correct if α significantly changes with *a* between the limits a_i and a_f.

Crack Growth Life Integration

$$N_f = \frac{a_f^{(-n/2)+1} - a_i^{(-n/2)+1}}{(-n/2+1)A(\Delta S)^n (\pi)^{n/2} \alpha^n}$$

where

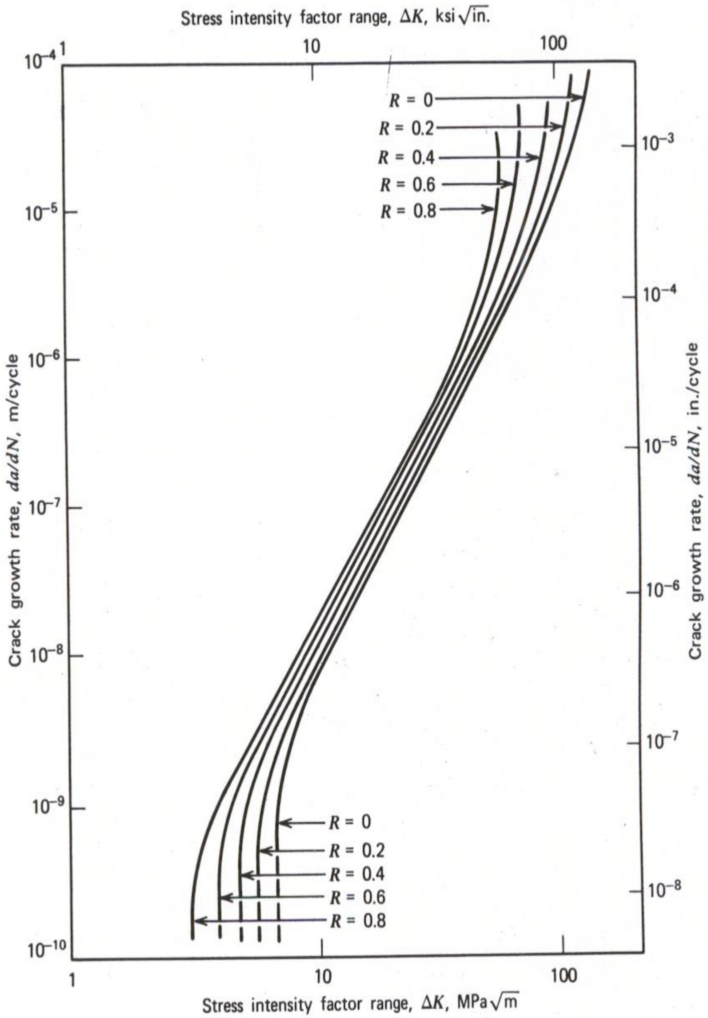
$$a_f = \frac{1}{\pi}\left(\frac{K_c}{S_{\max}\alpha}\right)^2$$

For cases where α is a function of *a*, integration is usually necessary using either standard numerical techniques or computer programs.

EXAMPLE PROBLEM

MEAN STRESS EFFECTS

- The general influence of mean stress on FCG behavior is shown schematically.
  - The **stress ratio R** = $K_{min}/K_{max} = S_{min}/S_{max}$ is used as the principal parameter.
  - Most mean stress effects on crack growth have been obtained for only positive stress ratio, that is, R≥0.
  - Increasing the R ratio (which means increasing the mean stress) has a tendency to increase the crack growth rates in all portions of the sigmoidal curve.
  - The increase in region II, however, may be small.
  - In region III, where fracture toughness K_c or K_{Ic} controls, substantial differences in crack growth rates occur for different R ratios.
- The effect of R on fatigue crack growth is very material dependent.



MEAN STRESS EFFECTS

- A commonly used equation depicting mean stress effects in regions II and III is the **Forman equation**:

$$\frac{da}{dN} = \frac{A'(\Delta K)^{n'}}{(1-R)K_c - \Delta K} = \frac{A'(\Delta K)^{n'}}{(1-R)(K_c - K_{\max})}$$

  – *A*' and *n*' are empirical material fatigue crack growth rate constants and K_c is fracture toughness for the material and thickness.

  – The Forman equation is a modification of the Paris equation to incorporate mean stress and region III fatigue crack growth behavior.

  – As K_{max} approaches K_c, the denominator approaches zero, thus the crack growth rate, da/dN, gets very large. This describes region III crack growth.

MEAN STRESS EFFECTS

- Another common empirical relationship used to describe mean stress effects with R≥0 is the **Walker relationship**

$$\frac{da}{dN} = \frac{A(\Delta K)^{n}}{(1-R)^{n(1-\lambda)}} = A''(\Delta K)^{n}$$

  - where A and n are the Paris coefficient and slope for R=0 and λ is a material constant.
  - The stress ratio, R, does not affect the slope, n.
  - Values of λ for various metals range from 0.3 to nearly 1 with a typical value around 0.5. Lower values of λ indicate a stronger influence of R on fatigue crack growth behavior.

MEAN STRESS EFFECTS

- Effect of mean stress on ΔK_{th} can be substantial (see Fig. 6.17 & Table A.4).

TABLE A.4 Fatigue Crack Growth Threshold, ΔK_{th}, for Selected Engineering Alloys

Material	S_u, MPa (ksi)	$R = K_{min}/K_{max}$	ΔK_{th} MPa$\sqrt{m}$ (ksi$\sqrt{in}$)
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		0.7	3.1 (2.8)
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		0.69	1.4 (1.3)
		0.80	1.3 (1.2)
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Nickel[c]	430 (62)	0	7.9 (7.2)
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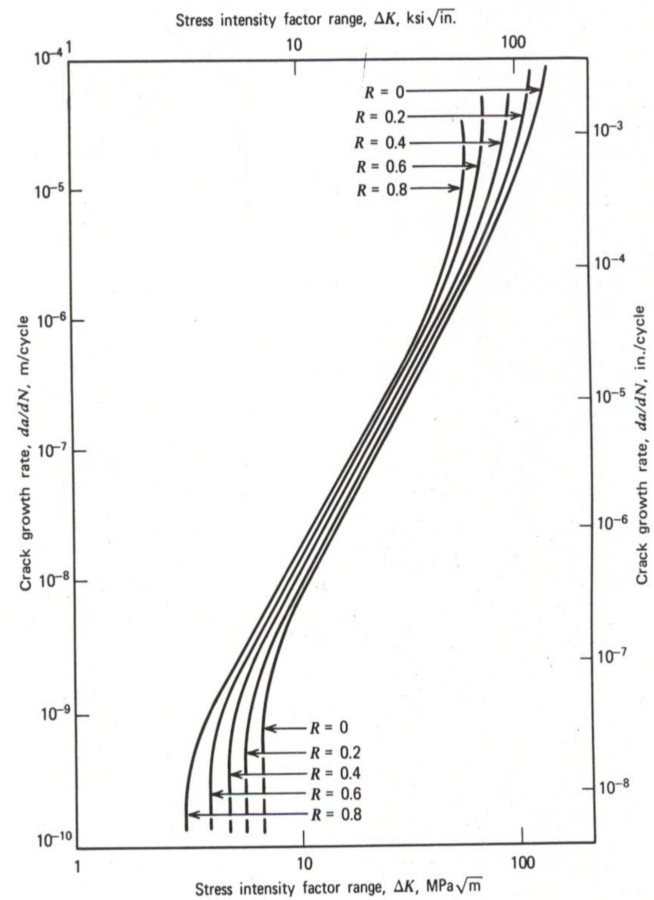
		0.57	5.2 (4.7)
		0.71	3.6 (3.3)



MEAN STRESS EFFECTS

– A form similar to the Walker equation can be used to describe the effect of R ratio on threshold:

$$\Delta K_{th\,(R \neq 0)} = \Delta K_{th\,(R=0)} (1 - R)^{1-\gamma}$$

where γ is an empirical constant.

– The effect of negative R ratios, which includes compression in the cycle on wrought and cast steels, cast irons, and aluminum alloys in regions II and III indicate crack growth rates based on ΔK values (which neglect compressive nominal stresses) are similar to R = 0 results.

EXAMPLE PROBLEM

CYCLIC PLASTIC ZONE

- The monotonic plastic zone size and stress distribution developed due to the maximum load applied in the loading cycle, point A, is shown in Fig. 6.18(b).

- As the maximum load is reduced during the loading cycle to the minimum load, point B, it causes the development of the cyclic plastic zone size, $2r_y'$, and corresponding stress distribution, shown in Fig. 6.18(c).

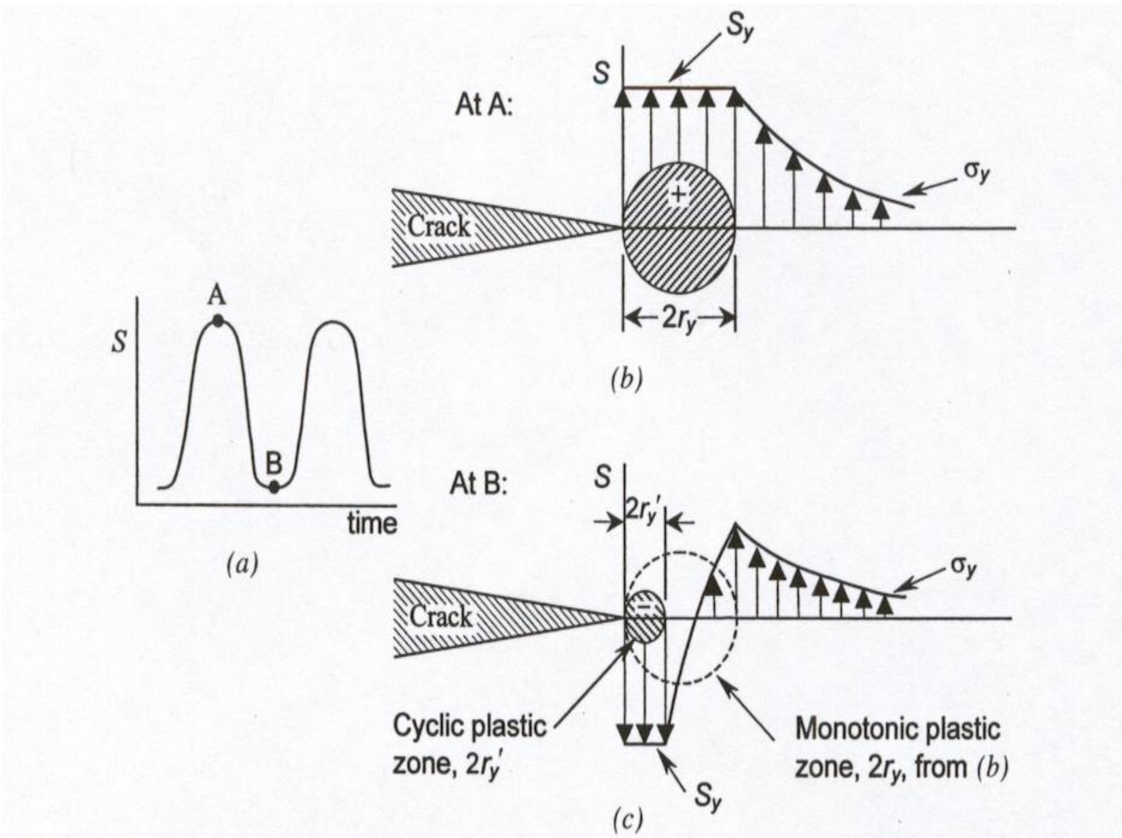


Figure 6.18 Schematic of the plastic zone at the tip of an advancing crack. (*a*) Loading cycle. (*b*) Monotonic plastic zone. (*c*) Cyclic plastic zone.

CYCLIC PLASTIC ZONE

- For the assumed elastic-perfectly plastic behavior, the maximum stress developed in Fig. 6.18(b) is S_y. The maximum stress change developed during unloading can be as large as $2S_y$.
- Fig. 6.18(c) shows the summation of the inelastic loading stress distribution plus the elastic unloading stress distribution.
- The stresses within the cyclic plastic zone, $2r_y'$, are compressive, while outside the cyclic plastic zone the compressive stress decreases and then becomes tensile.

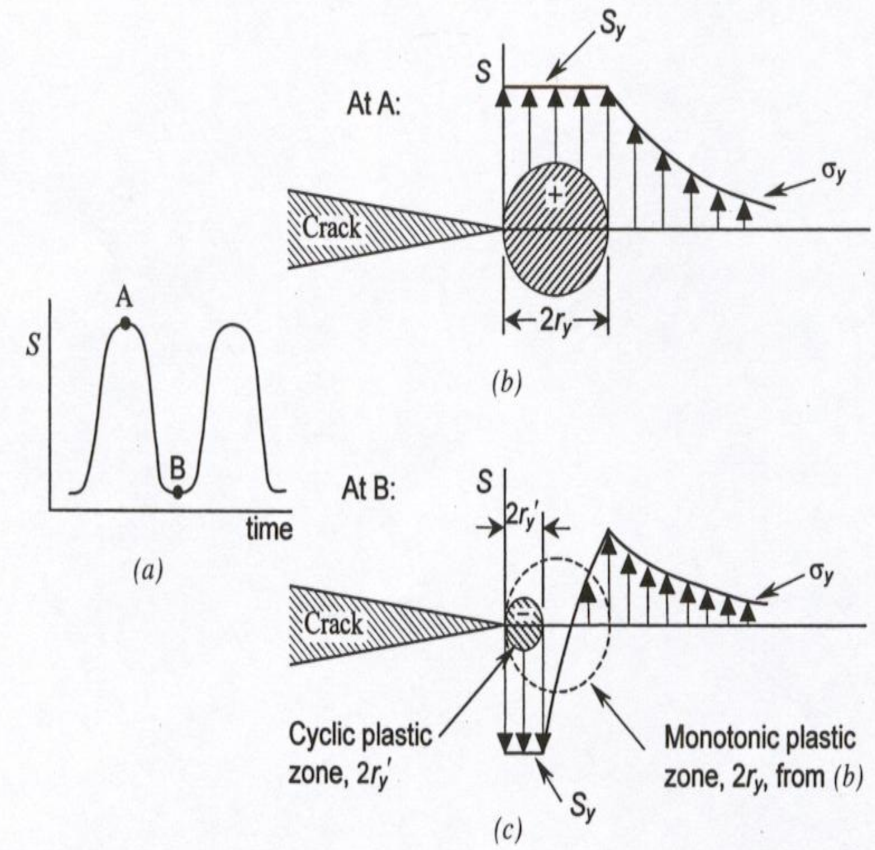


Figure 6.18 Schematic of the plastic zone at the tip of an advancing crack. (a) Loading cycle. (b) Monotonic plastic zone. (c) Cyclic plastic zone.

CYCLIC PLASTIC ZONE

- A key point to recognize is that the sign of the inelastic stress distribution associated with the cyclic plastic zone size is opposite to the sign of the applied stress during loading.

- Thus, if a region yields in tension during loading, as shown in Fig. 6.18(b), after unloading a portion of that region is in compression, as shown in Fig. 6.18(c).

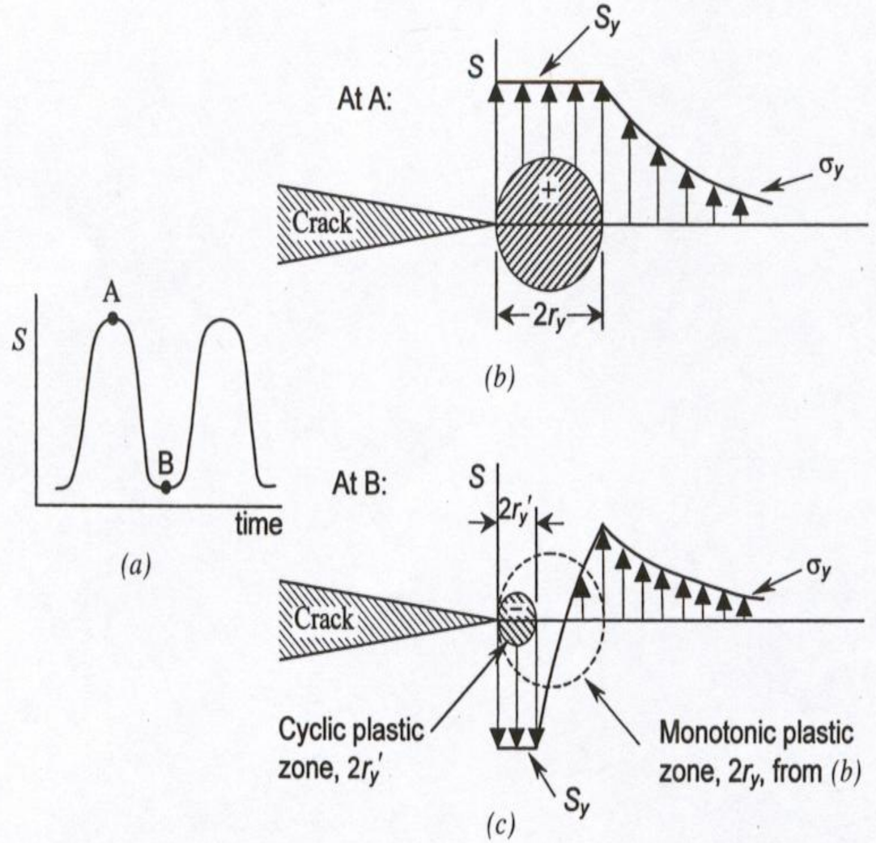


Figure 6.18 Schematic of the plastic zone at the tip of an advancing crack. (*a*) Loading cycle. (*b*) Monotonic plastic zone. (*c*) Cyclic plastic zone.

CYCLIC PLASTIC ZONE

- The size of the cyclic plastic zone where yielding occurs can be approximated by using $2S_y$ for S_y and ΔK for K in the monotonic plastic zone equation **for plane stress condition**:

$$2r_y = \frac{1}{\pi}\left(\frac{K}{S_y}\right)^2 \qquad\qquad 2r_y' \cong \frac{1}{\pi}\left(\frac{\Delta K}{2S_y}\right)^2 \cong \frac{1}{4\pi}\left(\frac{\Delta K}{S_y}\right)^2$$

where $2r_y'$ is the cyclic plastic zone size under plane stress conditions for $R \geq 0$.

- For $R = 0$, where $K_{max} = \Delta K$, the size of the plastic zone is only one quarter that of which existed at the peak tensile load.

CYCLIC PLASTIC ZONE

- The cyclic plastic zone **for plane strain conditions** is one third as large as the corresponding cyclic plastic zone for plane stress:

$$2r_y' \cong \frac{1}{3\pi}\left(\frac{\Delta K}{2S_y}\right)^2 \cong \frac{1}{12\pi}\left(\frac{\Delta K}{S_y}\right)^2$$

- If the material cyclic hardens or softens, S_y should be replaced with the cyclic yield strength, S_y'.

CYCLIC PLASTIC ZONE

- Excessive plasticity, where the plastic zone size or applied stress are a large fraction of the crack size or the yield strength respectively, violates the basic assumptions of LEFM, whether due to monotonic or cyclic loading.

- Because the cyclic plastic zone is usually much smaller than the monotonic plastic zone size, LEFM can often be applied to fatigue crack growth situations with good success even for materials that exhibit large plasticity or for region III crack growth where the magnitude of plasticity is greatest.

- In region III of crack growth, usually a very small fraction of the total fatigue life is involved, thus accuracy is not as important.

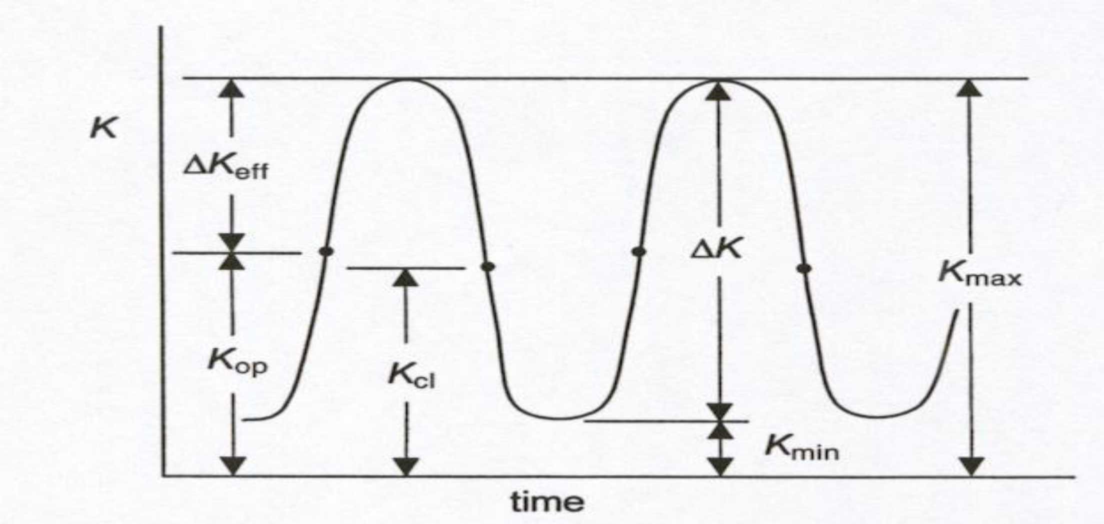
- Because r_y' is always less than r_y for fatigue loading, limitations of LEFM associated with r_y for monotonic loading can often be extended to r_y' as associated with fatigue crack growth.

- Excessive plasticity effects used in fatigue crack growth are addressed in Section 6.9.

CRACK CLOSURE

- Crack closure was first quantified and its importance made known by Elber in 1970 who showed that a fatigue crack closed even with a tensile load still applied (Figure 6.19).
- Based on his experimental results he argued that a reduction in the crack tip driving force occurred as a result of residual tensile deformation left in the wake of a fatigue crack tip.
- The residual tensile deformation caused the crack faces to close prematurely prior to the minimum load being reached.
- Since Elber's work, extensive research on crack closure has been performed and documented.
- Many fatigue crack growth effects have been explained, at least in part by closure concepts, however, many details of the mechanisms of crack closure are still only partly understood.

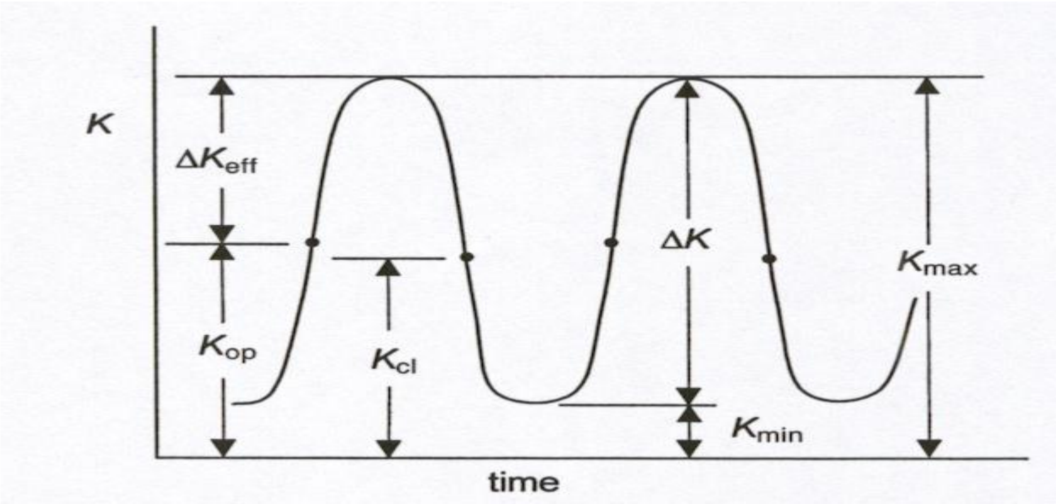
CRACK CLOSURE



- At K_{op} the crack opens on the loading portion of the cycle.
- At K_{cl} the crack closes on the unloading portion of the cycle.
- The opening stress intensity factor, K_{op}, is typically greater than the minimum stress intensity factor, K_{min}.

  $\Delta K = K_{max} - K_{min}$ But $\boldsymbol{\Delta K_{eff} = K_{max} - K_{op}}$

CRACK CLOSURE



- The effective crack tip driving force, ΔK_{eff}, is less than the nominal crack tip driving force, ΔK.
- The damaging portion of the loading cycle is restricted to the opening portion of the stress intensity range, ΔK_{eff}.

CRACK CLOSURE

- One parameter used to define the measurement of closure is the closure ratio:

$$U = \frac{\Delta K_{eff}}{\Delta K} = \frac{(1 - K_{op} / K_{\max})}{(1 - R)}$$

- For loading conditions were there is little or no closure, U approaches one.

- For loading conditions that show extensive crack closure, U is small.

- At high stress ratio (e.g. for R > 0.5) crack growth generally displays limited crack closure, while at low stress ratio crack growth tends to exhibit higher levels of crack closure.

CRACK CLOSURE

- The effect of crack closure on region I behavior is probably of greatest interest as:
  - It is this region where much of the crack growth life is expended,
  - In this region crack closure is most significant,
  - Region I crack growth is influenced by many variables, including stress ratio, environment, and microstructure, which have a direct influence on the effect of crack closure.
- The practical significance of crack closure is related to the growth, retardation, or arrest of fatigue cracks under in-service load histories. The effect of crack closure on variable amplitude loading is presented in Chapter 9.

CRACK CLOSURE

- Since **Elber's** pioneering work, additional crack closure mechanisms have been identified.

- The most extensively studied **forms of closure** include:
  - plasticity-induced closure
  - oxide-induced closure
  - roughness-induced closure

- Elber's work fundamentally described what is now referred to as plasticity-induced closure.

CRACK CLOSURE

- **Plasticity-induced closure** is most prevalent in metals at low stress ratios under plane stress conditions, although it can still be significant for plane strain.

- This form of closure arises from the presence of the compressive plastic zone.

- Based on plasticity-induced closure arguments, ΔK_{eff} has been shown to reasonably correlate fatigue crack growth curves for various stress ratios into a single narrow scatter band.

CRACK CLOSURE

- **Oxide formation** by way of various aggressive environments can affect crack closure levels in a number of ways:
  - The oxide can form as either a uniform layer on the crack faces or can form as rough bulky deposits.
  - At low stress ratio and near-threshold levels there is a greater propensity for repeated crack face contact to occur.
  - At low stress intensity levels there is a continual breaking and reforming of the oxide layer along the crack faces.
  - At high stress ratio where crack closure is less prevalent, oxide debris and build-up are not as significant.

CRACK CLOSURE

- Crack deflection due to microstructural contributions can have a large effect on threshold behavior.
- This form of crack closure is generally referred to as **roughness-induced closure.**
- An increase in surface roughness encourages crack deflection, resulting in premature crack face contact leading to higher crack opening levels.
- Materials with very small grain size show no crack closure, even at low stress ratio. This leads to low threshold values and no stress ratio dependence.
- For coarser grained materials, more serrated and faceted morphology is generally observed that lead to higher opening loads.
- In regions II and III, where crack opening displacements are higher, roughness-induced closure has much less effect.

CRACK CLOSURE

- Various forms of closure can operate synergistically.

- Methods to measure crack closure range from simple to complex and include:

  - Direct observation (using various microscopes, interferometry, lasers, or other detection devices).

  - Indirect observation (such as striation counting and spacing, crack growth rate changes, and high stress ratio tests to evaluate crack growth observations).

  - Compliance measurements:
    - They indicate a variation in the load-displacement response
    - They involve techniques that use clip gages, strain gages, electrical potential, ultrasound, and eddy current.
    - Compliance methods are generally the simplest and most often used.

SMALL FATIGUE CRACKS
AND
LEFM LIMITATIONS

SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

- In many design situations, LEFM analyses allow a direct comparison of fatigue crack growth behavior between engineering components or structures and laboratory specimens using the stress intensity factor range, ΔK.

- This holds true for conditions where LEFM is applicable, primarily based on small scale crack tip yielding.

- This is controlled primarily by
  - plastic zone size to crack length ratio and
  - the magnitude of the operating stress.

- LEFM assumptions are violated when operating stress levels are too high resulting in excessive plasticity or when cracks are small in comparison to either the plastic zone or microstructural dimensions.

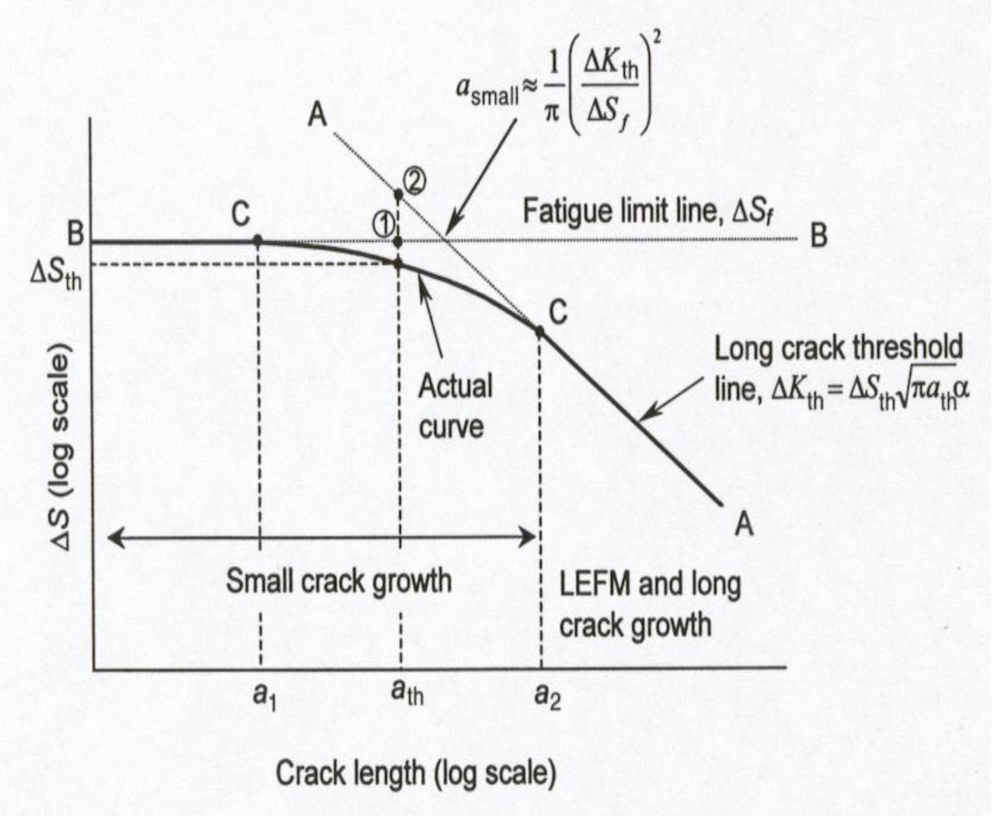
SMALL FATIGUE CRACKS & LEFM LIMITATIONS

- This figure provides a schematic comparison between stress range, ΔS, and crack length, *a*, on a log-log scale as first presented by Kitagawa and Takahashi.

- The sloped straight line labeled A-A, which defines the long crack threshold, represents the stress-crack length combination below which a crack should not grow using LEFM.

- At high stress range levels, extending beyond that shown, LEFM assumptions are violated due to excessive plasticity.

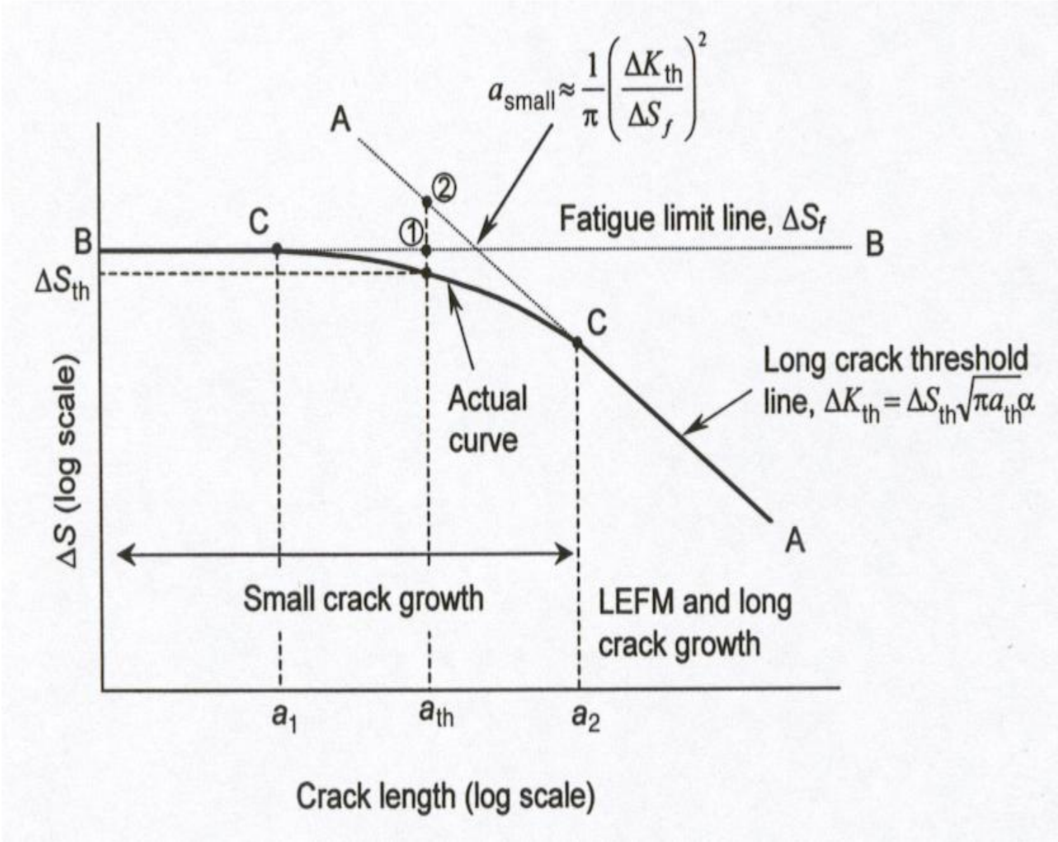
- The horizontal line labeled B-B identifies the fatigue limit or fatigue strength range, below which fatigue life is very long.



SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

- Experimental work has been shown to deviate from these two lines between certain small crack lengths, labeled a_1 and a_2 and follow the "solid curve" which merges the threshold and fatigue limit lines.

- For the solid curve segment labeled C-C, a_1 defines the smallest crack length capable of lowering the fatigue limit and a_2 defines the crack size at which small crack effects end.

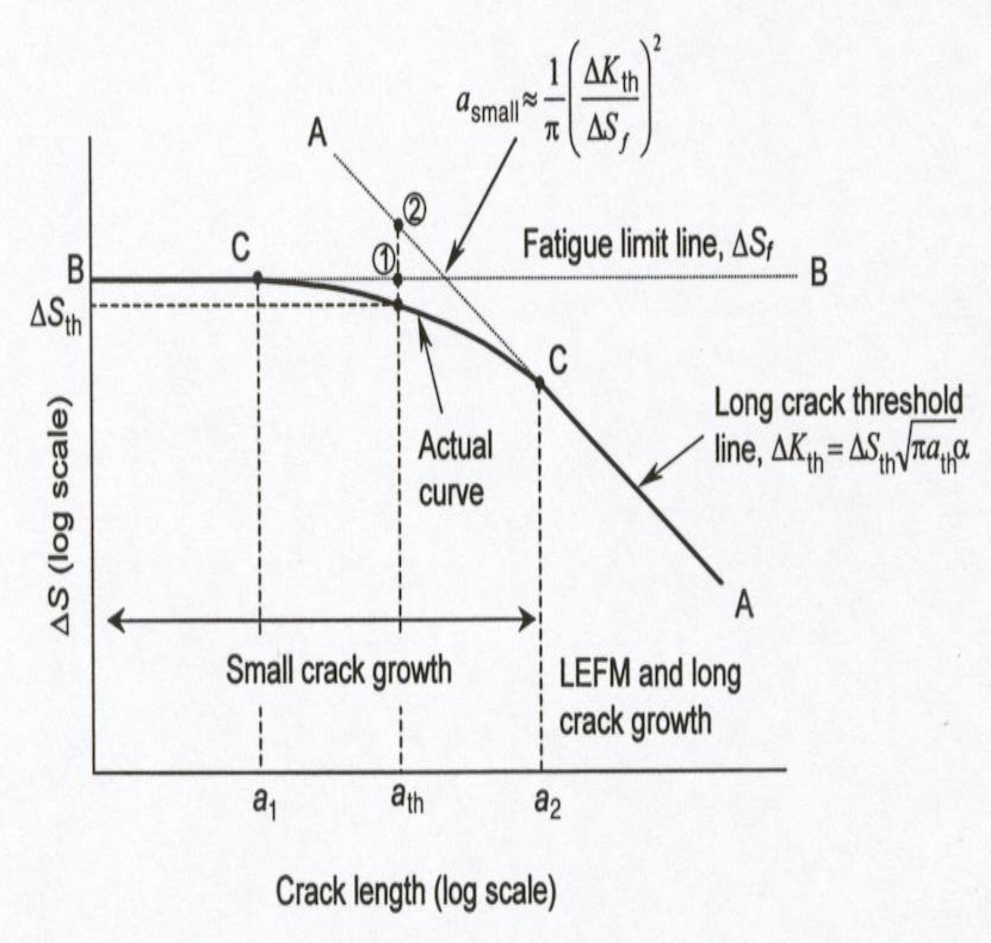


SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

- Thus, the experimentally determined threshold stress range, ΔS_{th}, shown for an arbitrary crack length (a_{th}) is less than the stress range predicted from either the fatigue limit (labeled point 1) or the long crack threshold (labeled point 2).

- This holds true up to a crack length of a_2.

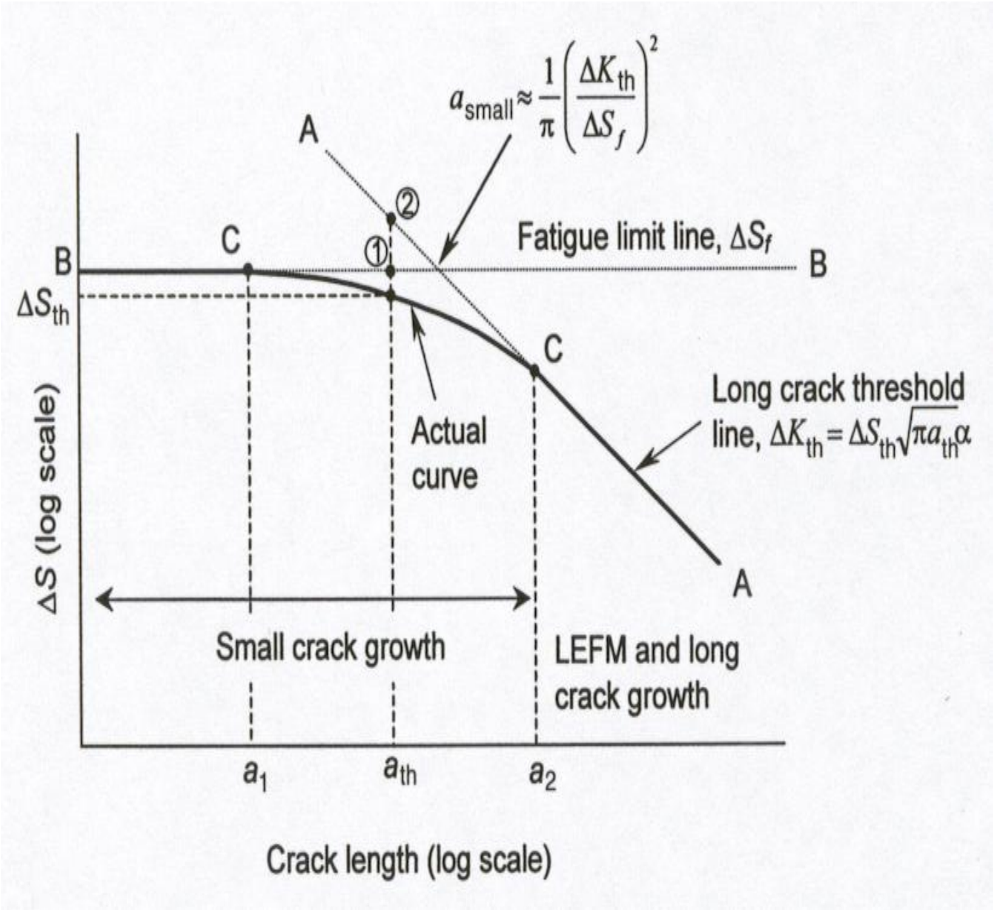
- This behavior leads to cracks growing below the long crack threshold and the fatigue limit and can lead to non-conservative life predictions based on LEFM and/or fatigue limit analysis



SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

- One simple method to approximate the crack length for which small crack growth behavior could pose a concern for a material can be obtained by setting the equations for the threshold line and fatigue limit line equal to each other and solving for the crack length.

- In the equation shown, a_{small} approximates small crack growth behavior and ΔS_f is the fatigue limit range, i.e. twice the fatigue limit, S_f.



SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

- Based on the general classification of small fatigue cracks as defined in ASTM Standard E647, cracks are defined as being small as follows:

  - **Microstructurally small** if their length is comparable to the scale of some microstructural dimension, for example the grain size.

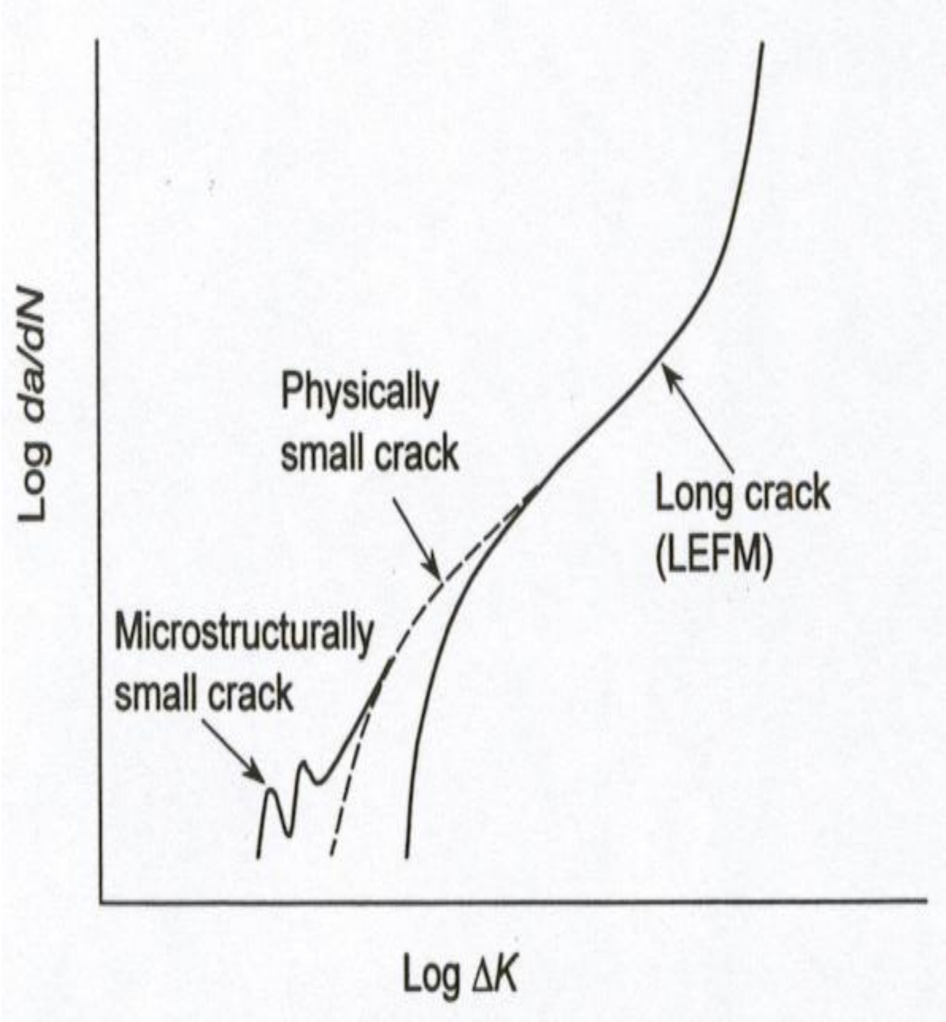
  - **Physically small** if their length is typically between 0.1 and 1 or 2 mm (0.004 to 0.04 or 0.08 in.),

  - **Mechanically small** if their length is small compared to the scale of local plasticity, for example a crack growing from a notch exhibiting local plasticity. Mechanically small cracks, typically those growing from notches, are discussed in Chapter 7.

SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

- Fatigue crack growth rates of small cracks can be significantly higher than the corresponding growth rates of long cracks for the same nominal stress intensity factor range ΔK.

- Also, small cracks grow at values of ΔK that are below the long crack threshold stress intensity factor range, ΔK_{th}. This in effect shifts the region I “small” crack growth curve to the left of the sigmoidal fatigue crack growth curve, as shown schematically for both microstructurally and physically small cracks.



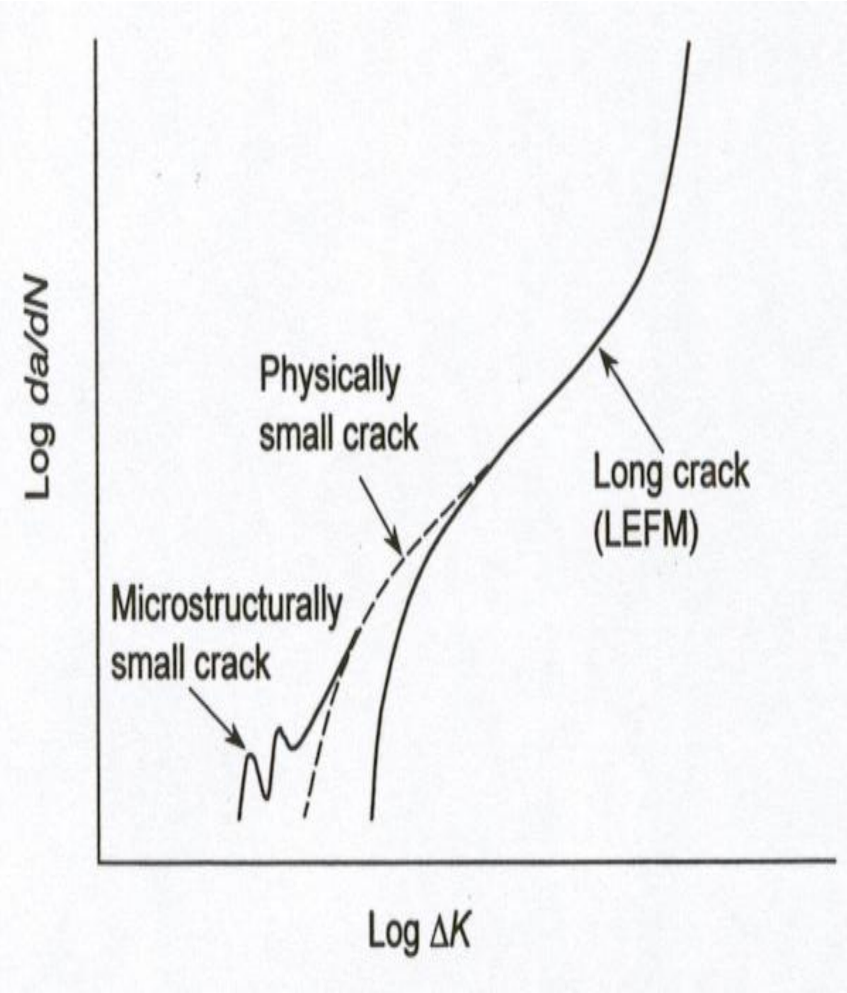
SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

- In general, most of the fatigue life of a crack occurs when the crack is small.
- Current design philosophy using LEFM analysis for long cracks usually provides accurate fatigue life estimates for crack sizes that are typically greater than about 1 mm (0.04”)
- Life predictions based on da/dN vs. ΔK curve, when extended to small crack behavior may lead to non-conservative estimates.
- This is of very importance for safety critical components, such as gas turbine engine discs and blades, where fatigue life is typically dominated by microstructurally and/or physically small crack growth and critical crack sizes can be very small because of high stress levels.

SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

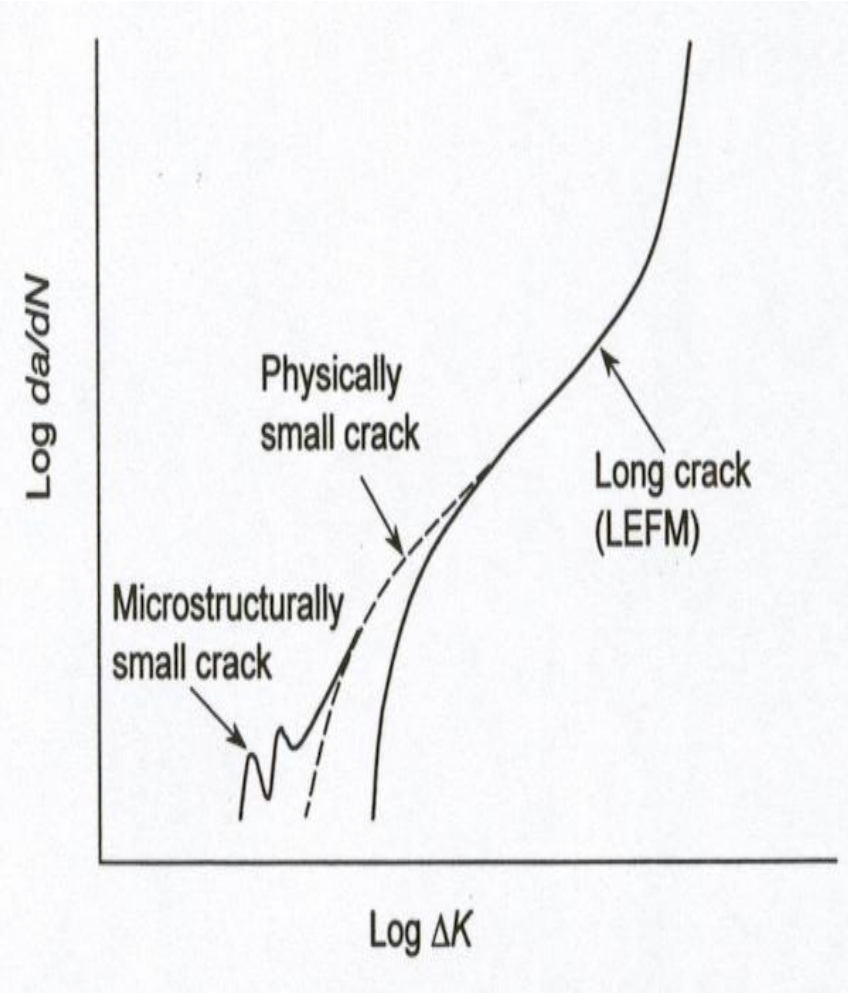
- **Microstructurally small cracks** are characterized as such because they can be influenced significantly by the microstructure.

- As the schematic shows, fatigue crack growth of a microstructurally small crack is usually **discontinuous** in nature, with accelerations and decelerations in crack growth rate during growth of the small crack.



SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

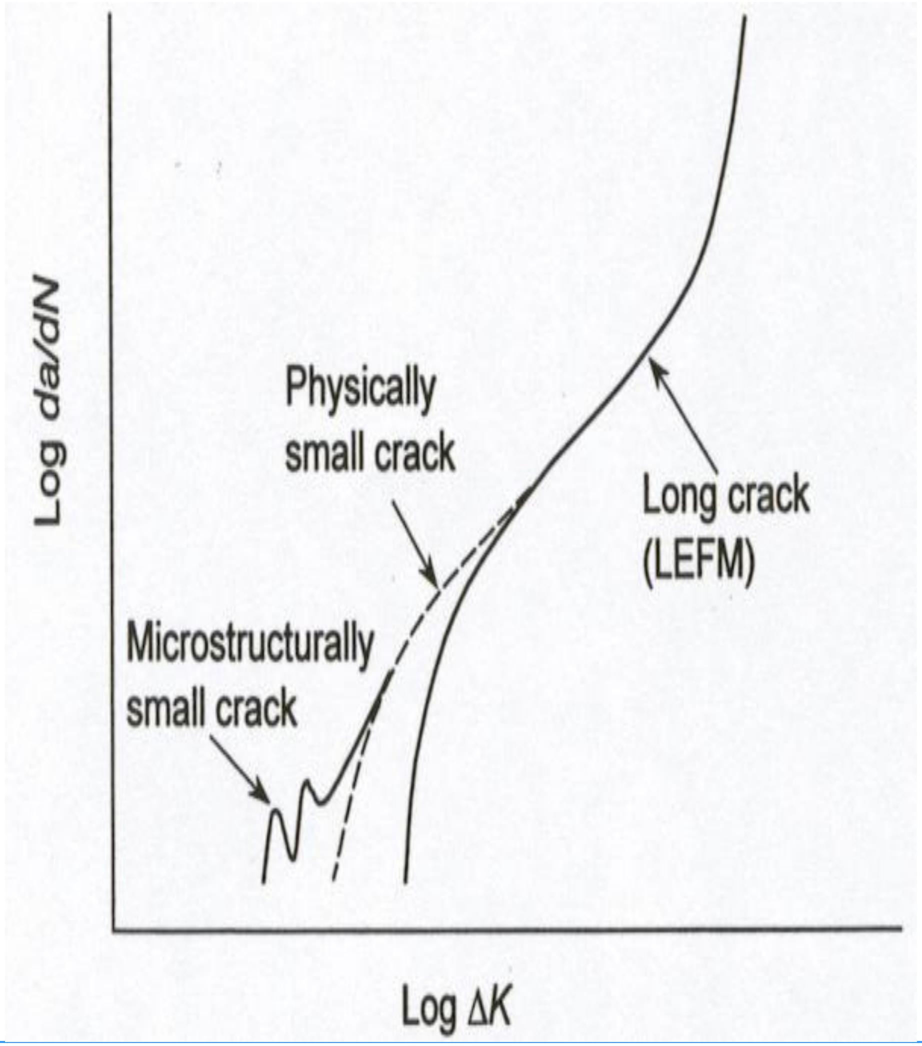
- This is associated with the interaction between the crack tip and microstructural barriers of the material such as grain boundaries.
  - As a small crack grows within a grain, the crack growth rate is typically higher than that observed for the general “long crack” da/dN-ΔK curve.
  - As the crack grows and approaches a grain boundary, the crack can retard, causing the crack growth rate to decrease.



SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

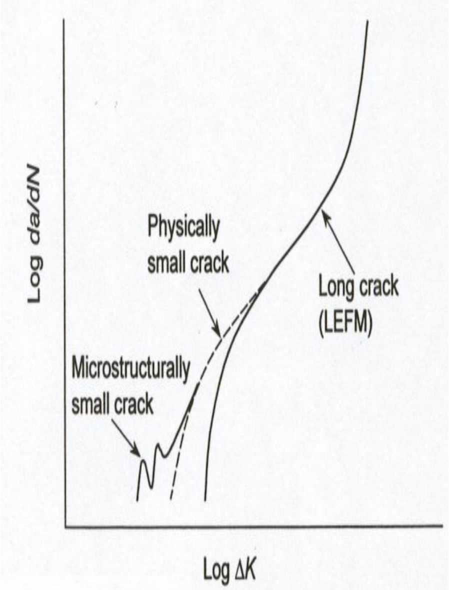
- Once the small crack grows to a size that is several times larger than the grain size, typically around ten grain sizes, the microscopically small crack curve generally merges with the physically small and then long crack curve as shown.

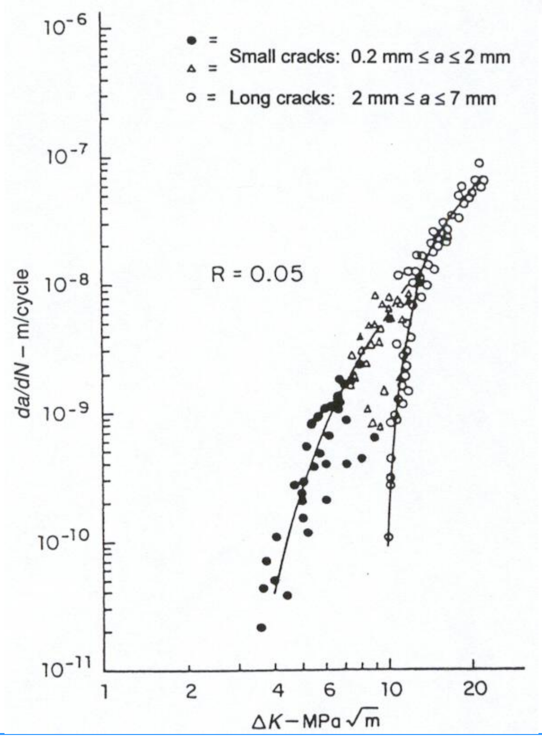
- The growth of microstructurally small cracks typically violates the basis of continuum mechanics and LEFM. The use of probabilistic and statistical methods, and/or damage mechanics, appears to be potential solutions for better characterizing the fatigue crack growth of microstructurally small cracks.



- **Physically small cracks**, typically less than about 1 or 2 mm, are such that in many cases they do not necessarily violate LEFM limitations based on stress level or near-tip plastic zone size, yet still propagate at rates faster than long cracks subjected to the same nominal crack tip driving force.

- The general fatigue crack growth behavior of physically small cracks is shown in the schematic of Fig. 6.21 and in Fig. 6.22.





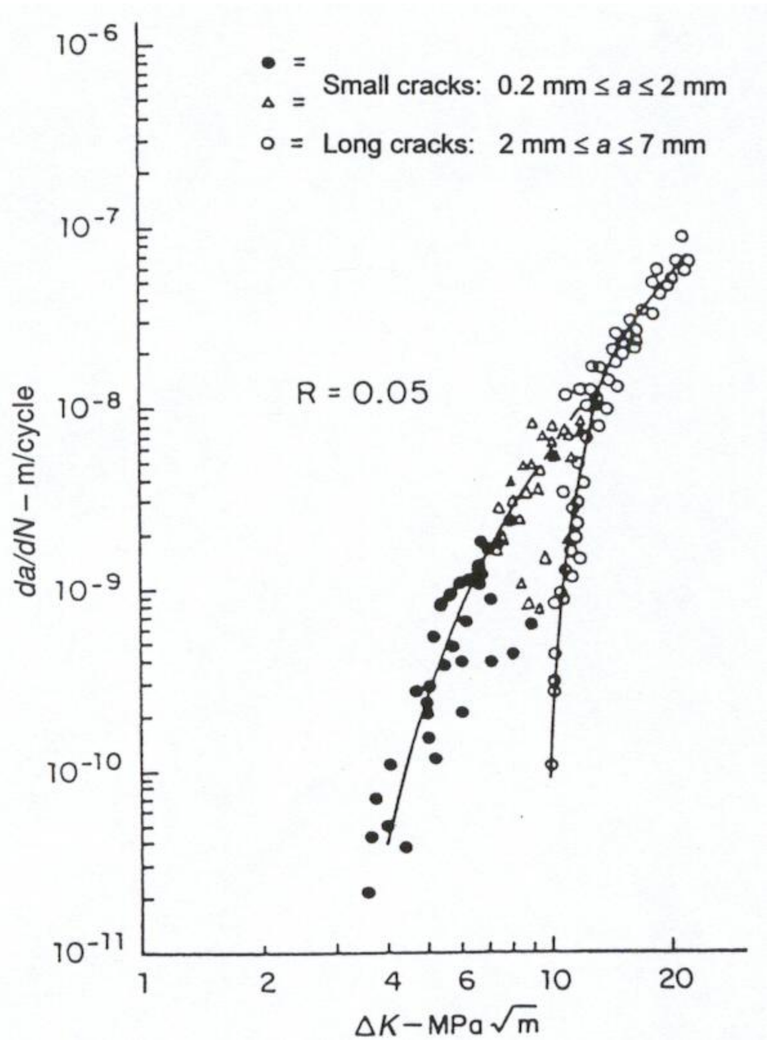
SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

- Crack growth behavior of physically small cracks is similar to the general long crack region I fatigue crack growth behavior except the growth rates are higher and ΔK_{th} is lower.

- Crack closure can contribute significantly to the effective ΔK level. The anomalous behavior of physically small cracks is associated with the lack of premature contact between the crack faces behind the crack tip as it advances, opposite of that typically observed at low stress ratio for long crack behavior.

- The crack growth behavior of physically small cracks is similar to the behavior of long cracks at higher stress ratio.

- In regions II and III, the crack growth rate increases rapidly thus a physically small crack operating in regions II or III becomes a "long crack" very quickly.

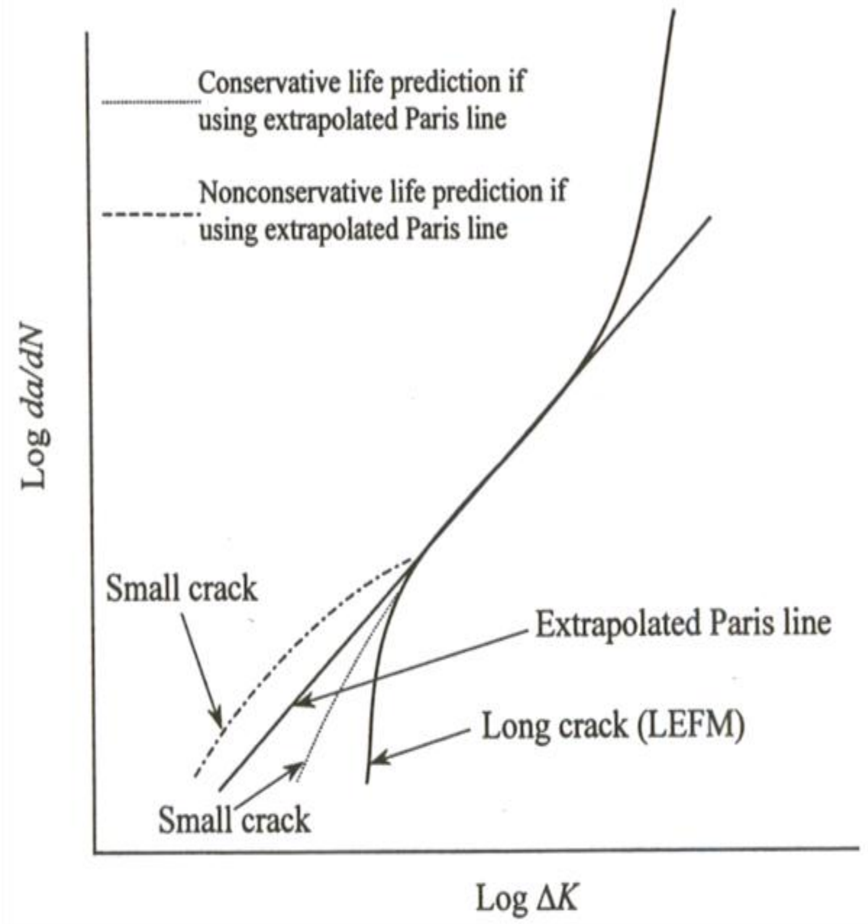


SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

- A **simple approximation** for this type of small crack growth behavior is extrapolation of the Paris equation to region I type growth.

- Although not an accurate fit to most long crack growth data in region I, extrapolation of the Paris equation may provide better life predictions by taking into account much of the non-conservative behavior associated with small crack growth.

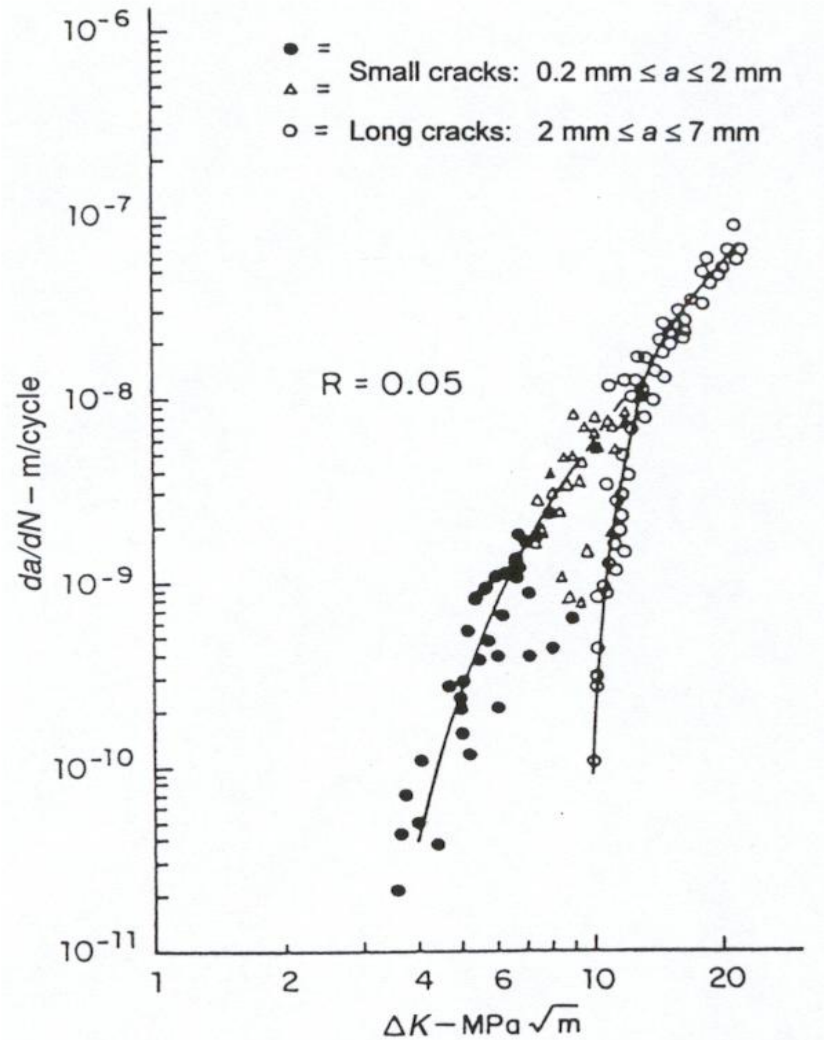
- However, one must be careful when performing this operation, as extrapolation could result in either conservative or non-conservative behavior depending on the physically small crack behavior observed.



SMALL FATIGUE CRACKS AND LEFM LIMITATIONS

- Examination of the data in Fig. 6.22 reveals the physically small crack data to fall below extrapolation of the Paris line into region I.

- Thus for this material, conservative life predictions would prevail using this simple approximation.



PLASTICITY EXTENSION OF LEFM

- Linear elastic fracture mechanics (LEFM) was originally developed for mostly elastic conditions.

- LEFM concepts can be used where **limited plasticity** exists, but only in the crack tip region.

- To account for **limited crack tip plasticity**, the crack length is enlarged slightly from *a* to $\boldsymbol{a + r_y}$. The stress intensity factor equation is then modified to:

$$K_{eff} = S\sqrt{a + r_y}\, f\left(\frac{a + r_y}{w}\right)$$

which is simply a modification of $S\sqrt{\pi a}\, f\left(\frac{a}{w}\right)$ taking into account a plasticity correction.

PLASTICITY EXTENSION OF LEFM

- $\mathbf{K_{eff}}$ defined here is not the same as K_{eff} defined regarding crack closure.
- Since calculation of r_y requires the value of K_{eff}, a **trial and error procedure** is often necessary to obtain K_{eff}, unless f [(a + r_y)/w] does not change significantly for the new crack length (a + r_y).
- The plastic zone correction is typically small when $S \ll S_y$, but increases appreciably when the applied stress, S, exceeds approximately $0.8S_y$.
- Therefore, conditions that exhibit appreciable plasticity, i.e. when a plasticity correction of about 20% is calculated, violate the basic assumptions of LEFM. This is typical for metals with low strength and high ductility at fracture and may occur with fatigue. Thus, the evolution of elastic-plastic fracture mechanics (EPFM).

ELASTIC-PLASTIC FRACTURE MECHANICS

- The parameter used in EPFM is the **J-integral**.

- The crack opening displacement (**COD**) approach, introduced prior to the introduction of the J-integral, is another technique used to characterize fracture and fatigue crack growth for conditions with excessive plasticity.

- Consideration of elastic-plastic fracture and nonlinear behavior is prominent in the **nuclear power industry**, specifically for pressure vessels and piping.
  - Metals used to fabricate these structures are usually steels with high ductility toughness, where fracture or FCG is accompanied by extensive plastic deformation.
  - Similar to the extensive use of K in the aerospace industry, because of the catastrophic consequences of failure in the nuclear power industry, much research has focused on the development of nonlinear (elastic-plastic) fracture mechanics methods to assess fracture risks in nuclear power plant structures and components.

- Elastic-plastic fracture concepts also are used in **elevated temperature applications**, offshore petrochemical rigs, structures, and components, as well as applications to safety critical components and structures in other industries.

SUMMARY

- Development of LEFM provides an understanding of the reduced strength of components in the presence of flaws or cracks.

- The stress intensity factor, K, is the governing parameter in LEFM and is a function of applied stress, crack length, and geometry.

- For various crack geometries and configurations K takes on the form of one of the following:

$$S\sqrt{a}\,Y \quad \text{OR} \quad S\sqrt{\pi a}\, f\left(\frac{a}{w}\right)$$

- Determination of fracture toughness, K_c or K_{Ic}, of metals provides a quantitative design criteria to prevent failure of cracked components.

SUMMARY

- Limitations on the use of LEFM due to plasticity have been established that
  - restrict the net (nominal) stress to less than approximately 80% of the yield strength, and
  - the plastic zone size be small relative to the crack length and local geometry.

- LEFM applications to fatigue crack growth and damage tolerant design have allowed life estimations and predictions to be made.

- The total life is more sensitive to initial crack size than fracture toughness, which illustrates the importance of minimizing initial crack or discontinuity size.

SUMMARY

- Empirical relationships describing fatigue crack growth developed by Paris, Forman, Walker, and others, are used to perform fatigue life calculations.

- Region II fatigue crack growth essentially shows a linear relationship between log da/dN and log ΔK which corresponds to the equation:

$$\frac{da}{dN} = A(\Delta K)^n = A(\Delta S)^n (\pi a)^{n/2} \alpha^n$$

- Resulting in:

$$N_f = \int_0^{N_f} dN = \frac{1}{A(\Delta S)^n (\pi)^{n/2}} \int_{a_i}^{a_f} \frac{da}{\alpha^n a^{n/2}}$$

SUMMARY

- The threshold stress intensity range, ΔK_{th}, defines a lower limit of crack growth rate, typically around 10^{-10} m/cycle below which cracks are assumed to be non-propagating.
- Concepts such as the J-integral and EPFM, crack closure, and small crack growth have provided a great deal of stimulus to the engineering community during the past 25 years and more. Despite these, LEFM is still applicable to many fatigue situations.

DOS AND DON'TS IN DESIGN

- Do recognize that the presence of cracks or crack-like manufacturing and metallurgical discontinuities can significantly reduce the strength of a component or structure.

- LEFM can aid both qualitatively and quantitatively in estimating static strength as well as fatigue crack growth life and final fracture.

- Do consider that fracture toughness depends much more on metallurgical discontinuities and impurities than does ultimate or yield strength. Low impurity alloys have better fracture toughness.

- Don't expect doubling thickness or doubling ultimate strength of a component to double the fracture load. Cracks can exist and fracture toughness may drop appreciably with both thickness and ultimate strength increases.

- Do recognize the importance of distinguishing between plane stress and plane strain in fracture mechanics analysis as fracture toughness, crack tip plasticity, and LEFM limitations can be significantly different for the two conditions.

DOS AND DON'TS IN DESIGN

- Don't neglect the importance of nondestructive flaw or crack inspection for both initial and periodic inspection periods.

- Do note that most fatigue crack growth usually occurs in mode I even under mixed-mode conditions, and hence the opening mode stress intensity factor range ΔK_I is often the predominant controlling factor in FCG.

- Do investigate the possibility of using LEFM principles in fatigue crack growth life predictions even in low strength materials; crack tip plasticity can be small even in low strength materials under fatigue conditions. If plasticity is large, EPFM may be required.

- Do consider the possibility of inspection before fracture. High fracture toughness materials may not provide appreciable increases in fatigue crack growth life, but they do permit longer cracks before fracture, which makes inspection and detection of cracks more reliable.