Fatigue and Fracture

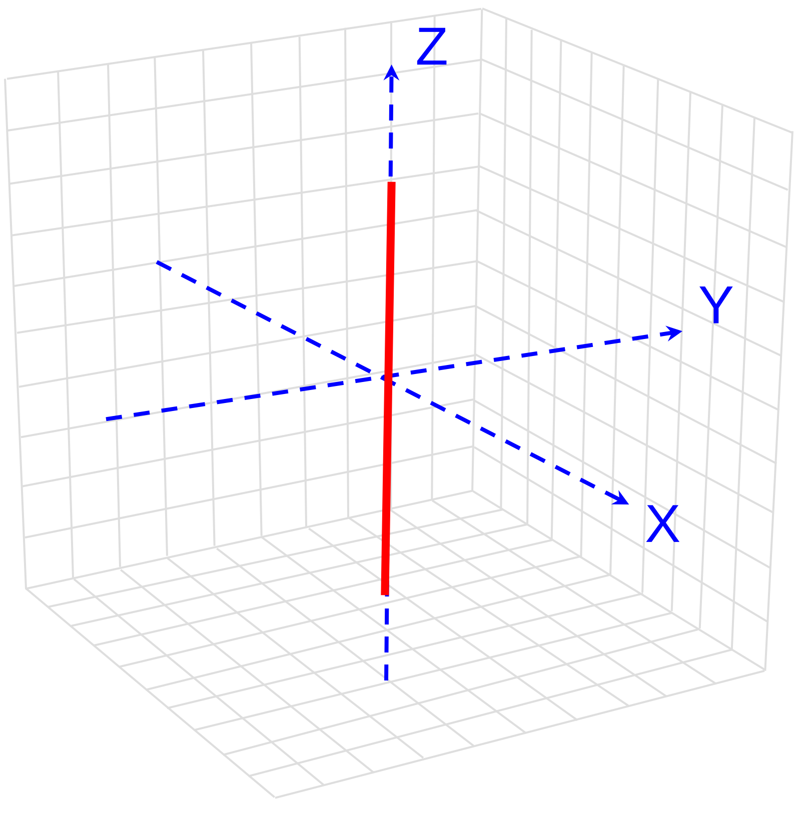
Multiaxial Fatigue

Professor Darrell F. Socie
Mechanical Science and Engineering
University of Illinois

When is Multiaxial Fatigue Important ?

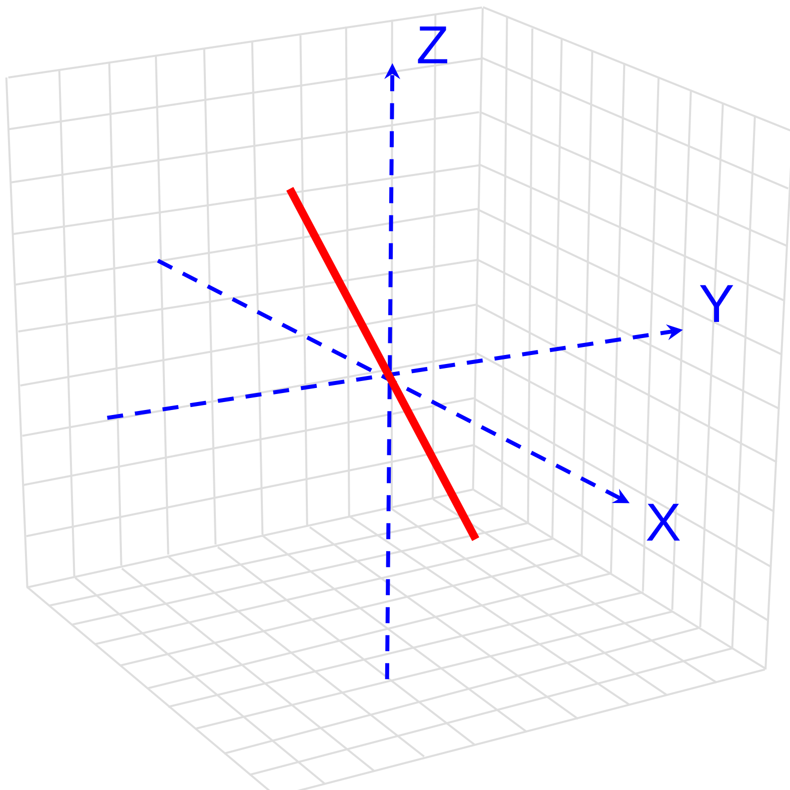
- Complex state of stress
- Complex out of phase loading

Uniaxial Stress



one principal stress
one direction

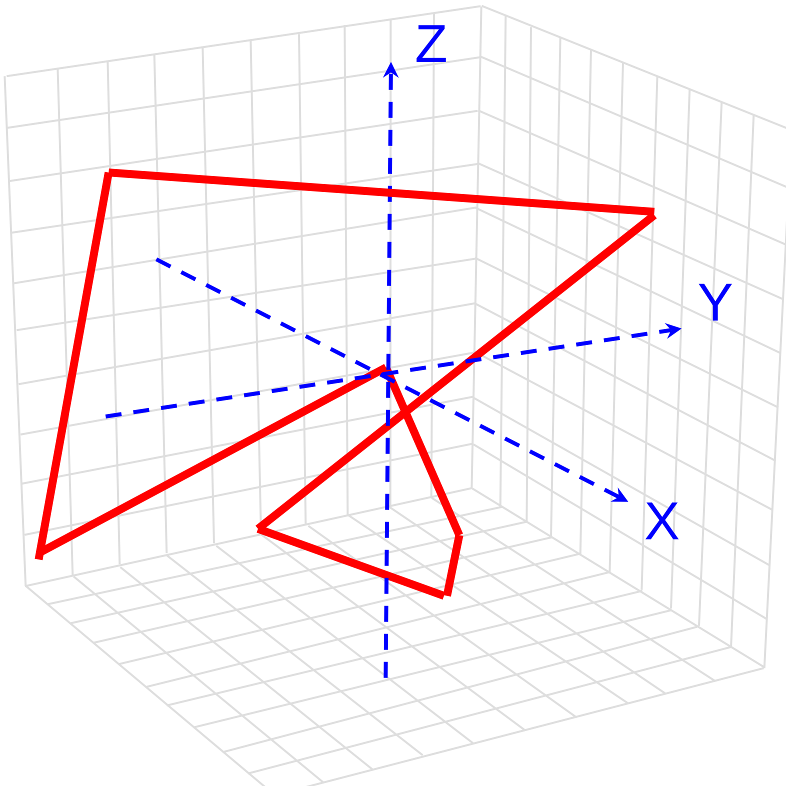
Proportional Biaxial



principal stresses vary proportionally but do not rotate

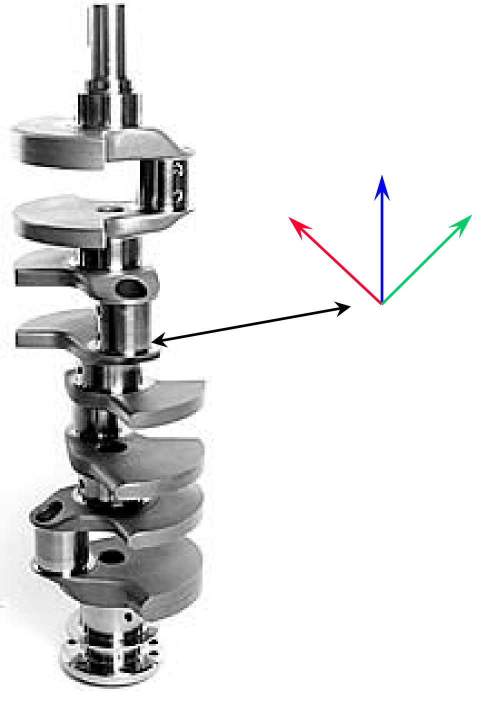
$$\sigma_1 = \alpha\sigma_2 = \beta\sigma_3$$

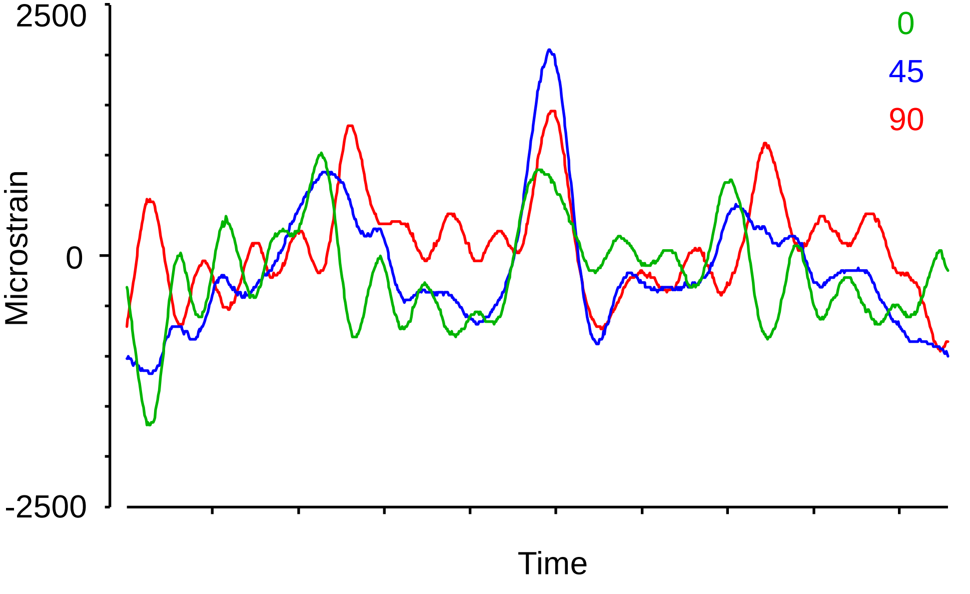
Nonproportional Multiaxial



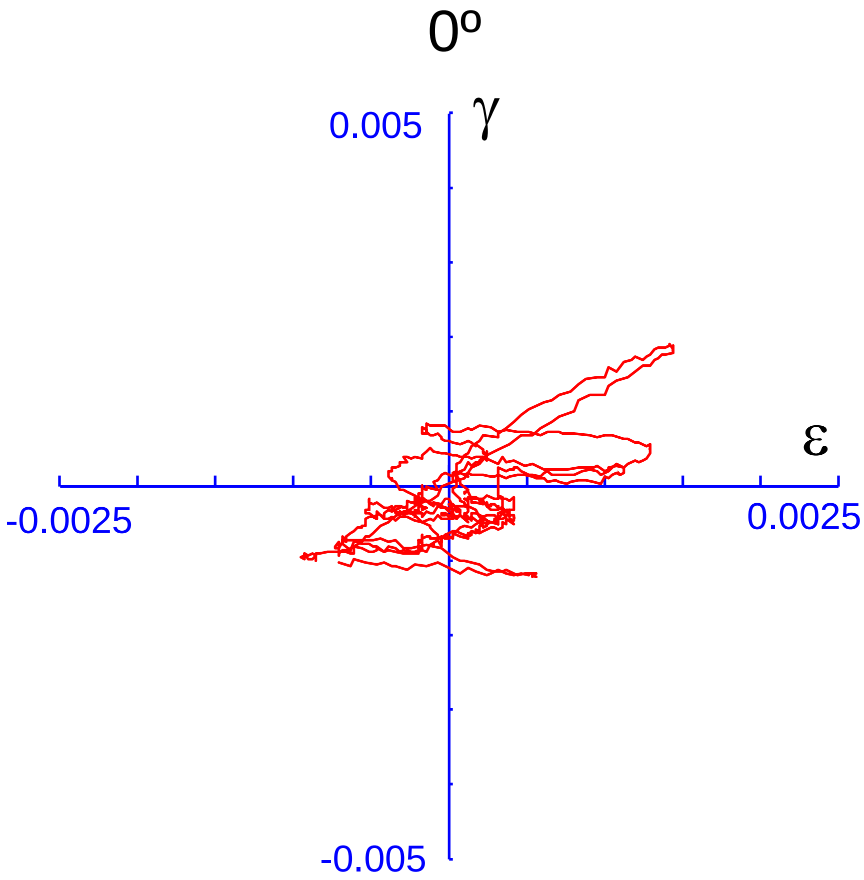
Principal stresses may vary nonproportionally and/or change direction

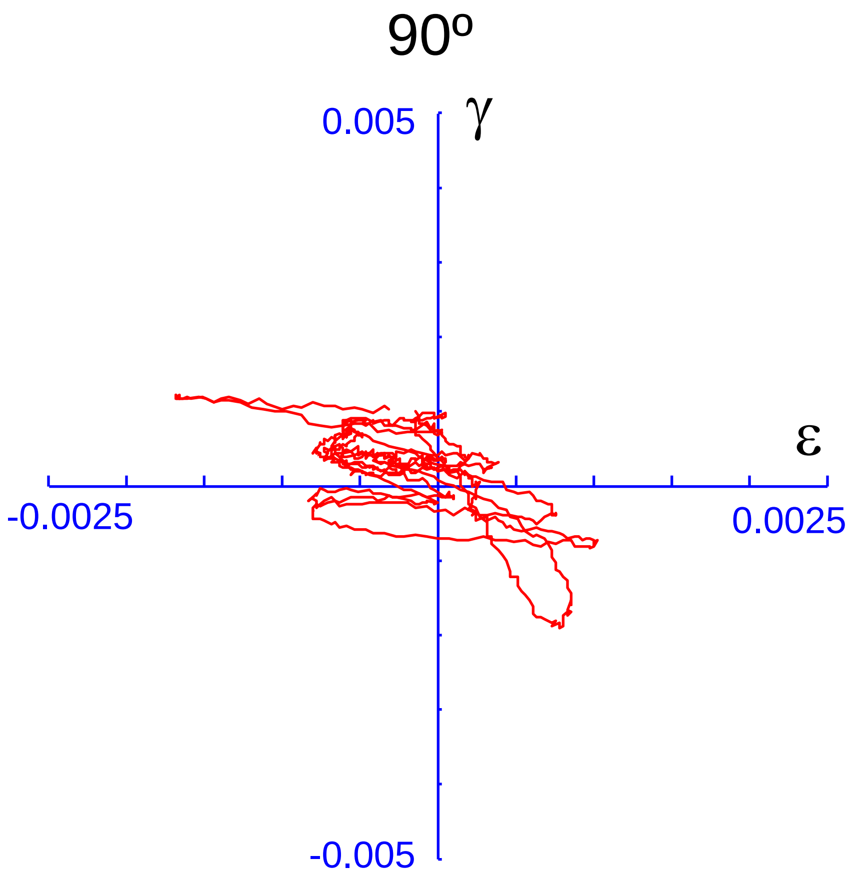
Crankshaft



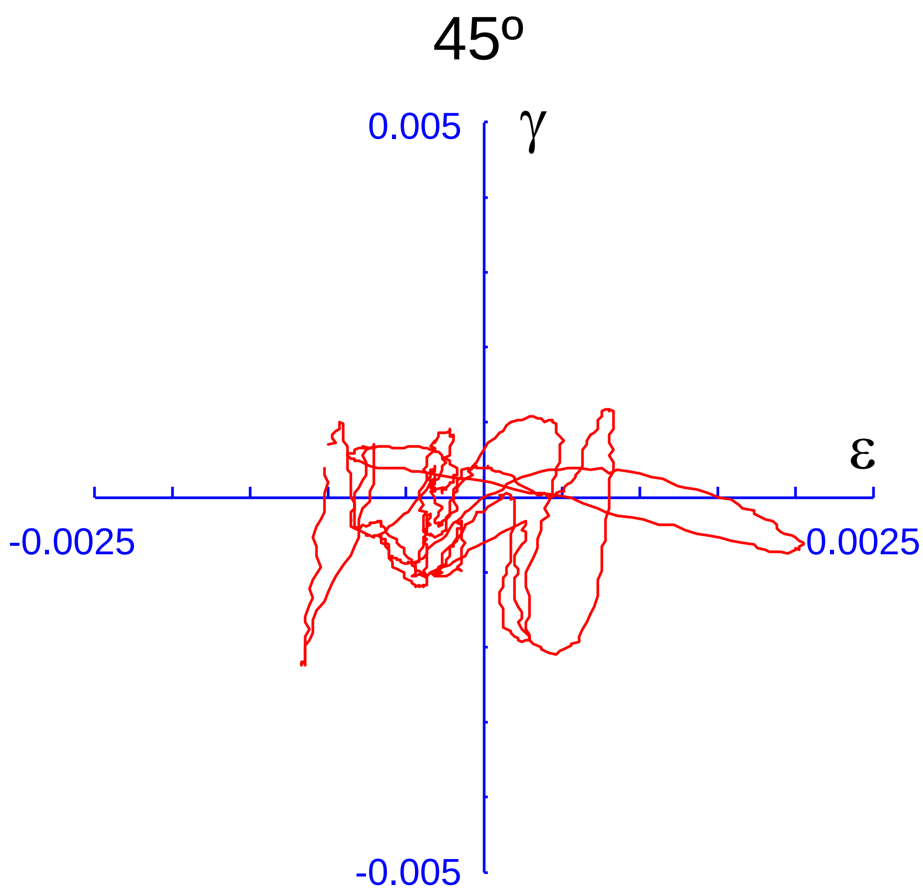


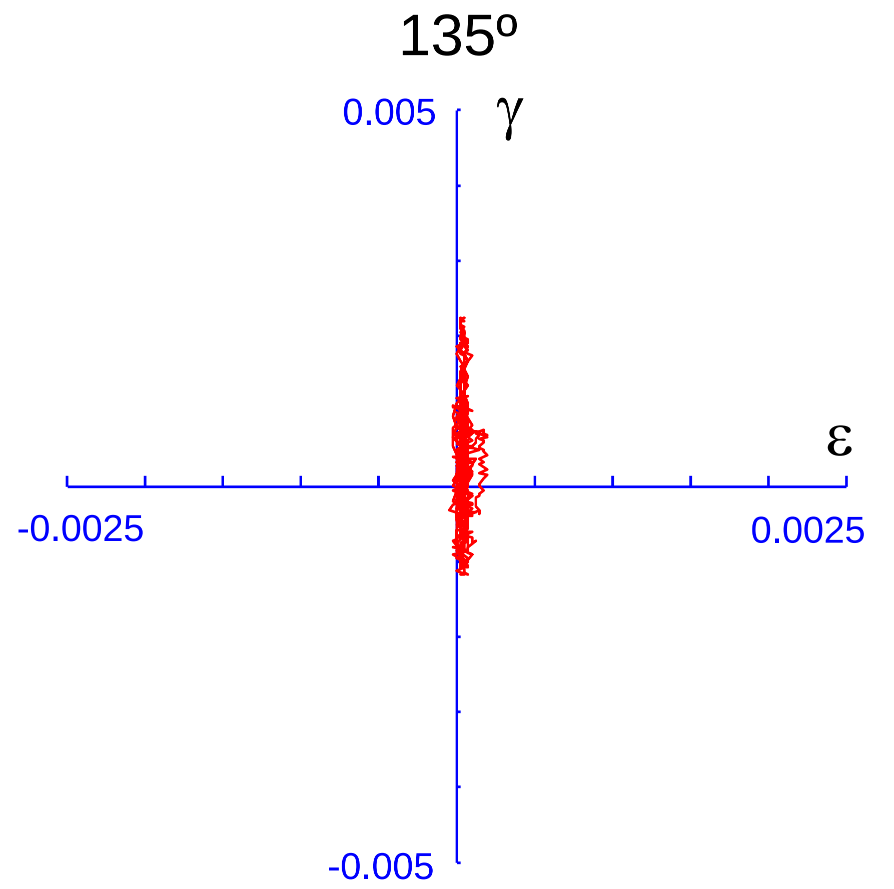
Shear and Normal Strains



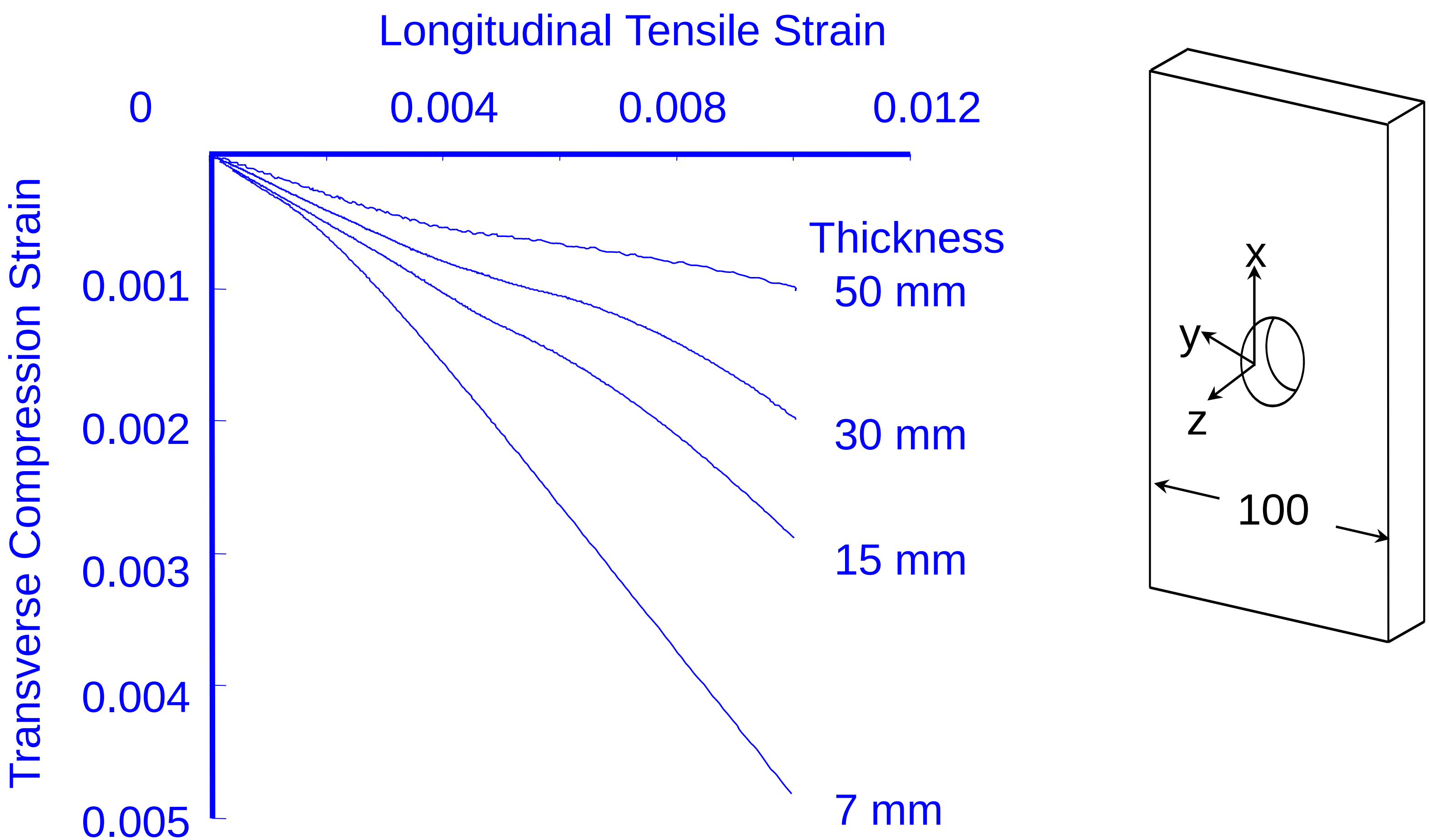


Shear and Normal Strains

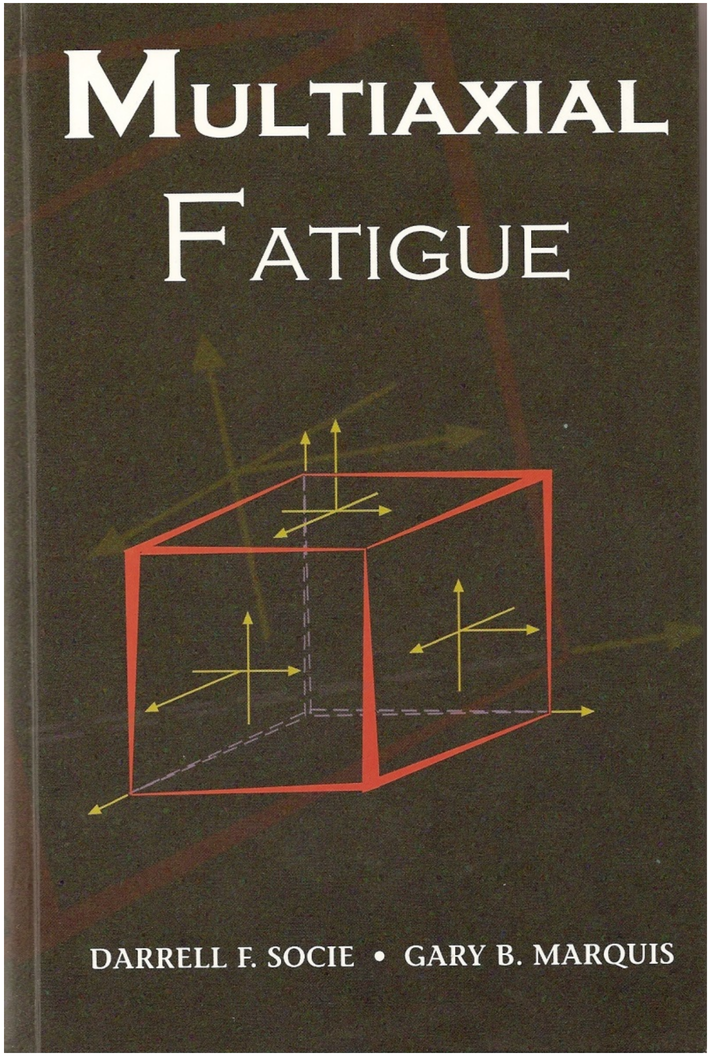




3D stresses


Longitudinal Tensile Strain
0
0.004
0.008
0.012
Transverse Compression Strain
0.001
0.002
0.003
0.004
0.005
Thickness
50 mm
30 mm
15 mm
7 mm
x
y
z
100

Book



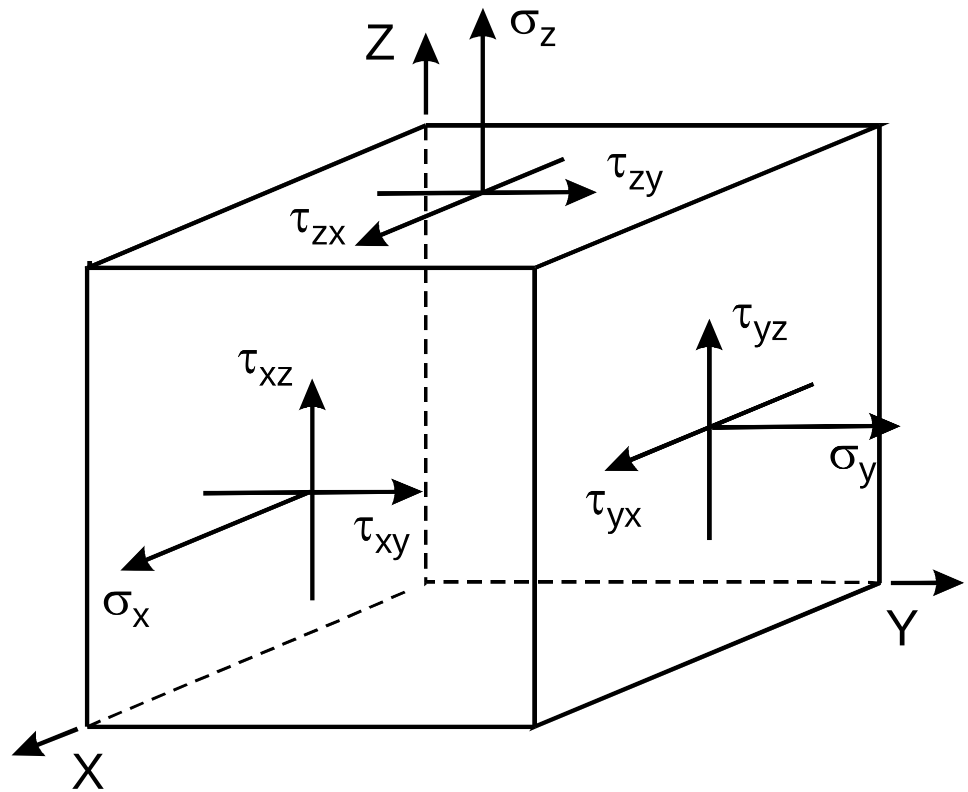
Outline

- **State of Stress**
- Stress-Strain Relationships
- Fatigue Mechanisms
- Multiaxial Testing
- Stress Based Models
- Strain Based Models
- Fracture Mechanics Models
- Nonproportional Loading
- Stress Concentrations

State of Stress

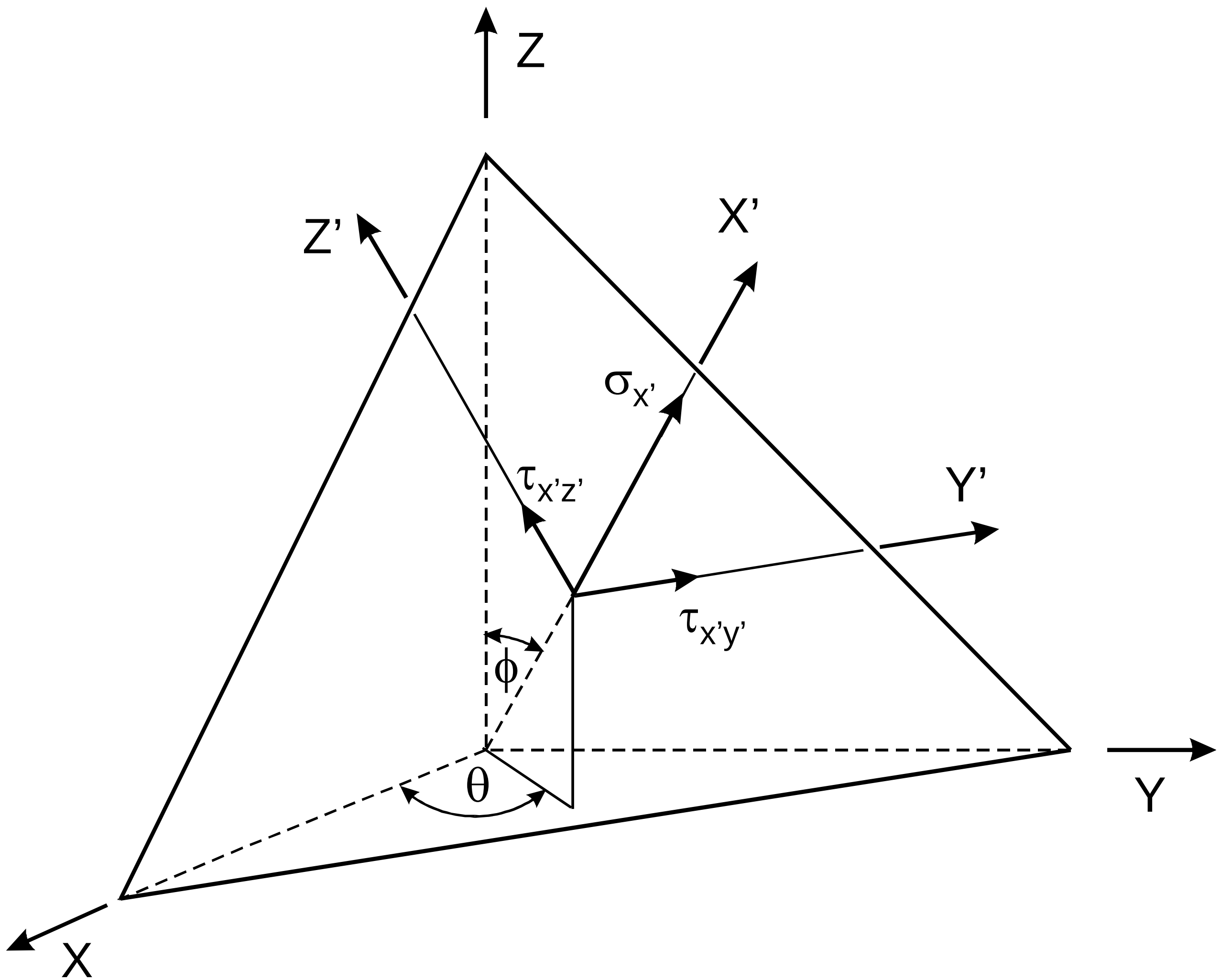
- Stress components
- Common states of stress
- Shear stresses

Stress Components

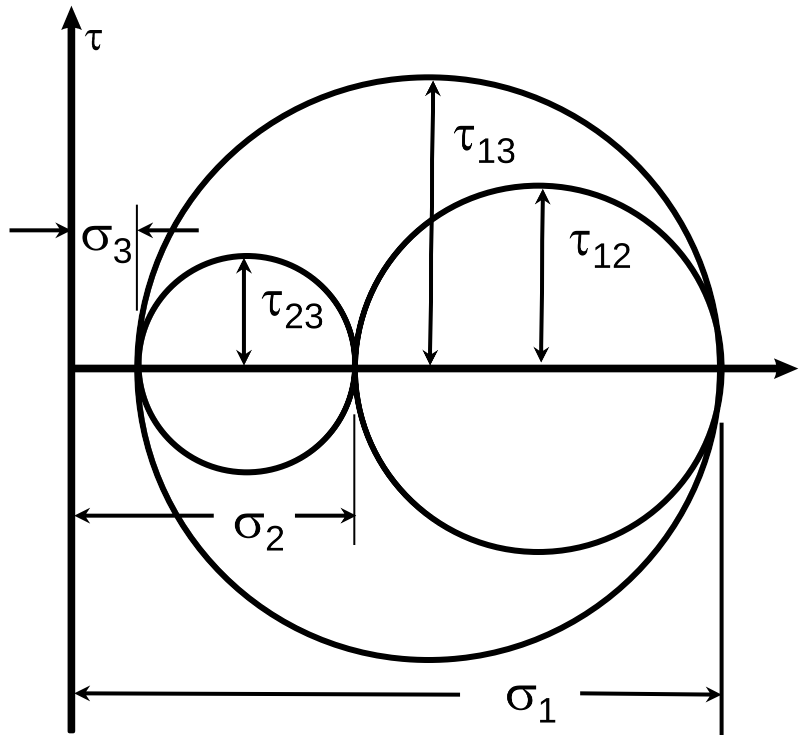


Six stresses and six strains

Stresses Acting on a Plane

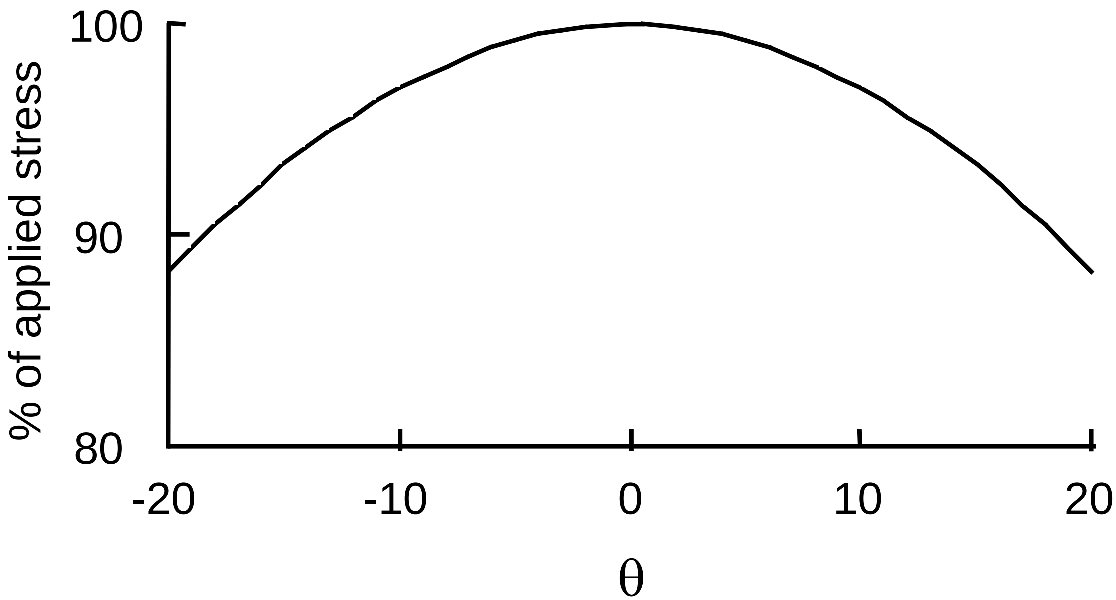


Principal Stresses



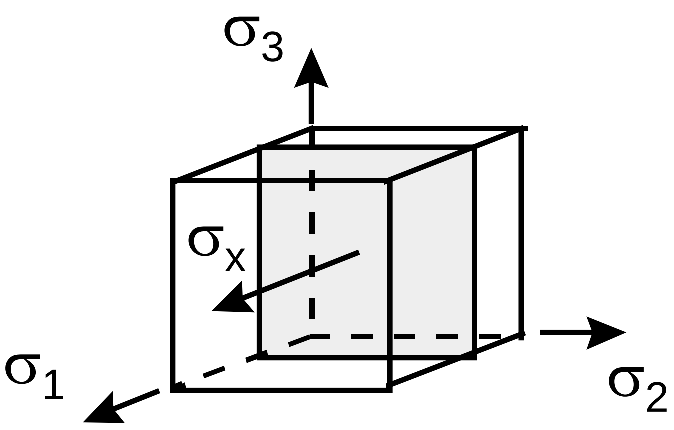
$$\sigma^3 - \sigma^2(\sigma_X + \sigma_Y + \sigma_Z) + \sigma(\sigma_X\sigma_Y + \sigma_Y\sigma_Z\sigma_X\sigma_Z - \tau^2_{XY} - \tau^2_{YZ} - \tau^2_{XZ})$$
$$- (\sigma_X\sigma_Y\sigma_Z + 2\tau_{XY}\tau_{YZ}\tau_{XZ} - \sigma_X\tau^2_{YZ} - \sigma_Y\tau^2_{ZX} - \sigma_Z\tau^2_{XY}) = 0$$

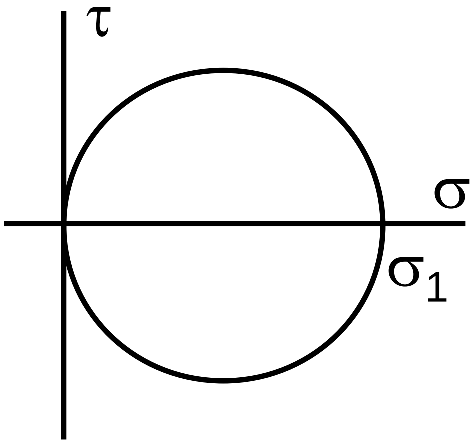
Stress and Strain Distributions



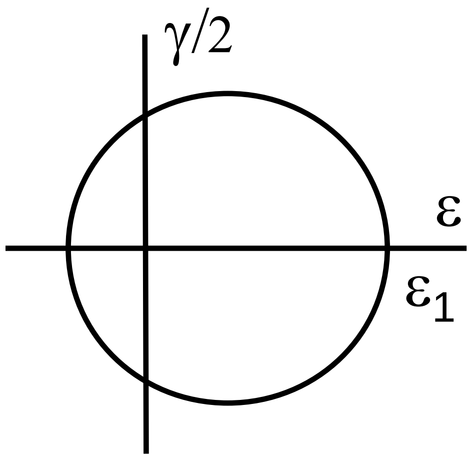
Stresses are nearly the same over a 10° range of angles

Tension



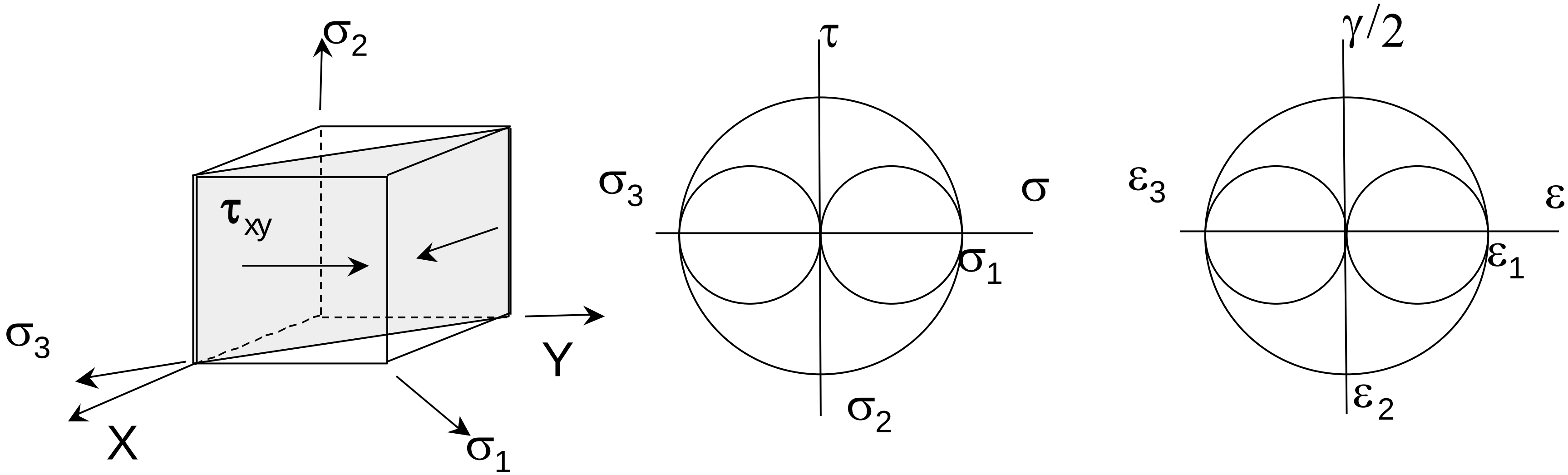


$$\sigma_2 = \sigma_3 = 0$$



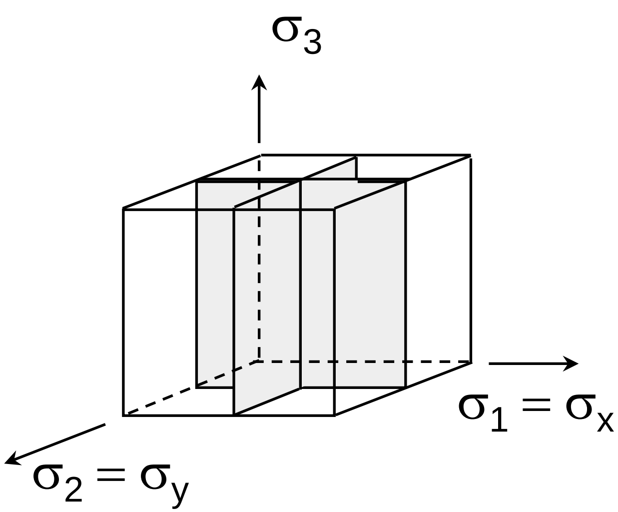
$$\varepsilon_2 = \varepsilon_3 = -\nu\varepsilon_1$$

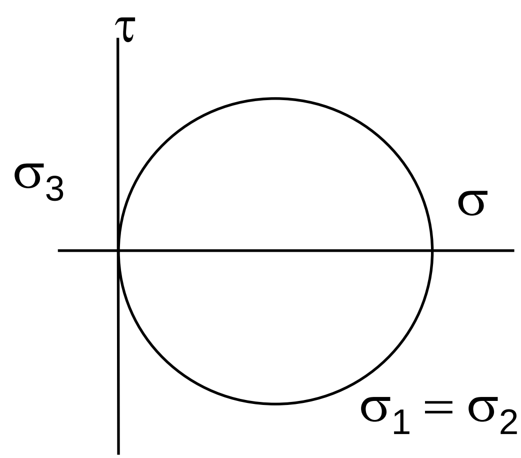
Torsion

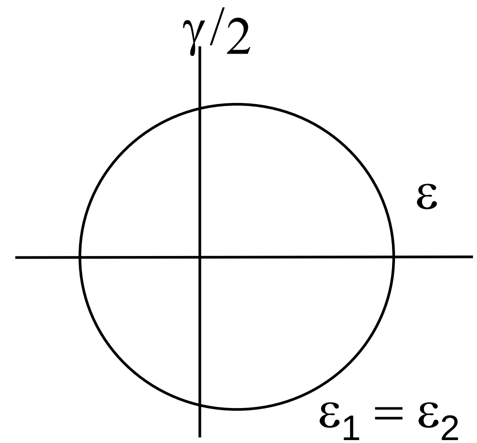


$\sigma_1 = \tau_{xy}$

Biaxial Tension

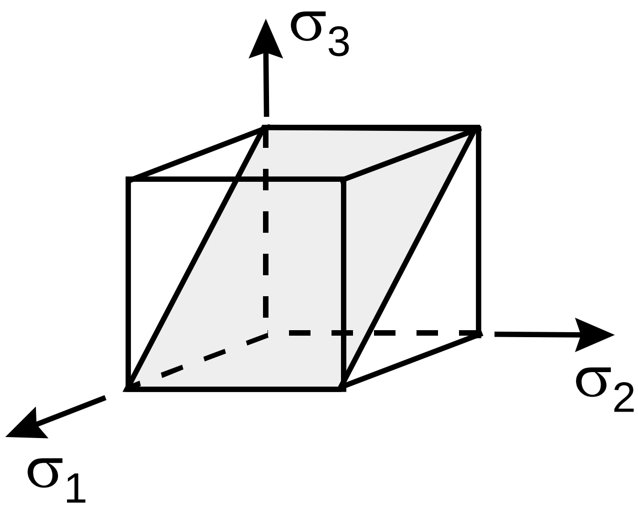


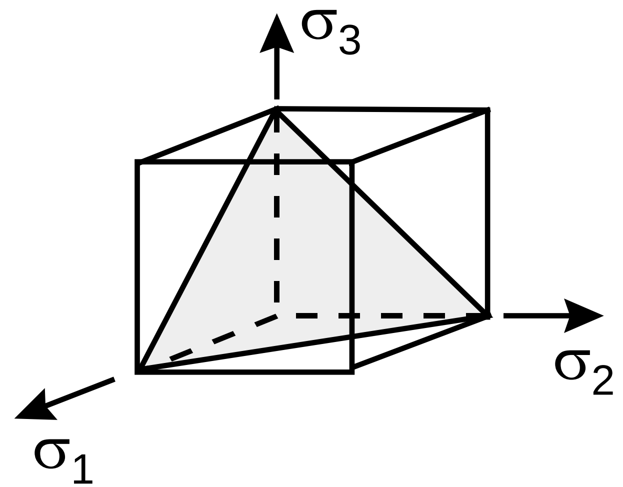




$$\varepsilon_3 = -\frac{2\nu}{1-\nu}\varepsilon$$

Shear Stresses





Maximum shear stress

Octahedral shear stress

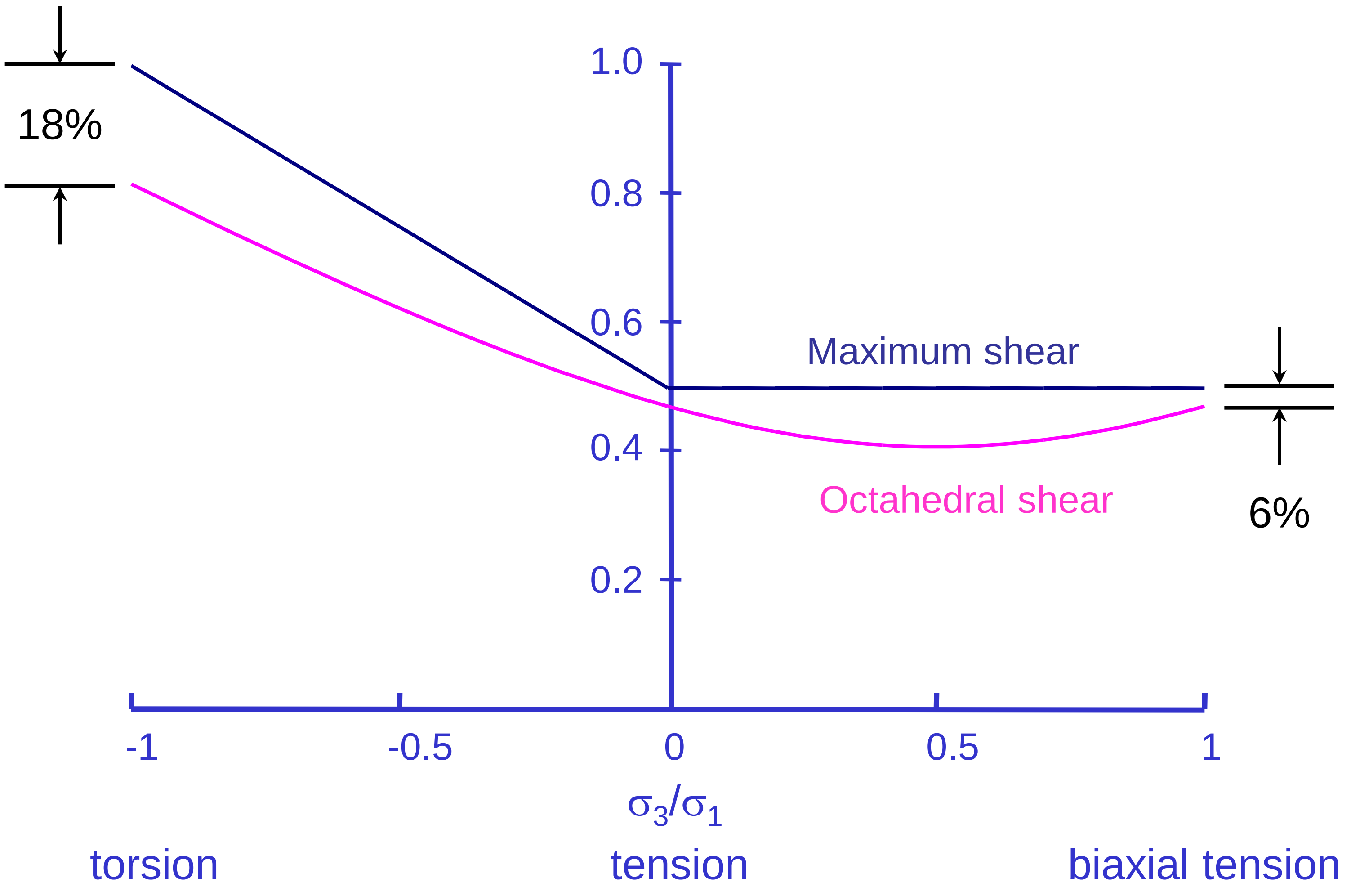
$$\tau_{13} = \frac{|\sigma_1 - \sigma_3|}{2}$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2}$$

Mises: $\bar{\sigma} = \frac{3}{\sqrt{2}}\tau_{oct}$

$$\tau_{oct} = \frac{3}{2\sqrt{2}}\tau_{13} = 0.94\,\tau_{13}$$

Maximum and Octahedral Shear



State of Stress Summary

- Stresses acting on a plane
- Principal stress
- Maximum shear stress
- Octahedral shear stress

Outline

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- **Fatigue Mechanisms**
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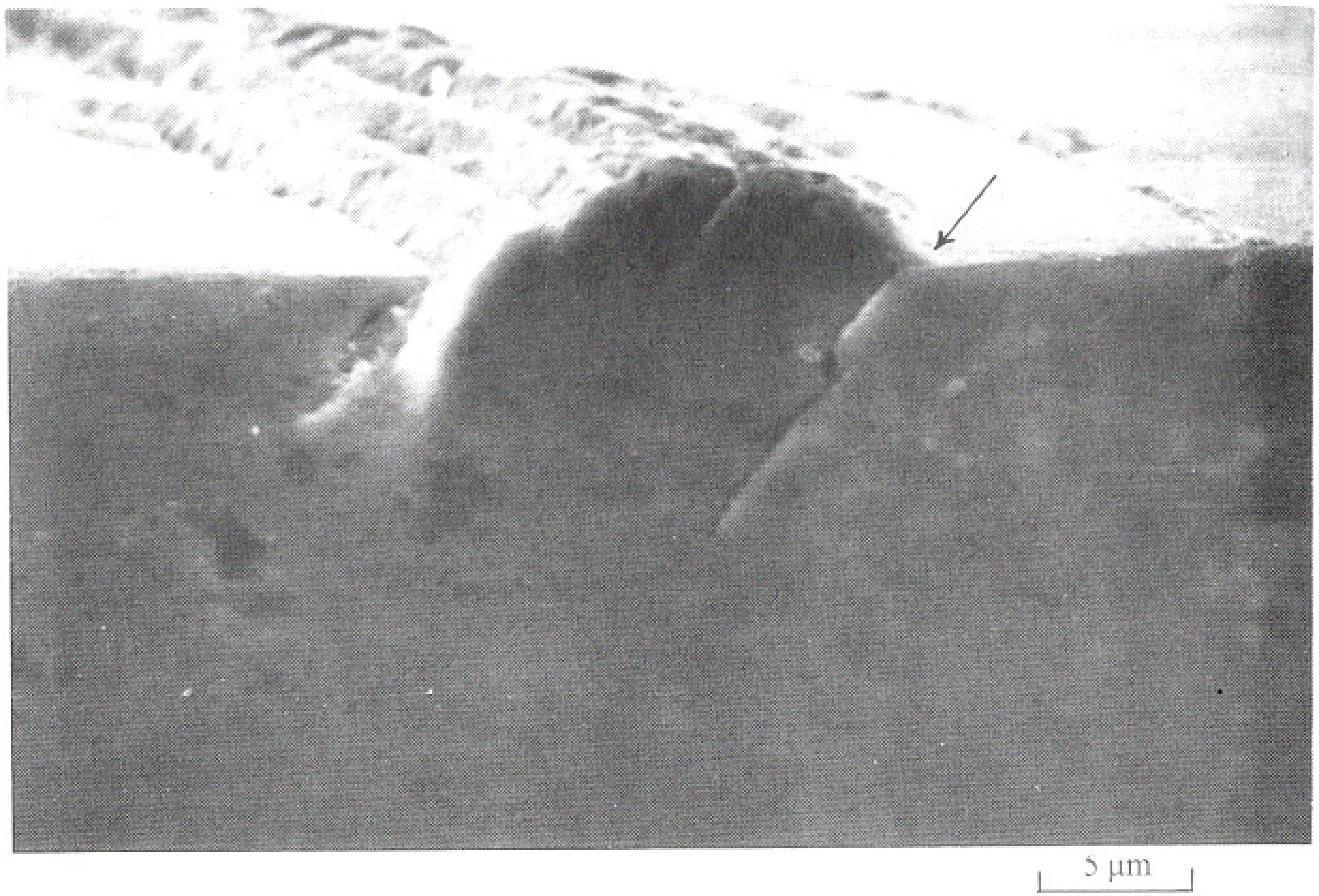
The Fatigue Process

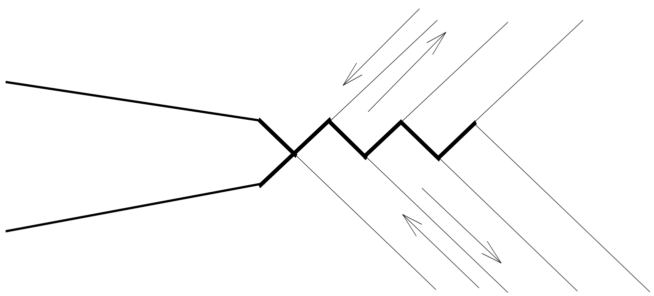
- Crack nucleation
- Small crack growth in an elastic-plastic stress field
- Macroscopic crack growth in a nominally elastic stress field
- Final fracture

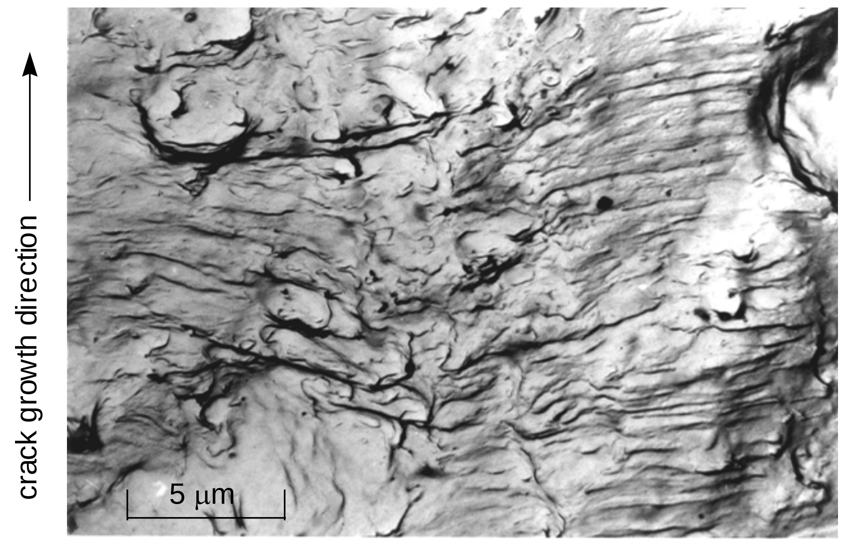
Slip Bands



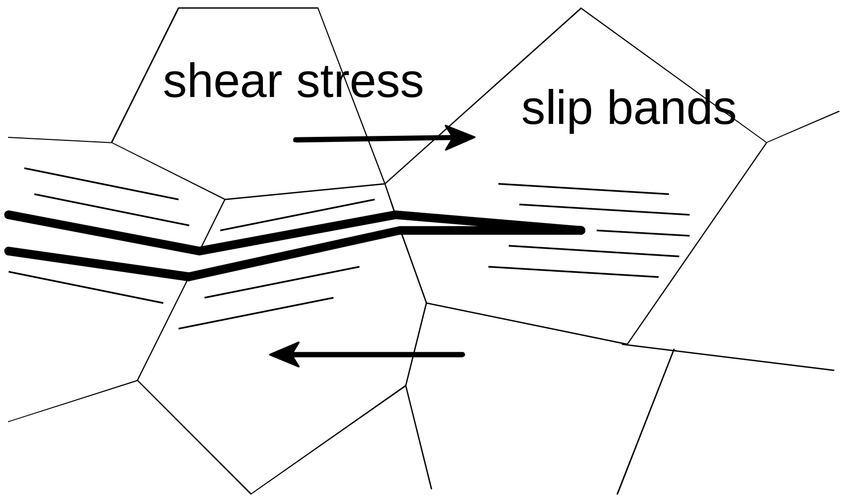
Ma, B-T and Laird C. "Overview of fatigue behavior in copper sinle crystals –II Population, size, distribution and growth Kinetics of stage I cracks for tests at constant strain amplitude", Acta Metallurgica, Vol 37, 1989, 337-348

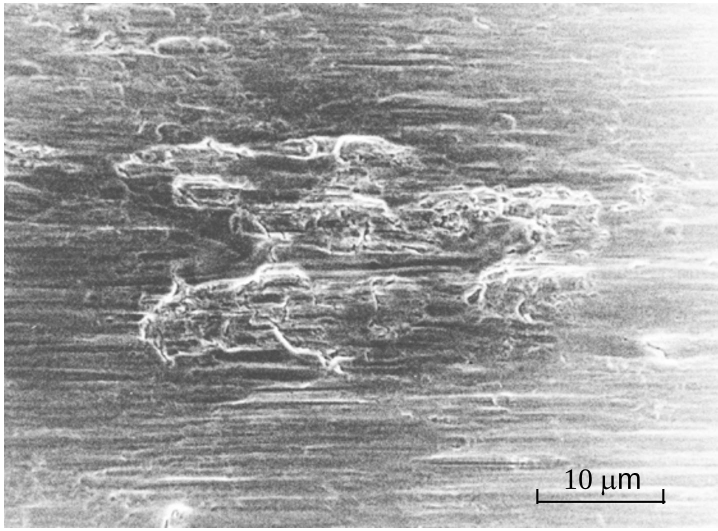
Mode I Growth





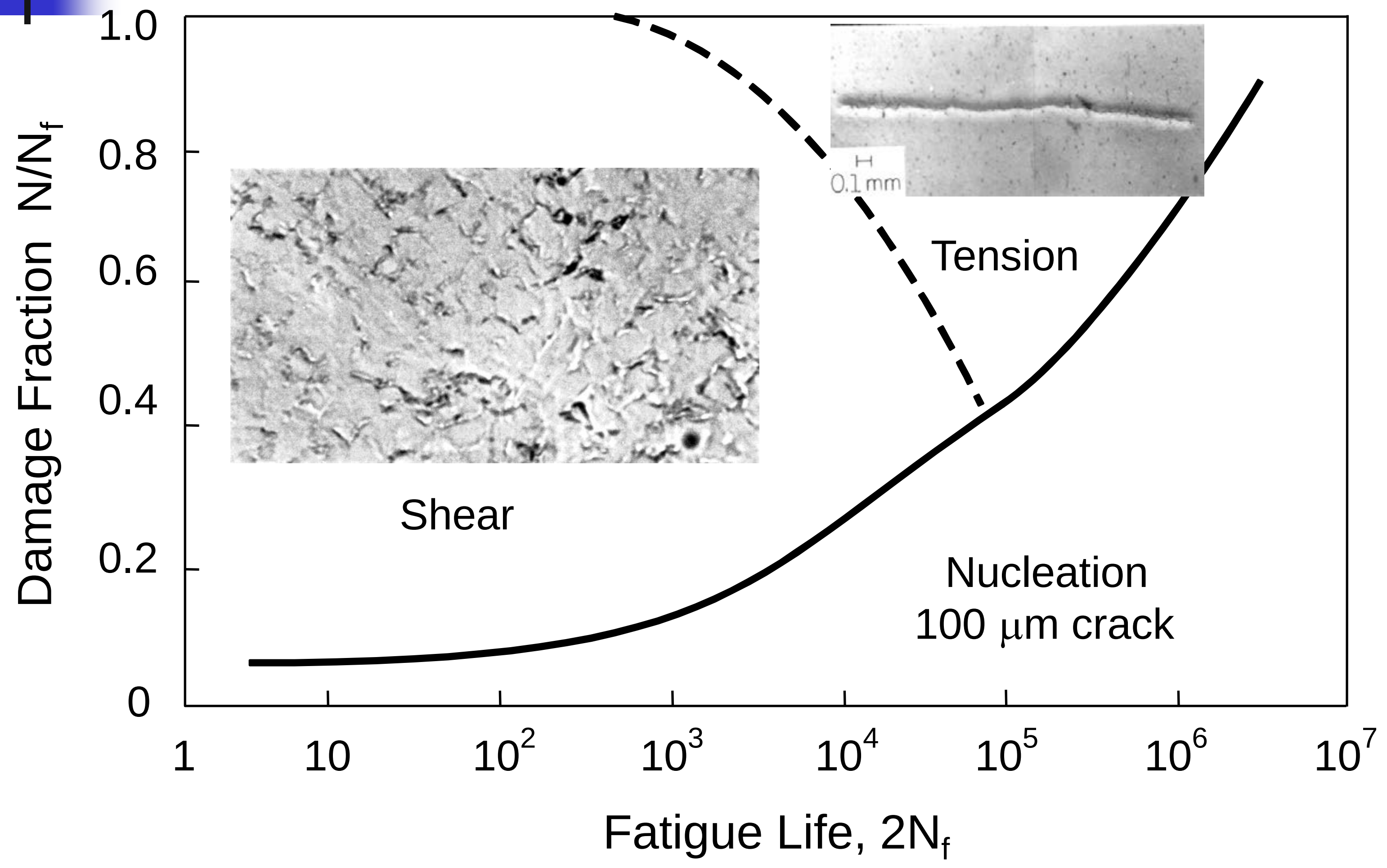
Mode II Growth



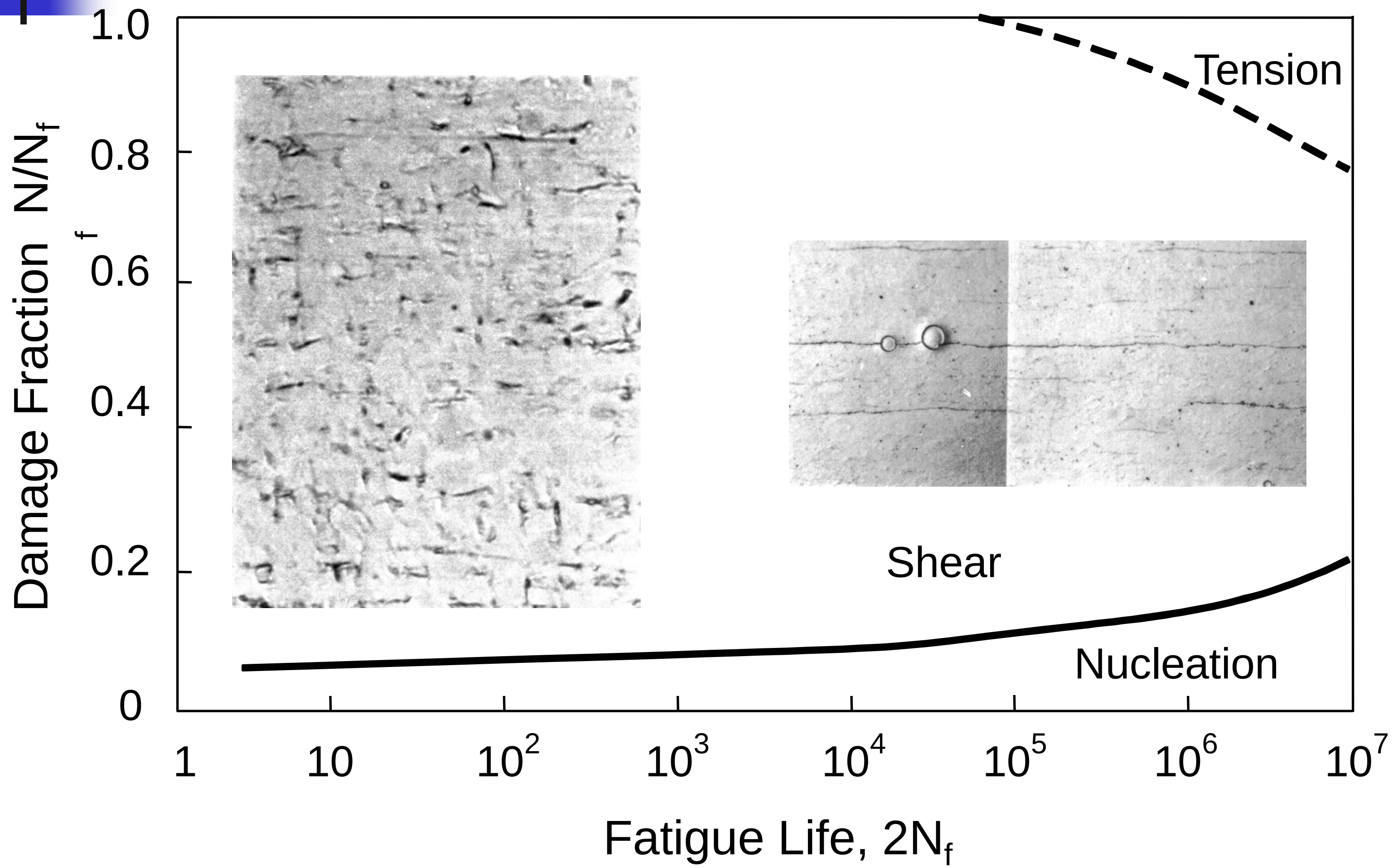


crack growth direction

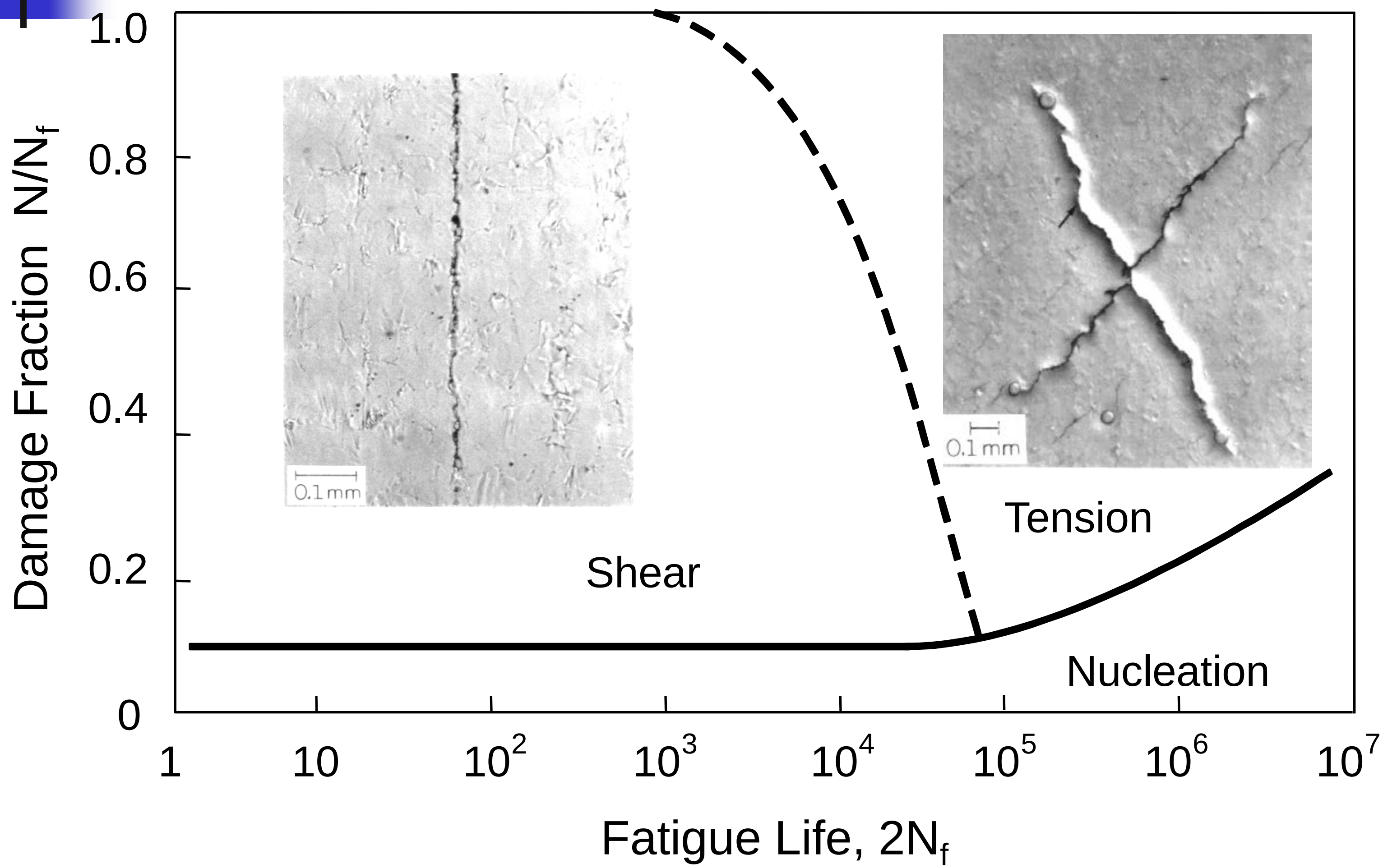
1045 Steel - Tension



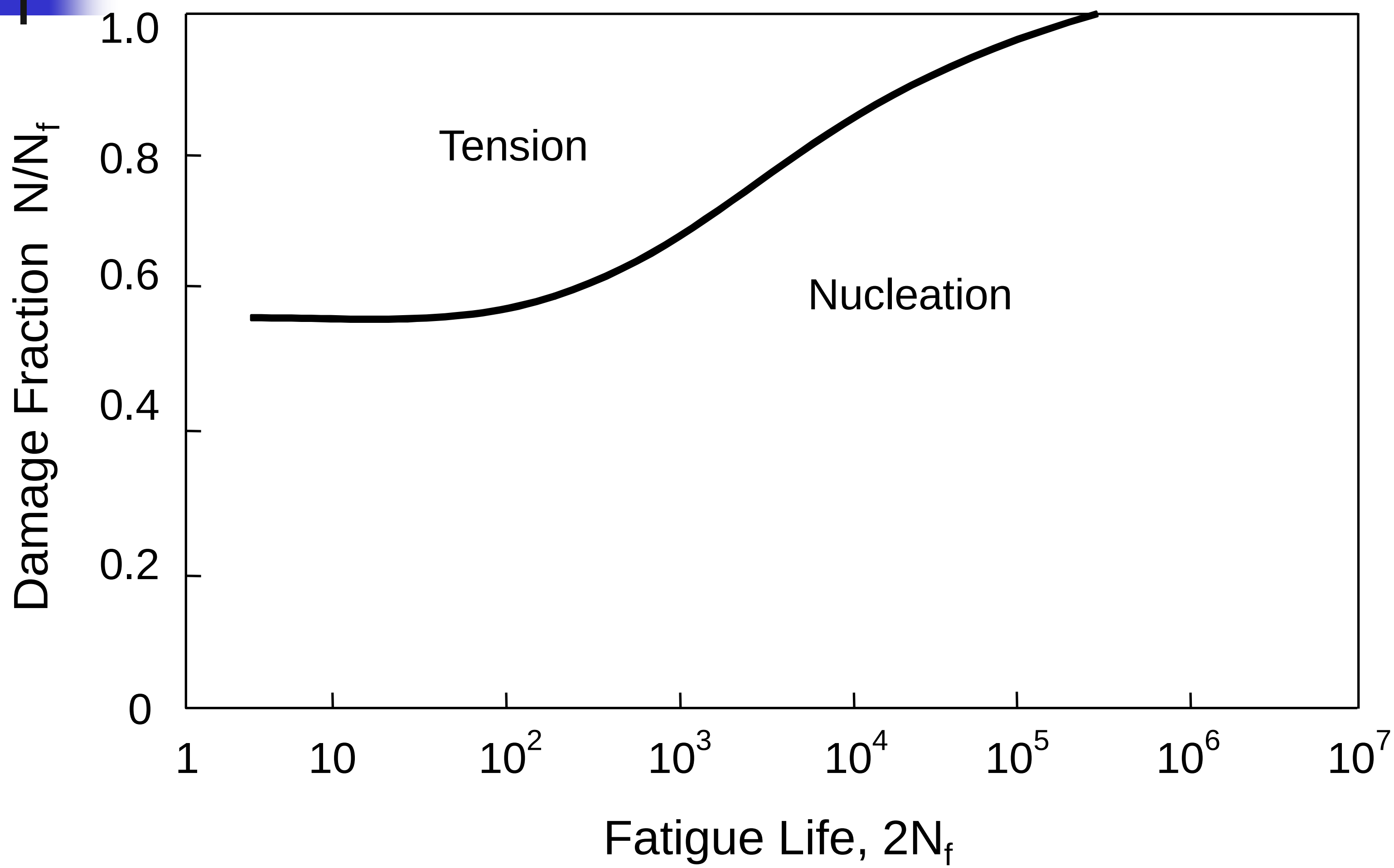
1045 Steel - Torsion



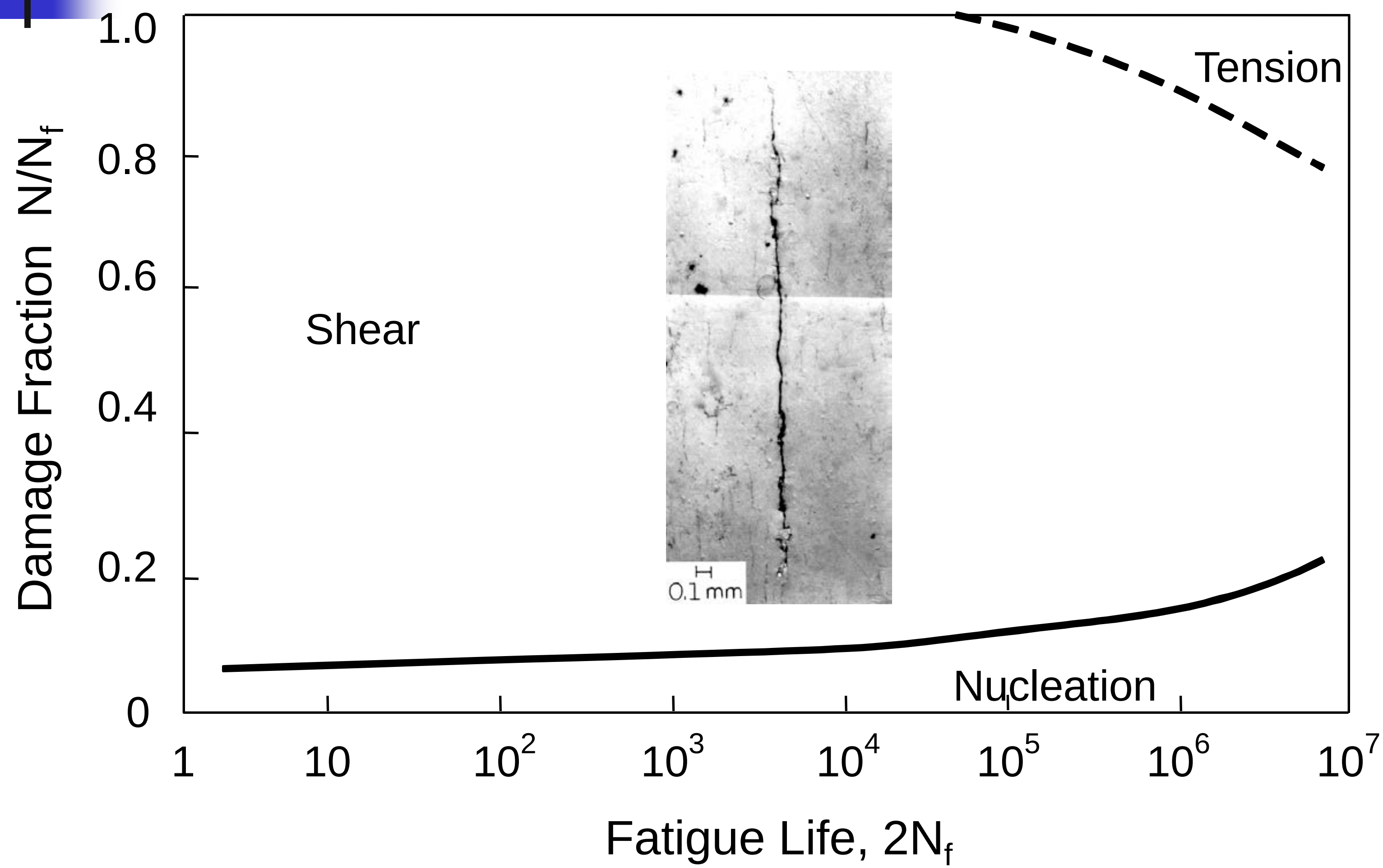
304 Stainless Steel - Torsion



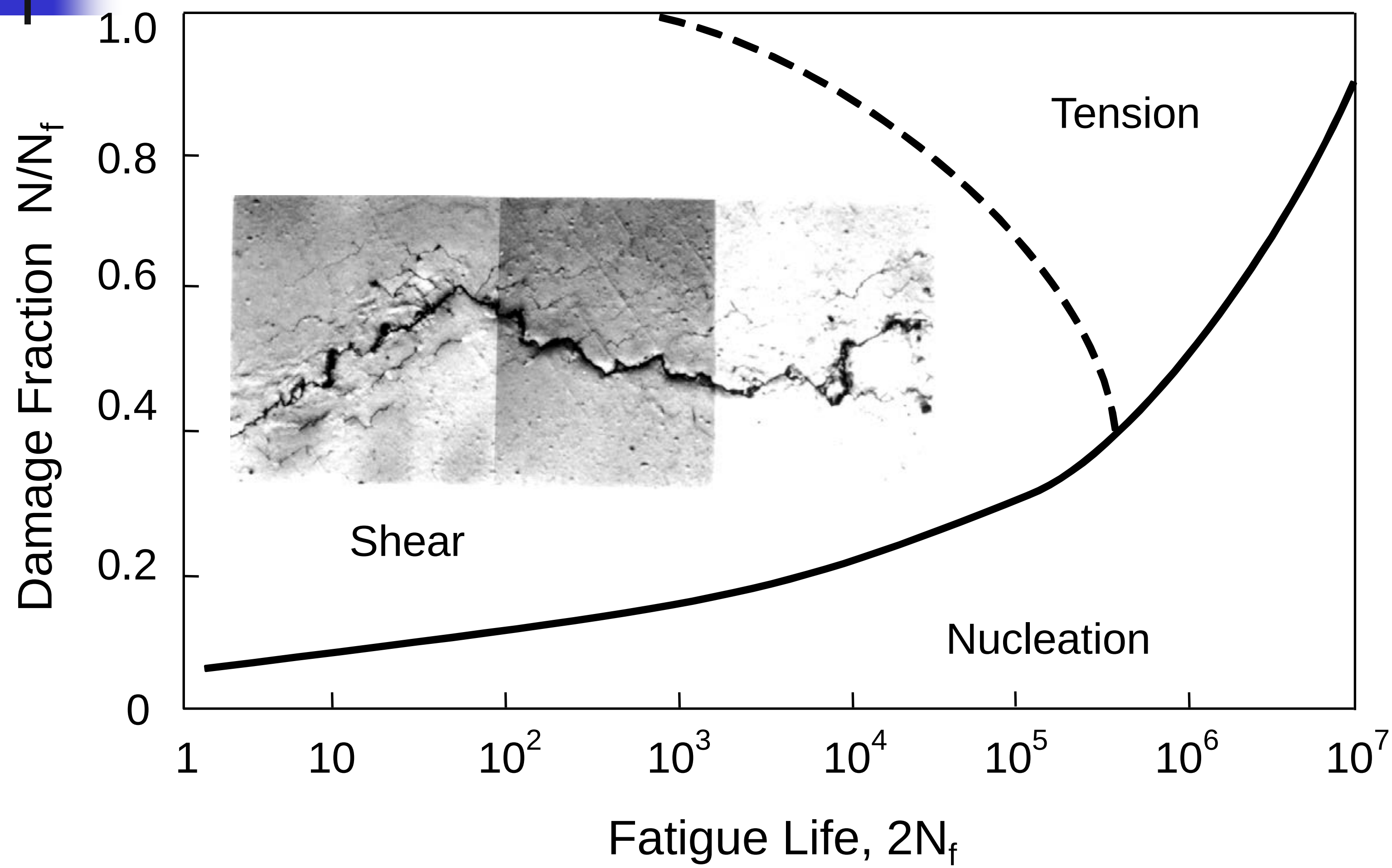
304 Stainless Steel - Tension



Inconel 718 - Torsion



Inconel 718 - Tension



Outline

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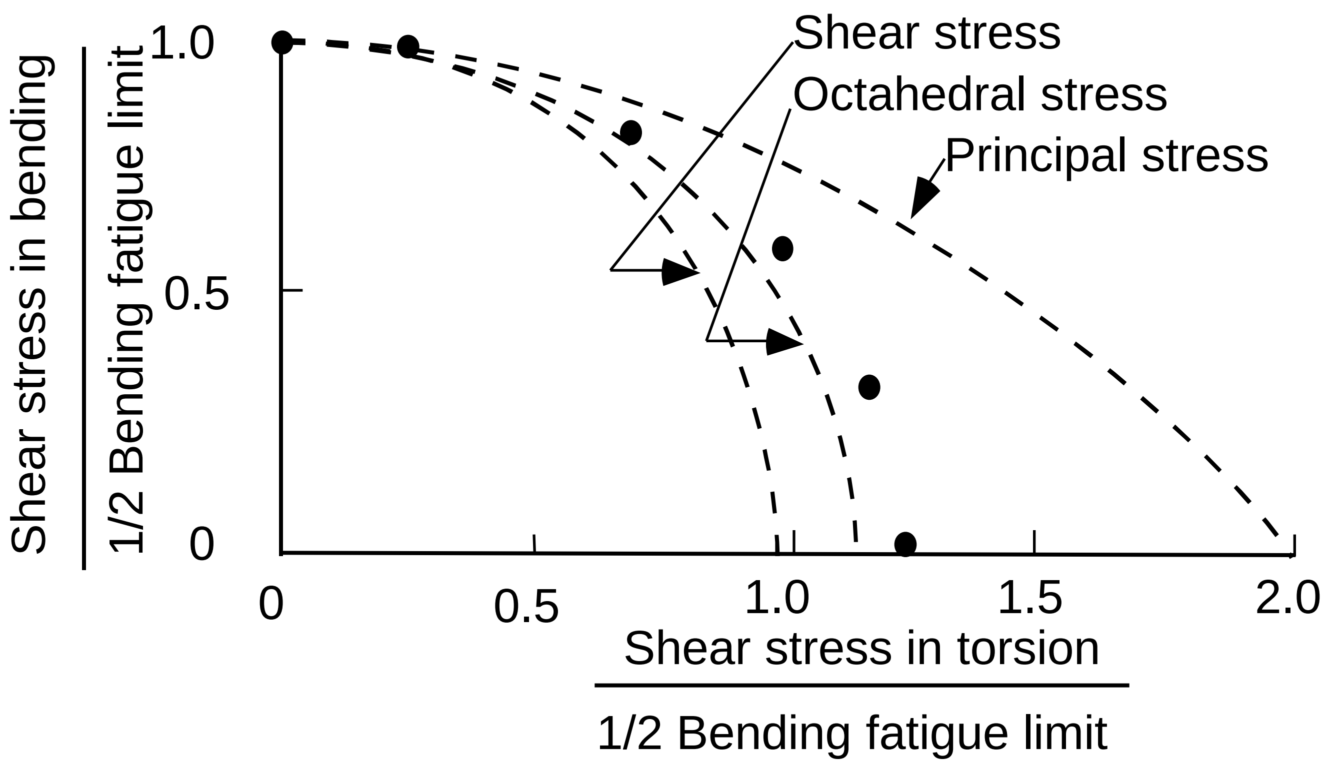
Fatigue Mechanisms Summary

- Fatigue cracks nucleate in shear
- Fatigue cracks grow in either shear or tension depending on material and state of stress

Stress Based Models

- Sines
- Findley
- Dang Van

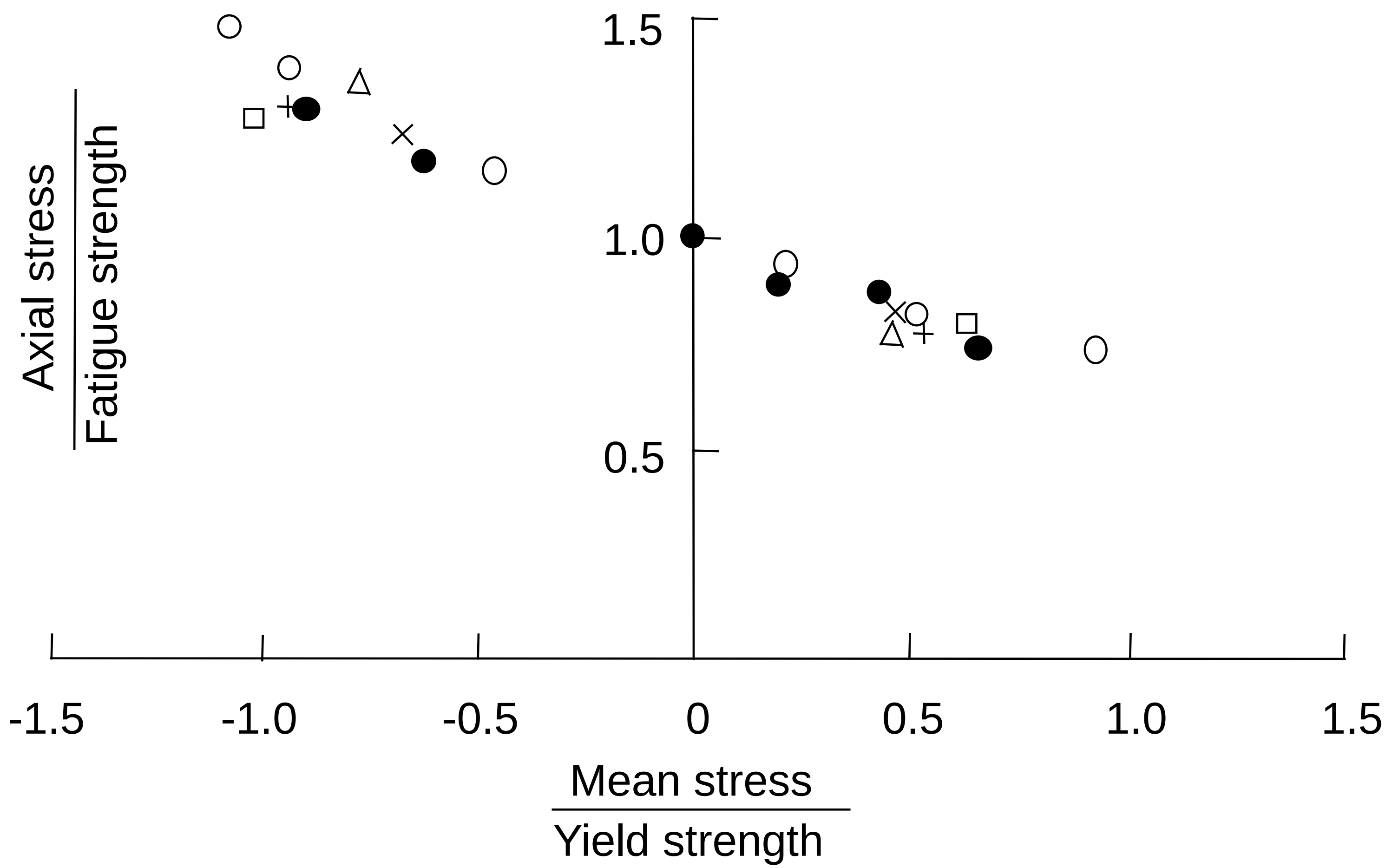
Bending Torsion Correlation



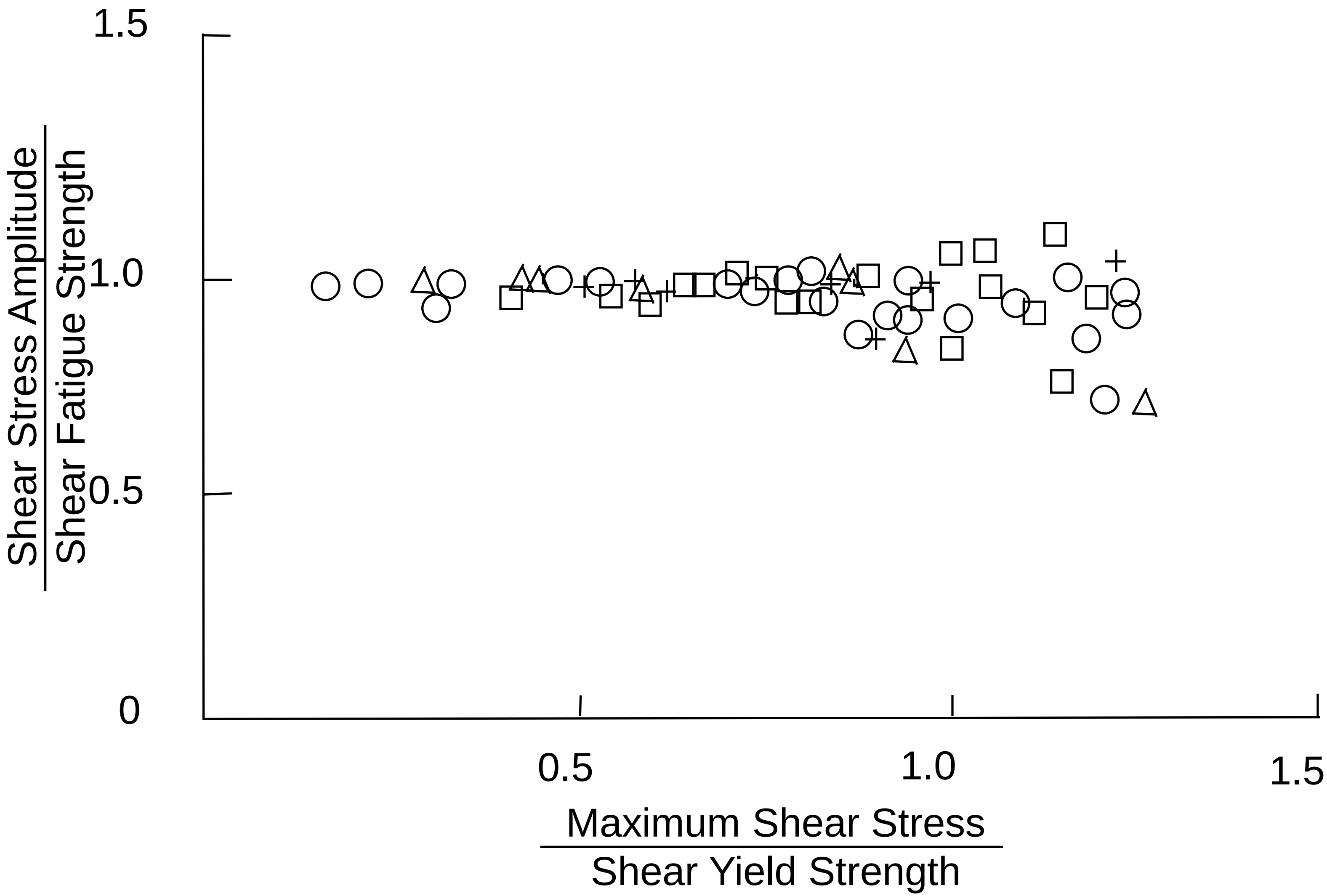
Test Results

- Cyclic tension with static tension
- Cyclic torsion with static torsion
- Cyclic tension with static torsion
- Cyclic torsion with static tension

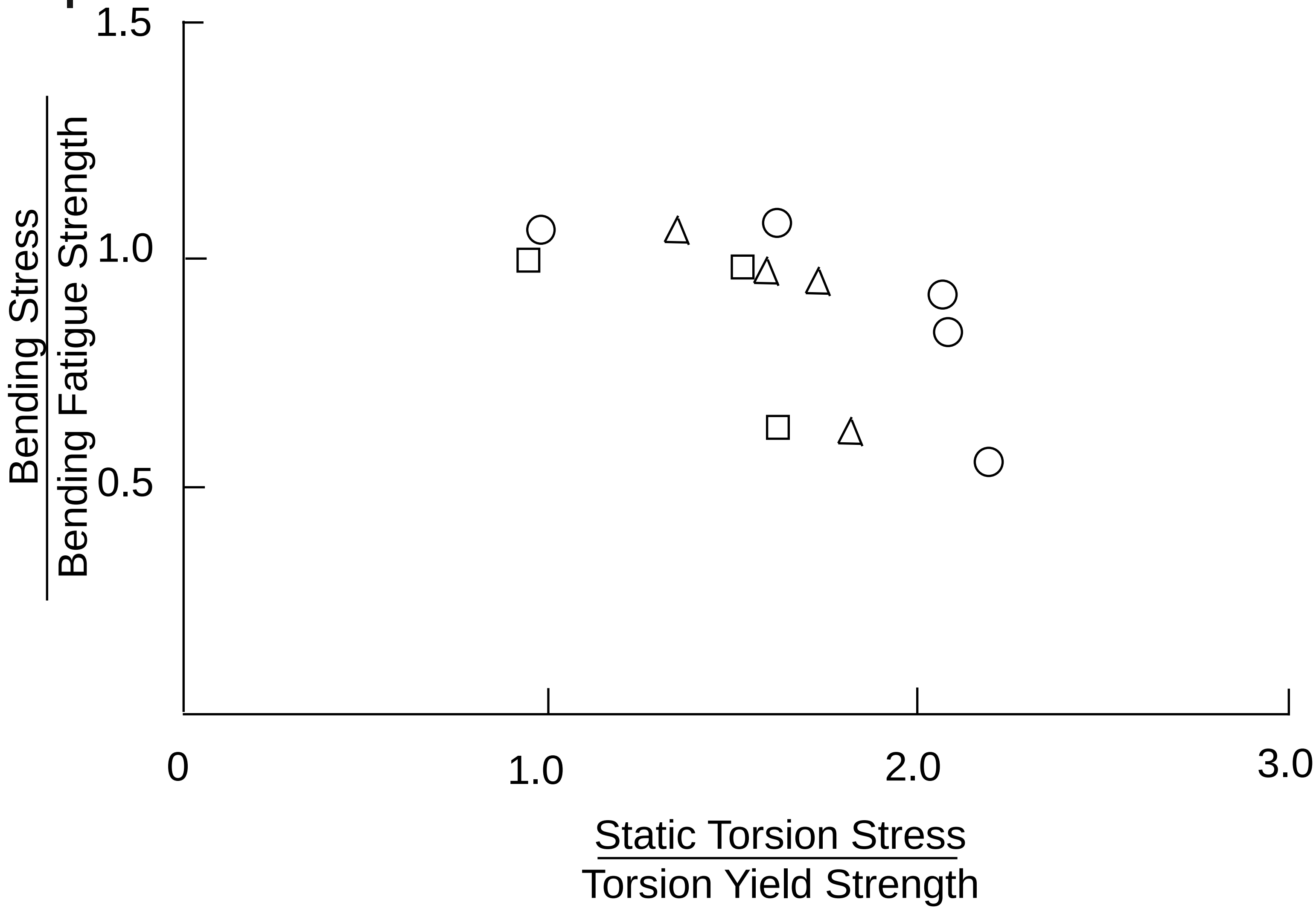
Cyclic Tension with Static Tension



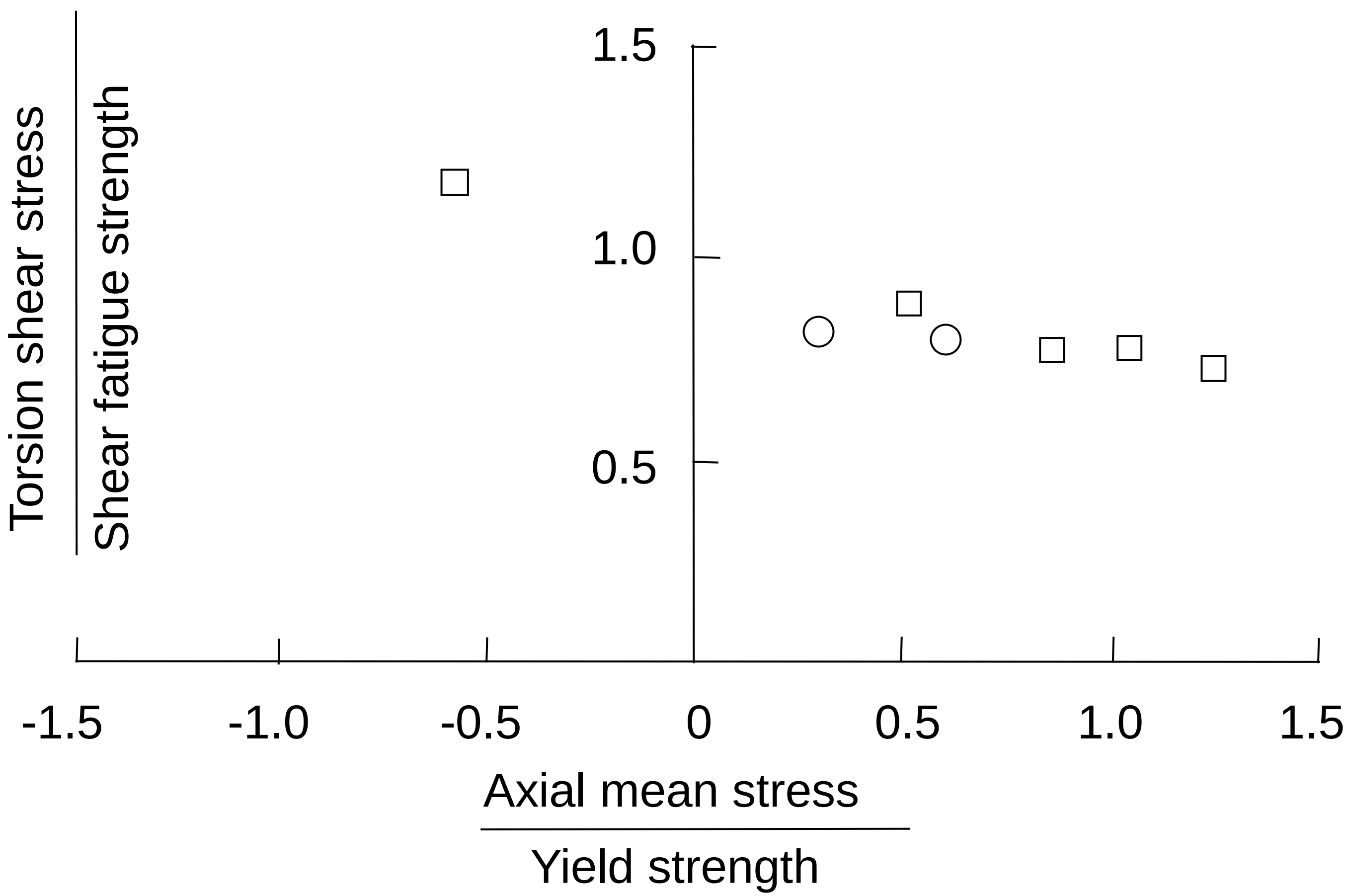
Cyclic Torsion with Static Torsion



Cyclic Tension with Static Torsion



Cyclic Torsion with Static Tension



Conclusions

- Tension mean stress affects both tension and torsion
- Torsion mean stress does not affect tension or torsion

Sines

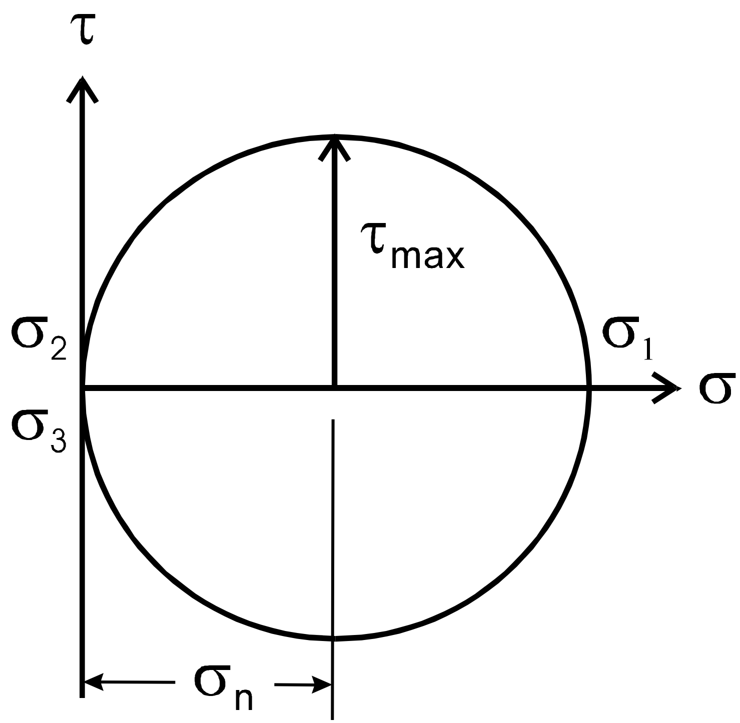
$$\frac{\Delta\tau_{oct}}{2} + \alpha(3\sigma_h) = \beta$$

$$\frac{1}{6}\sqrt{(\Delta\sigma_x - \Delta\sigma_y)^2 + (\Delta\sigma_x - \Delta\sigma_z)^2 + (\Delta\sigma_y - \Delta\sigma_z)^2 + 6(\Delta\tau_{xy}^2 + \Delta\tau_{xz}^2 + \Delta\tau_{yz}^2)} +$$

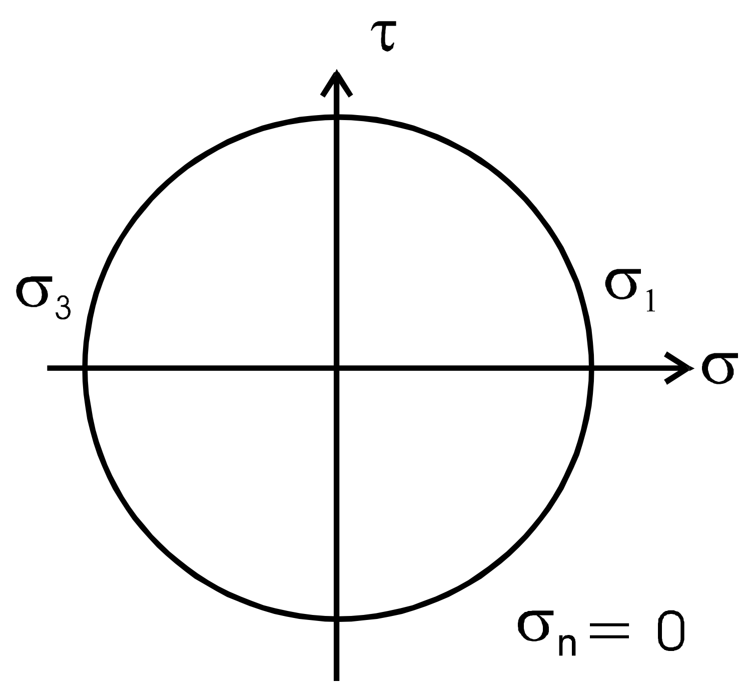
$$\alpha(\sigma_x^{mean} + \sigma_y^{mean} + \sigma_z^{mean}) = \beta$$

Findley

$$\left(\frac{\Delta\tau}{2} + k\sigma_n\right)_{max} = f$$

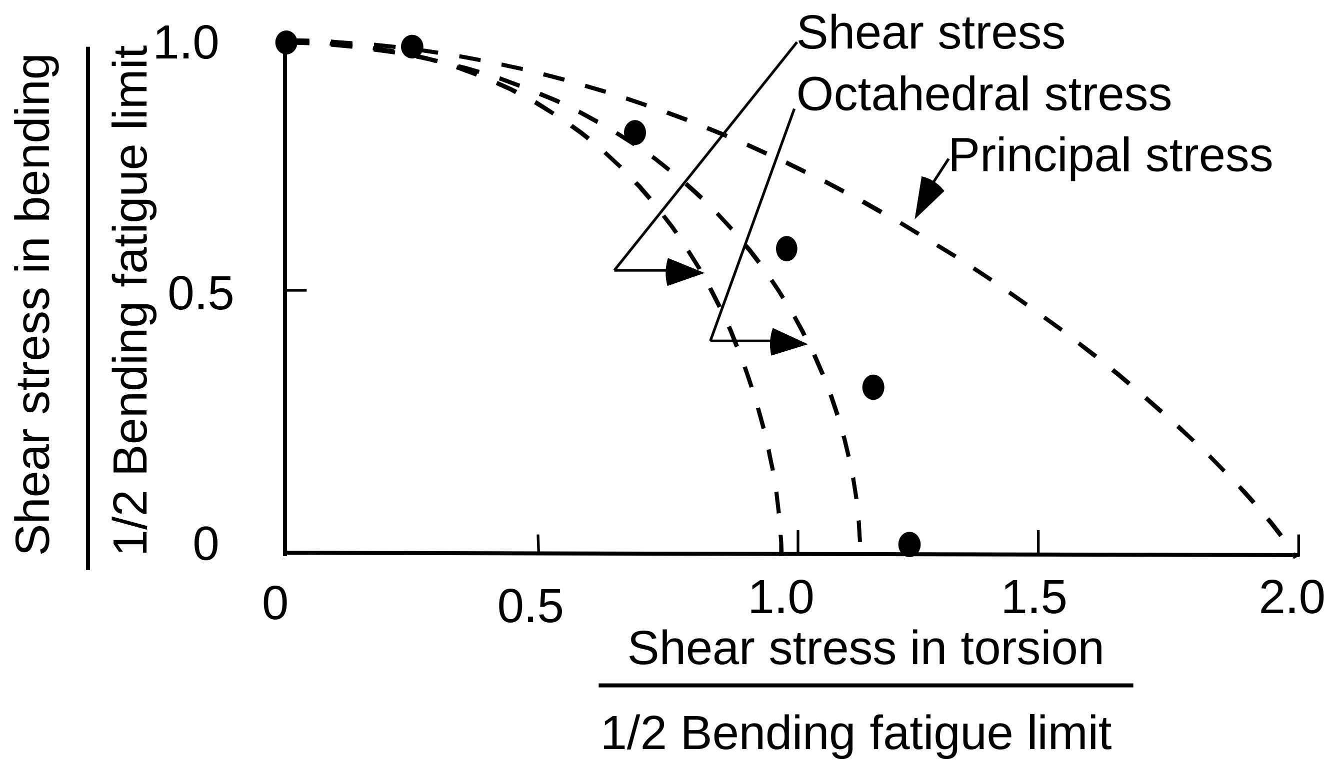


tension



torsion

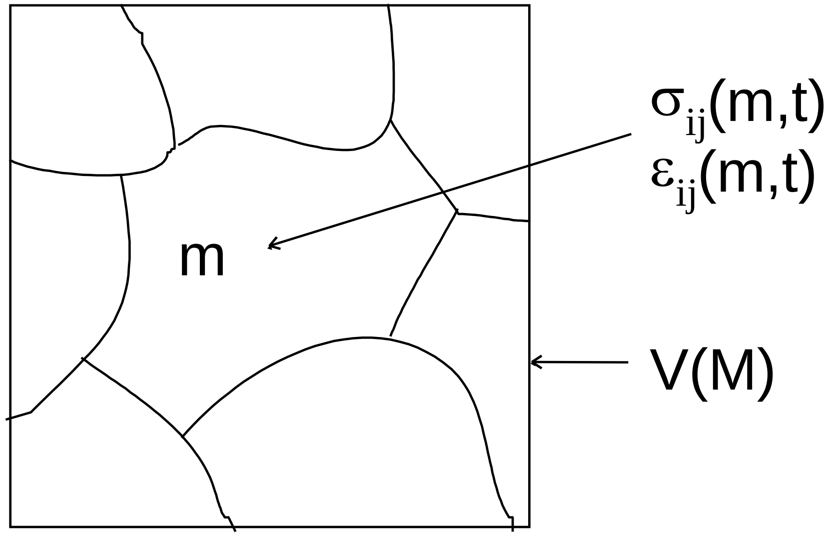
Bending Torsion Correlation



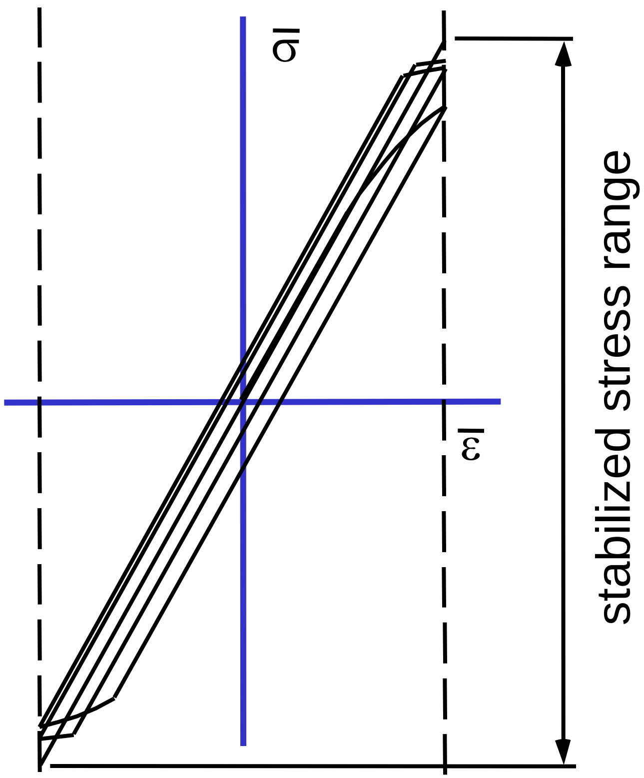
Dang Van

$$\tau(t) + a\sigma_h(t) = b$$

$\Sigma_{ij}(M,t)$ $E_{ij}(M,t)$

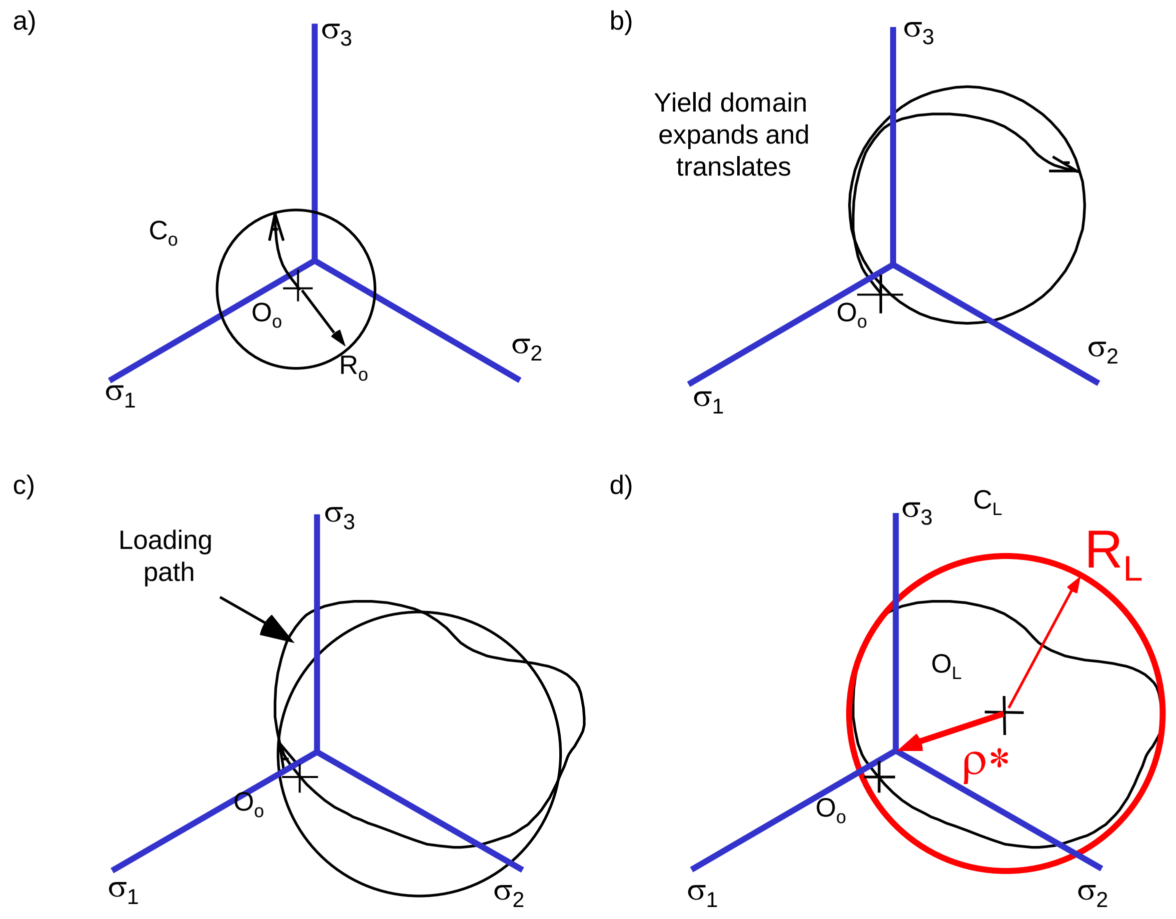


Isotropic Hardening



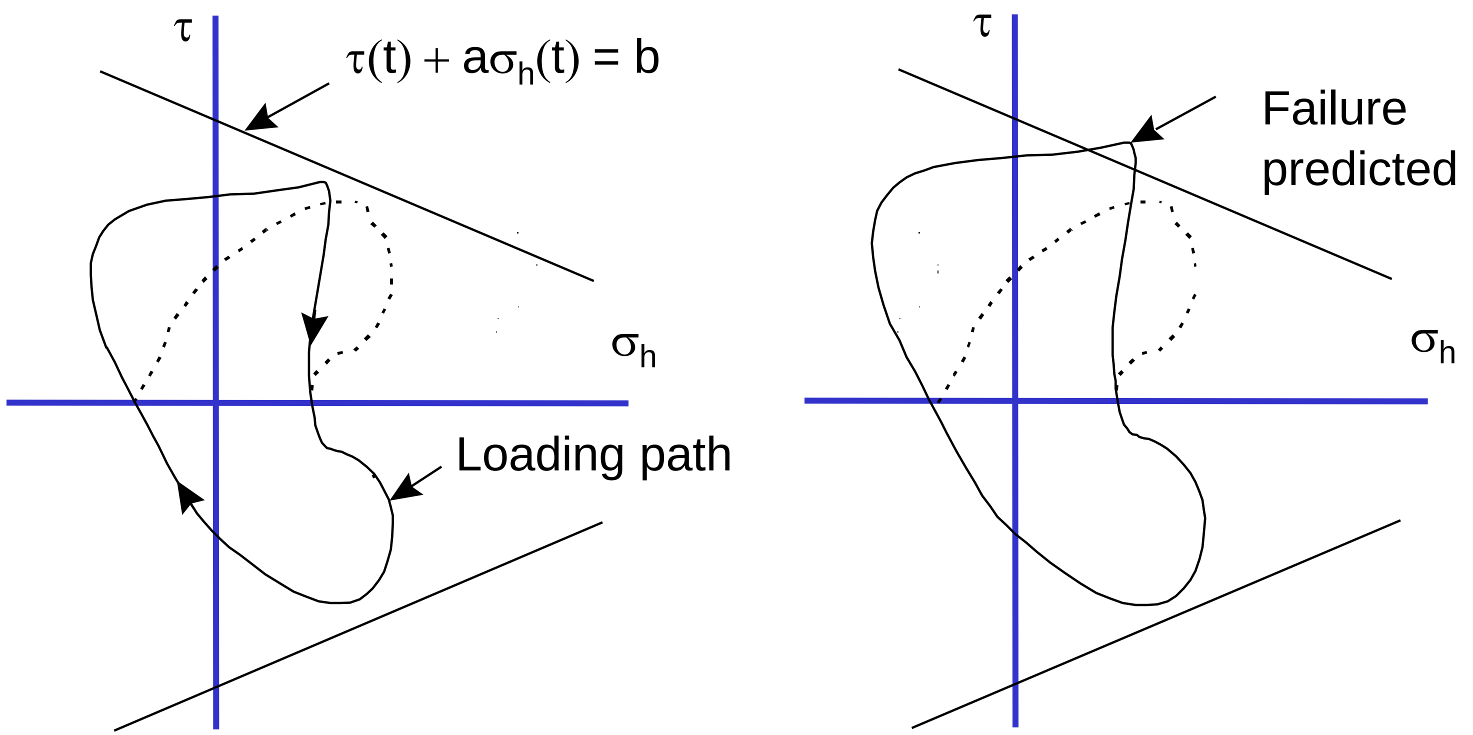
Failure occurs when the stress range is not elastic

Multiaxial Kinematic and Isotropic



ρ* stabilized residual stress

Dang Van (continued)



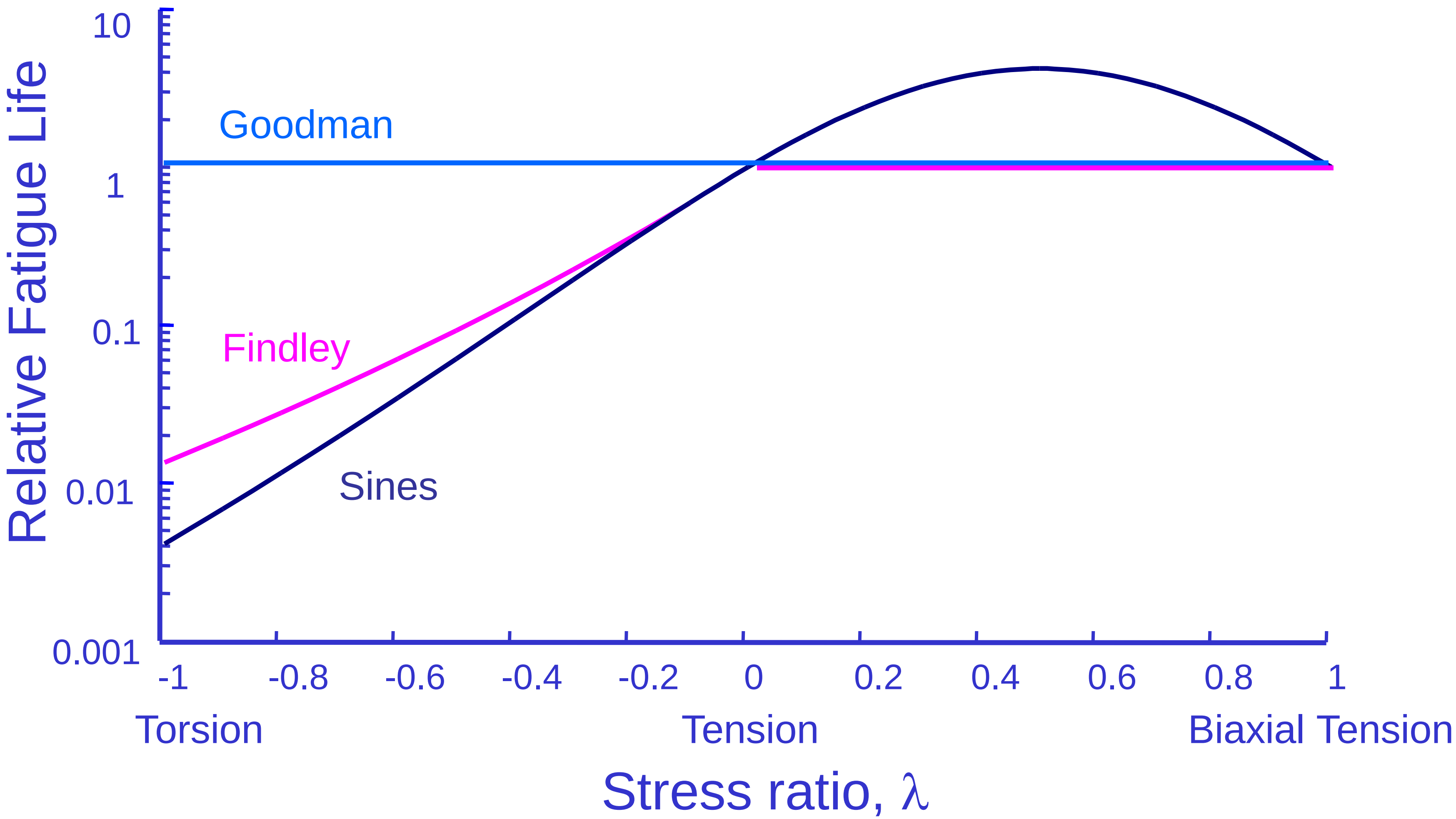
Stress Based Models Summary

Sines: $$\frac{\Delta\tau_{oct}}{2} + \alpha(3\sigma_h) = \beta$$

Findley: $$\left(\frac{\Delta\tau}{2} + k\sigma_n\right)_{max} = f$$

Dang Van: $$\tau(t) + a\sigma_h(t) = b$$

Model Comparison R = -1



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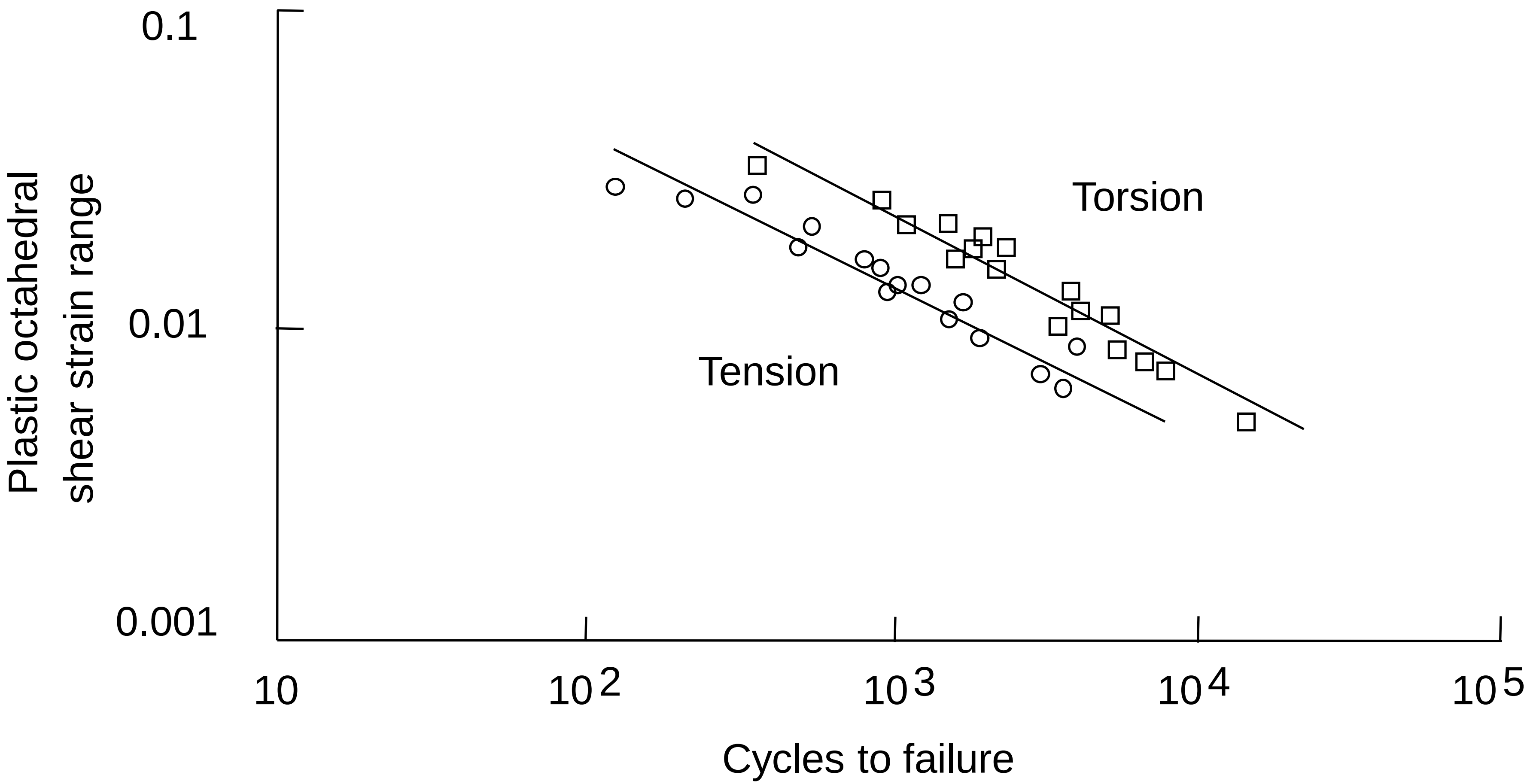
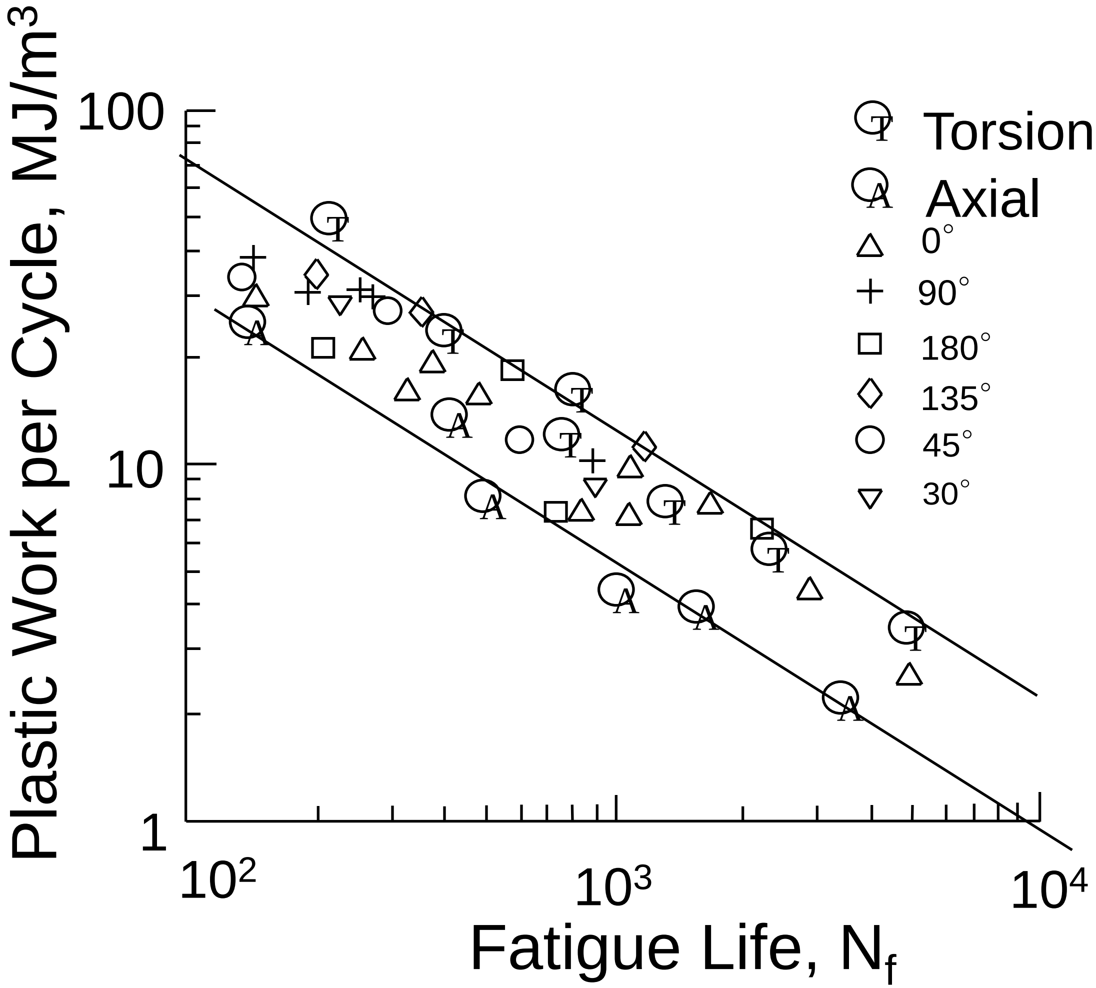
Strain Based Models

- Plastic Work
- Brown and Miller
- Fatemi and Socie
- Smith Watson and Topper
- Liu

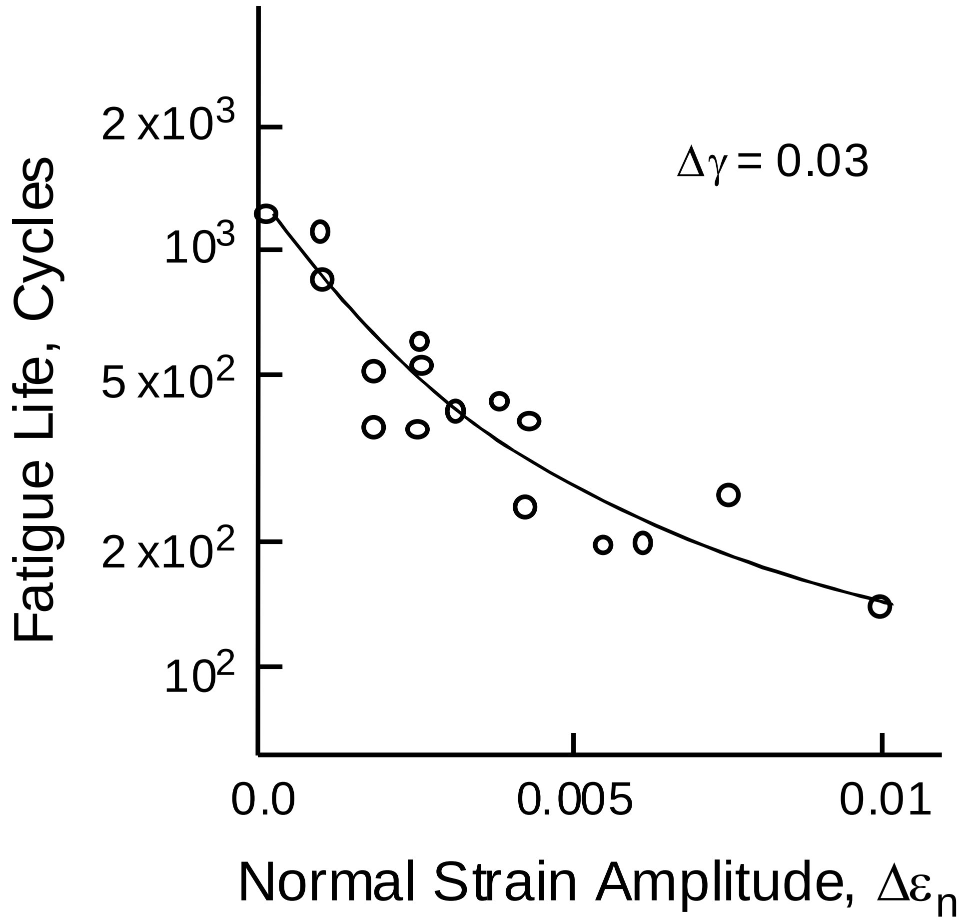
Octahedral Shear Strain



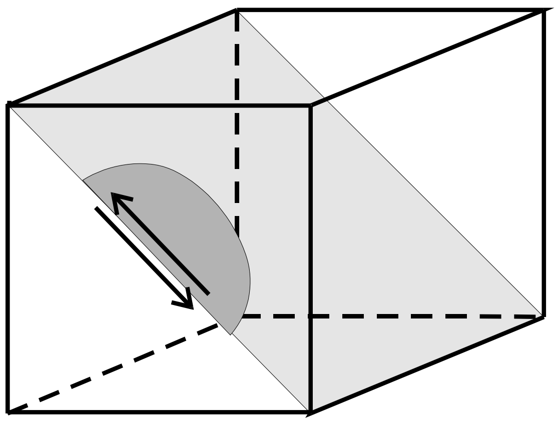
Plastic Work



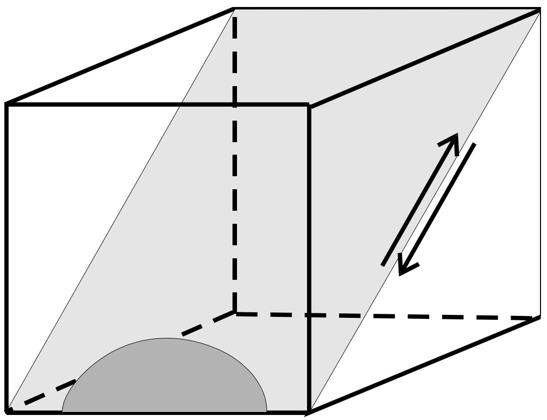
Brown and Miller



Case A and B



Case A

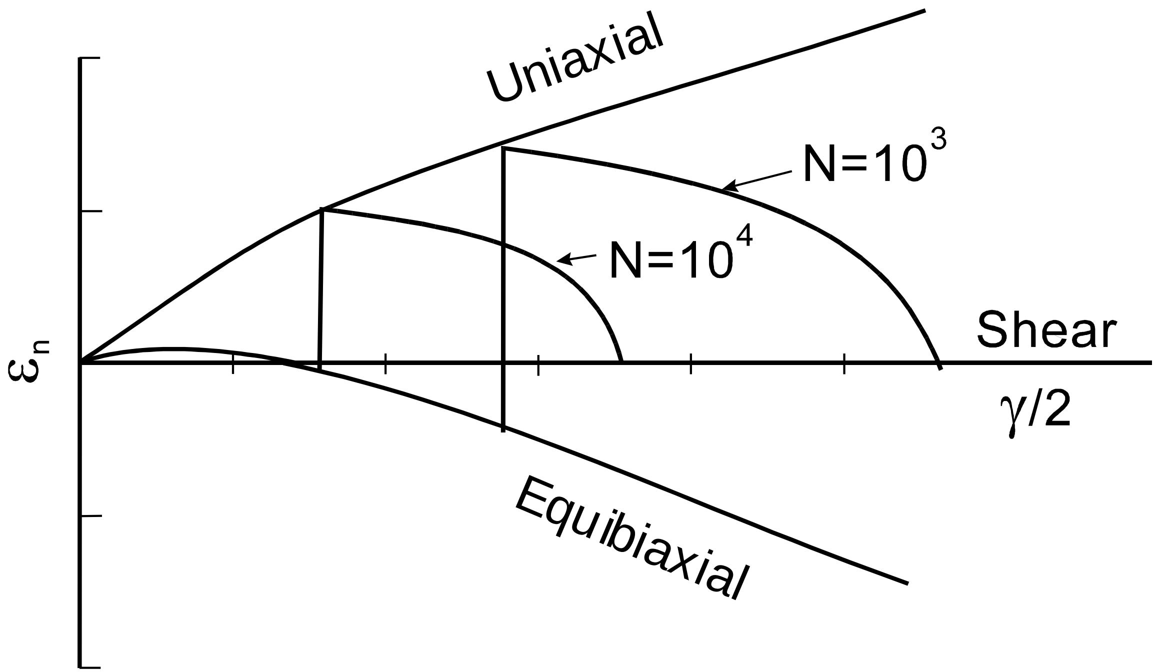


Case B

Growth along the surface

Growth into the surface

Brown and Miller (continued)

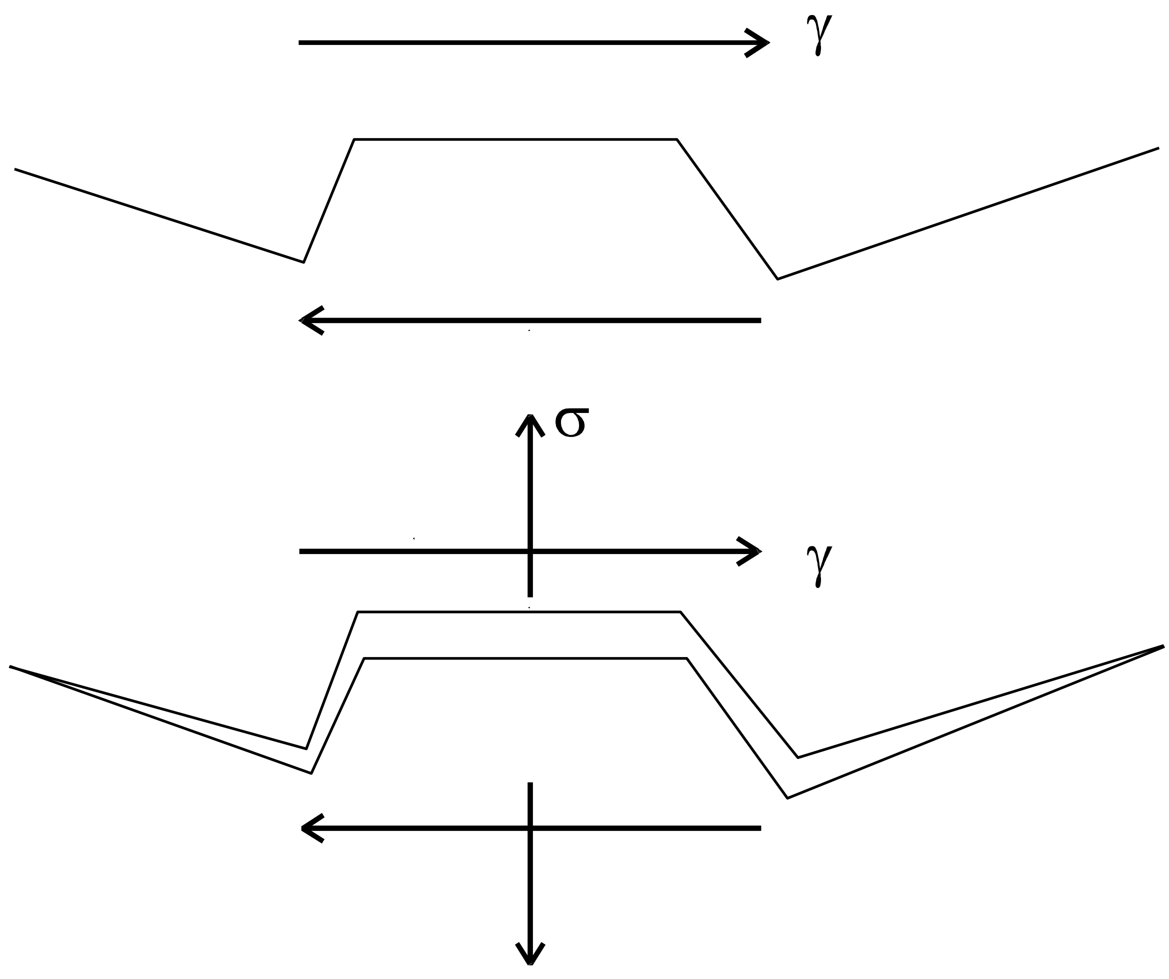


Brown and Miller (continued)

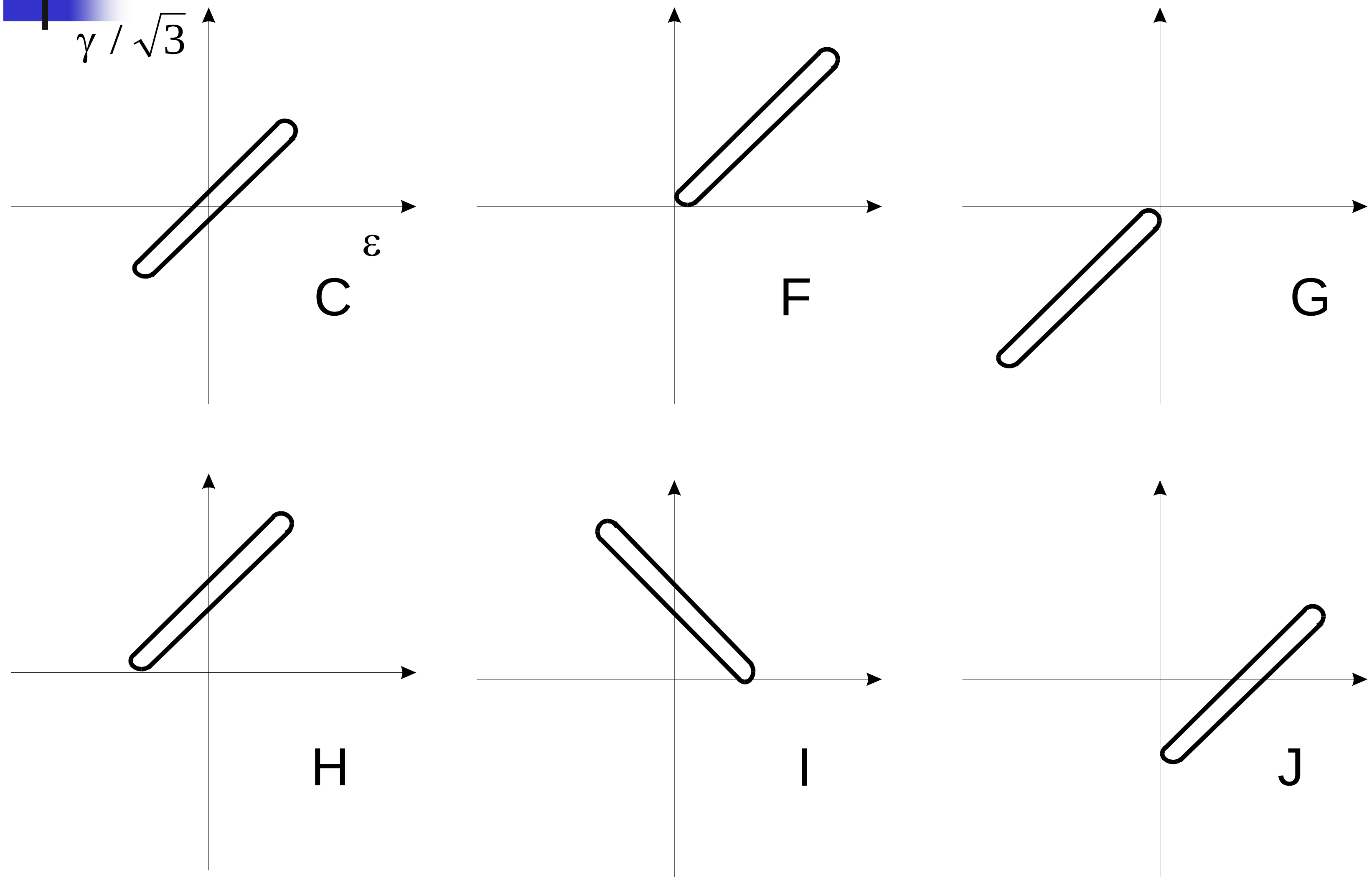
$$\Delta\hat{\gamma} = \left(\Delta\gamma_{max}^{\alpha} + S\Delta\varepsilon_{n}^{\alpha}\right)^{\frac{1}{\alpha}}$$

$$\frac{\Delta\gamma_{max}}{2} + S\,\Delta\varepsilon_{n} = A\,\frac{\sigma_{f}^{'} - 2\sigma_{n,mean}}{E}\left(2N_{f}\right)^{b} + B\,\varepsilon_{f}^{'}\left(2N_{f}\right)^{c}$$

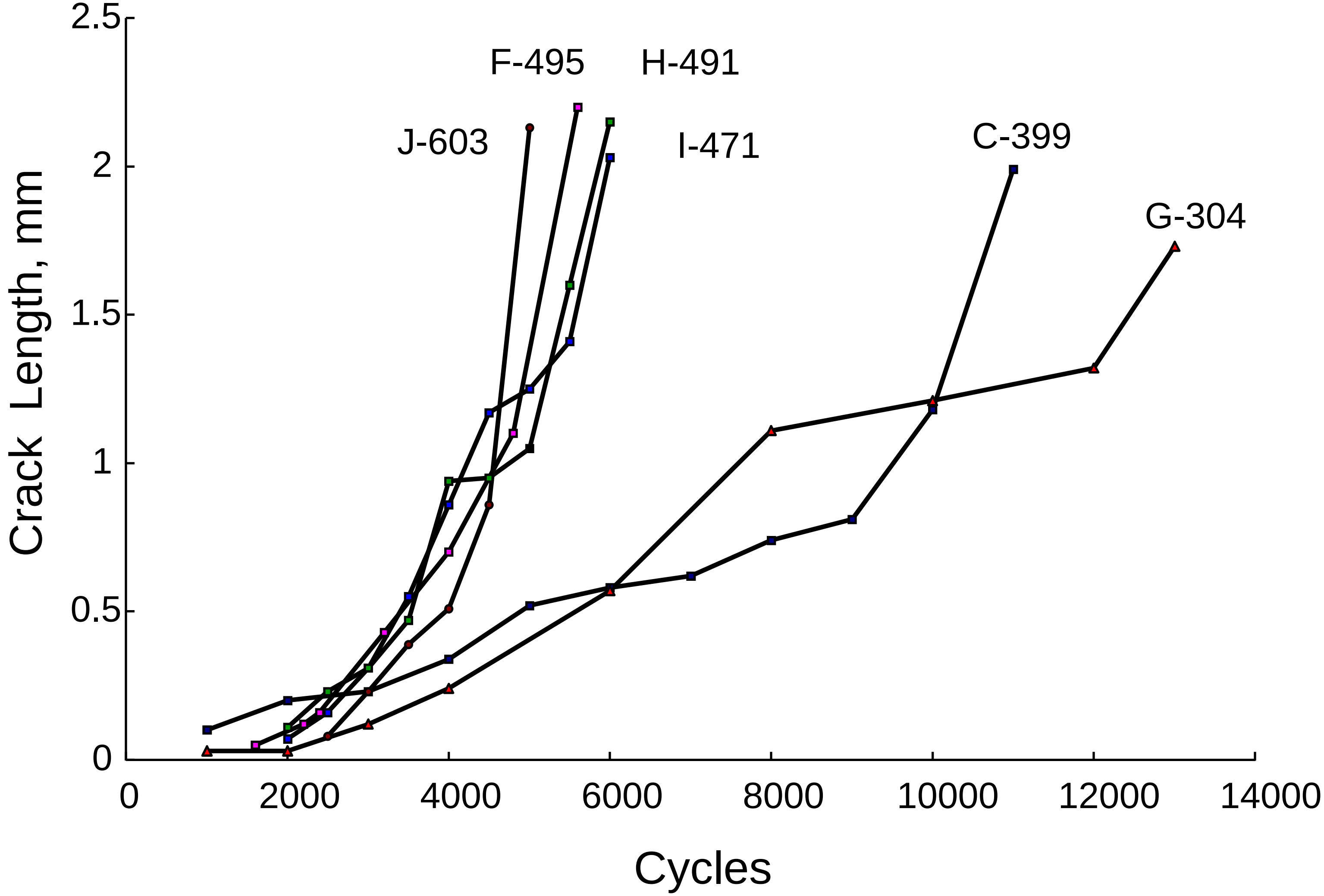
Fatemi and Socie



Loading Histories



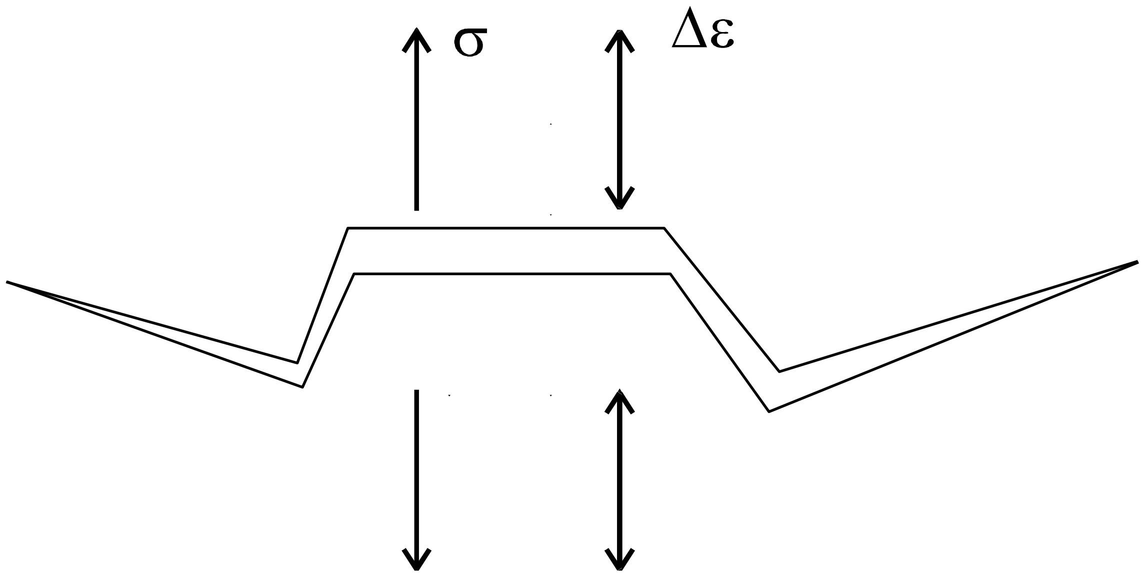
Crack Length Observations



Fatemi and Socie

$$\frac{\Delta\gamma}{2}\left(1+k\frac{\sigma_{n,max}}{\sigma_y}\right)=\frac{\tau_f^{'}}{G}(2N_f)^{bo}+\gamma_f^{'}(2N_f)^{co}$$

Smith Watson Topper



SWT

$$\sigma_n \frac{\Delta\varepsilon_1}{2} = \frac{\sigma_f^{'2}}{E}(2N_f)^{2b} + \sigma_f^{'}\varepsilon_f^{'}(2N_f)^{b+c}$$

Liu

Virtual strain energy for both mode I and mode II cracking

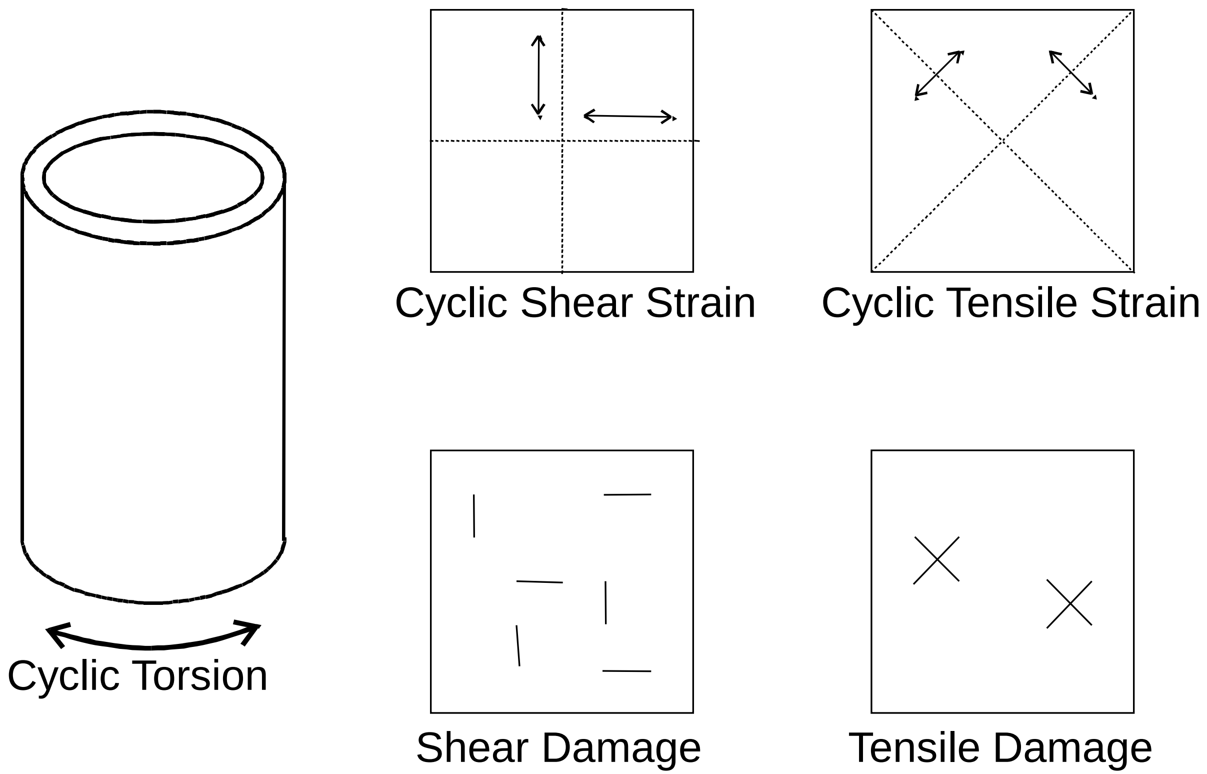
$$\Delta W_I = (\Delta\sigma_n \, \Delta\varepsilon_n)_{max} + (\Delta\tau \, \Delta\gamma)$$

$$\Delta W_I = 4\sigma'_f \varepsilon'_f (2N_f)^{b+c} + \frac{4\sigma'^2_f}{E}(2N_f)^{2b}$$

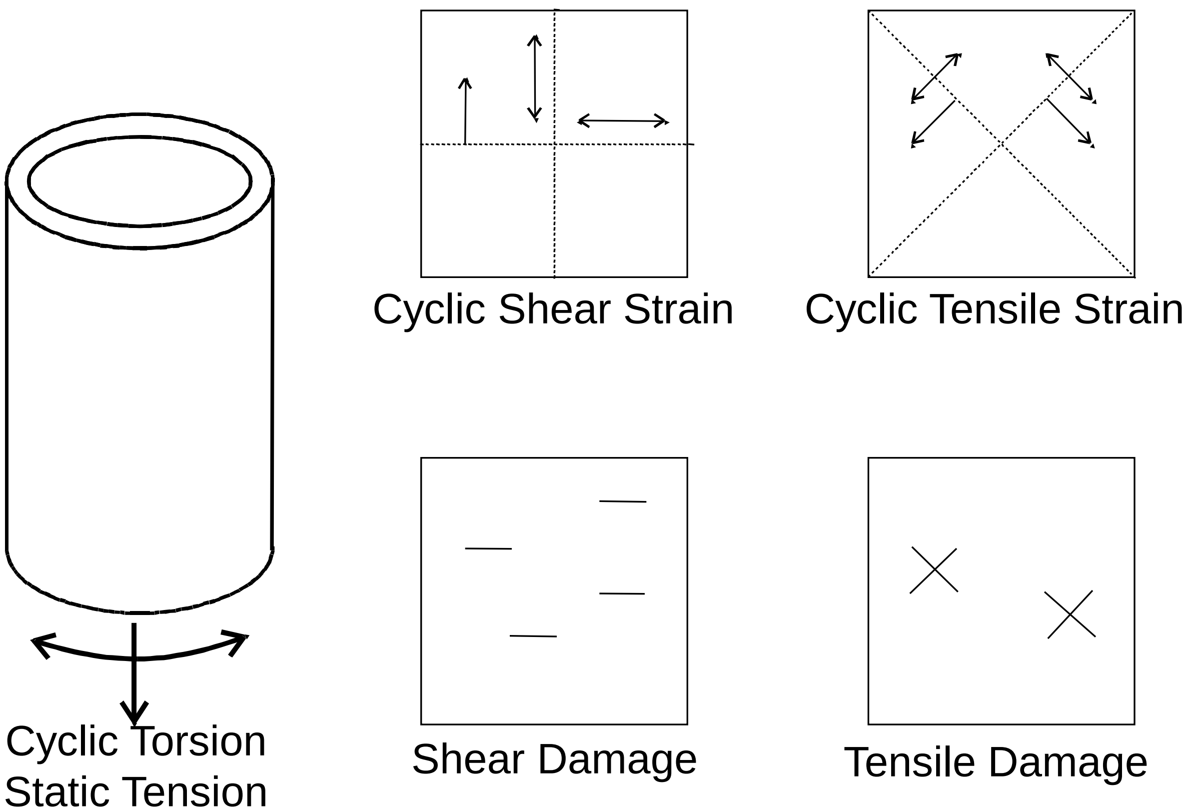
$$\Delta W_{II} = (\Delta\sigma_n \, \Delta\varepsilon_n) + (\Delta\tau \, \Delta\gamma)_{max}$$

$$\Delta W_{II} = 4\tau'_f \gamma'_f (2N_f)^{bo+co} + \frac{4\tau'^2_f}{G}(2N_f)^{2bo}$$

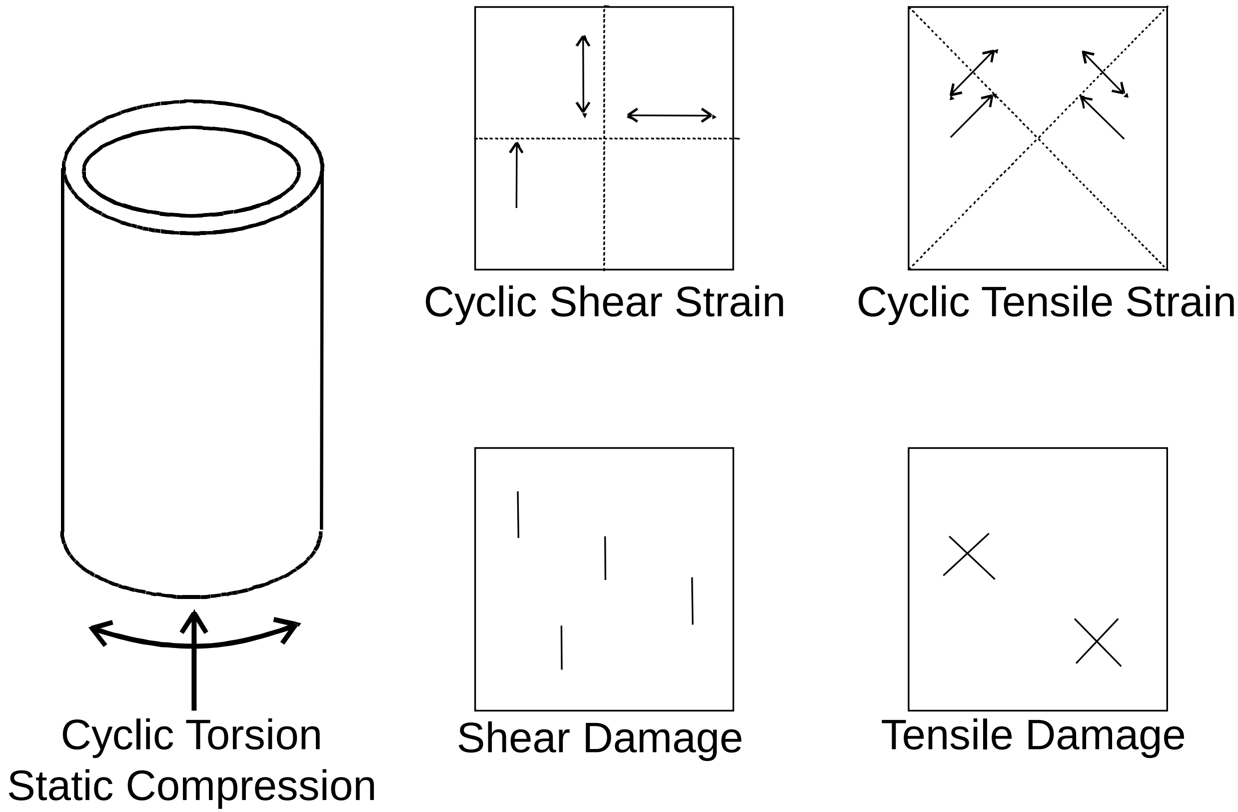
Cyclic Torsion



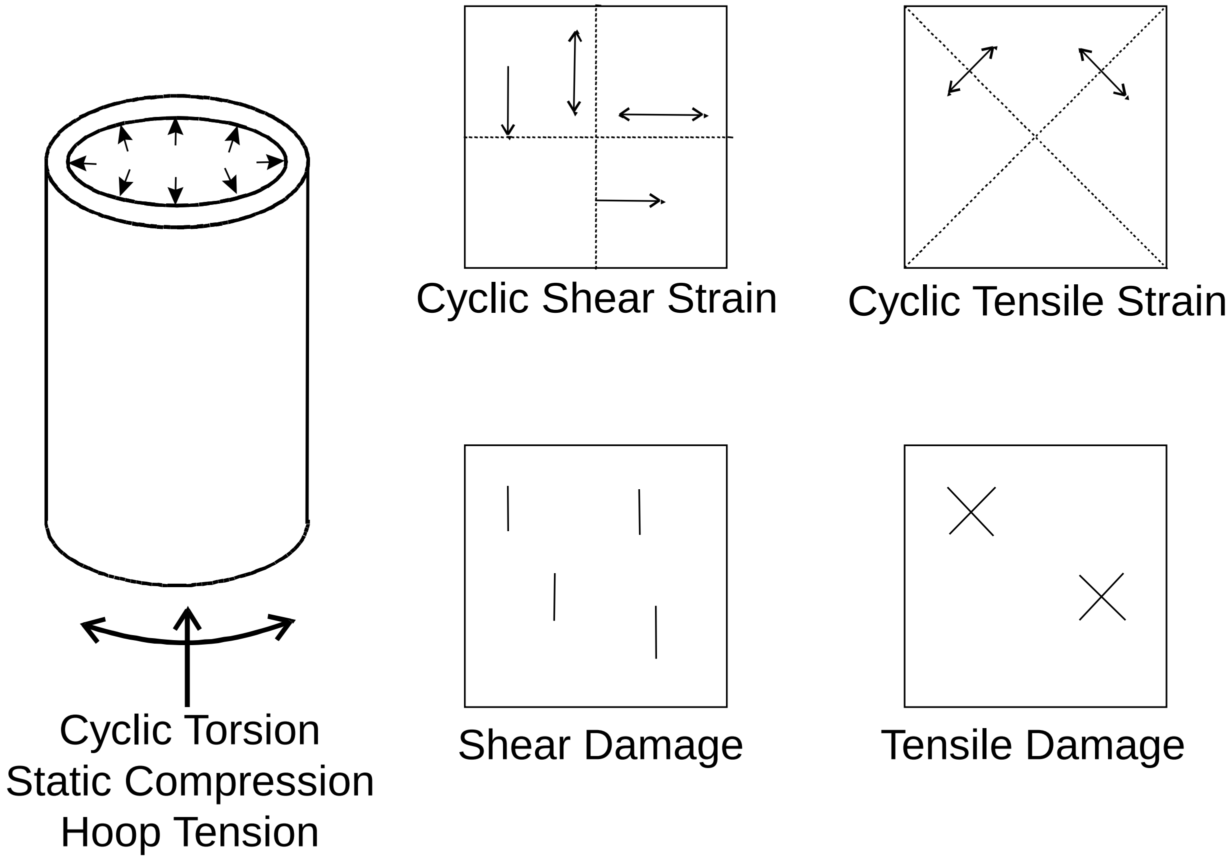
Cyclic Torsion with Static Tension



Cyclic Torsion with Compression



Cyclic Torsion with Tension and Compression



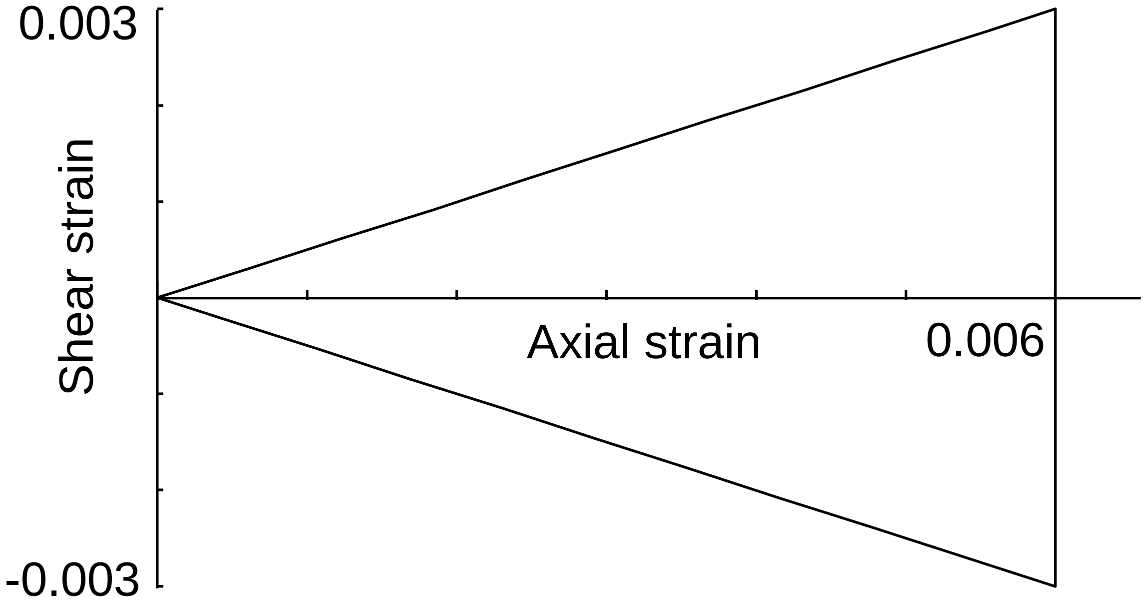
Test Results

Load Case	$\Delta\gamma/2$	σ_{hoop} MPa	σ_{axial} MPa	N_f
Torsion	0.0054	0	0	45,200
with tension	0.0054	0	450	10,300
with compression	0.0054	0	-500	50,000
with tension and compression	0.0054	450	-500	11,200

Conclusions

- All critical plane models correctly predict these results
- Hydrostatic stress models can not predict these results

Loading History



Model Comparison

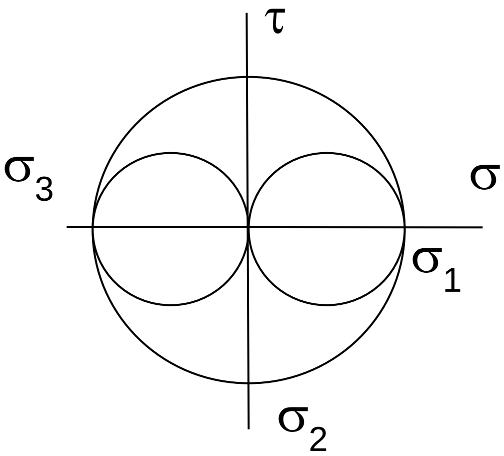
Summary of calculated fatigue lives

Model	Equation	Life
Epsilon	6.5	14,060
Garud	6.7	5,210
Ellyin	6.17	4,450
Brown-Miller	6.22	3,980
SWT	6.24	9,930
Liu I	6.41	4,280
Liu II	6.42	5,420
Chu	6.37	3,040
Gamma		26,775
Fatemi-Socie	6.23	10,350
Glinka	6.39	33,220

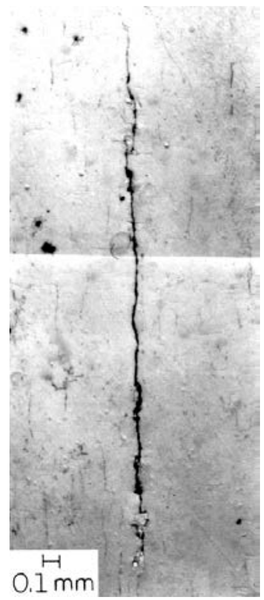
Strain Based Models Summary

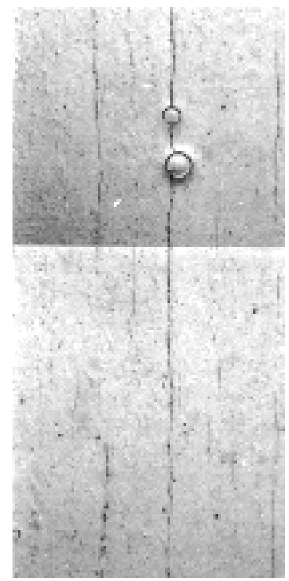
- Two separate models are needed, one for tensile growth and one for shear growth
- Cyclic plasticity governs stress and strain ranges
- Mean stress effects are a result of crack closure on the critical plane

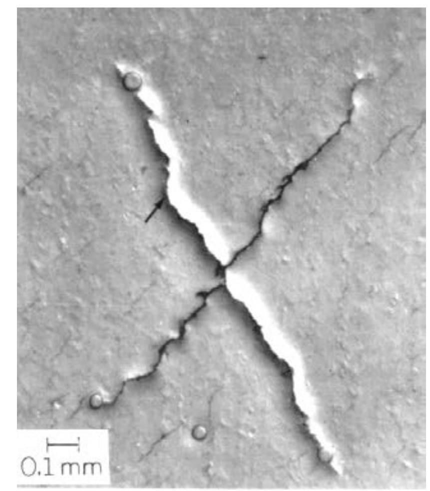
Separate Tensile and Shear Models



$\sigma_1 = \tau_{xy}$







Inconel 1045 steel stainless steel

Cyclic Plasticity

$\Delta\varepsilon$

$\Delta\gamma$

$\Delta\varepsilon^p$

$\Delta\gamma^p$

$\Delta\varepsilon\Delta\sigma$

$\Delta\gamma\Delta\tau$

$\Delta\varepsilon^p\Delta\sigma$

$\Delta\gamma^p\Delta\tau$

Mean Stresses

$$\Delta\varepsilon_{eq} = \frac{\sigma'_f - \sigma_{mean}}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c$$

$$\frac{\Delta\gamma_{max}}{2} + S\Delta\varepsilon_n = (1.3 + 0.7S)\frac{\sigma'_f - 2\sigma_n}{E}(2N_f)^b + (1.5 + 0.5S)\varepsilon'_f(2N_f)^c$$

$$\frac{\Delta\gamma}{2}\left(1 + k\frac{\sigma_{n,max}}{\sigma_y}\right) = \frac{\tau'_f}{G}(2N_f)^{bo} + \gamma'_f(2N_f)^{co}$$

$$\sigma_n\frac{\Delta\varepsilon_1}{2} = \frac{\sigma'^2_f}{E}(2N_f)^{2b} + \sigma'_f\varepsilon'_f(2N_f)^{b+c}$$

$$\Delta W_I = \left[(\Delta\sigma_n\Delta\varepsilon_n)_{max} + (\Delta\tau\Delta\gamma)\right]\left(\frac{2}{1-R}\right)$$

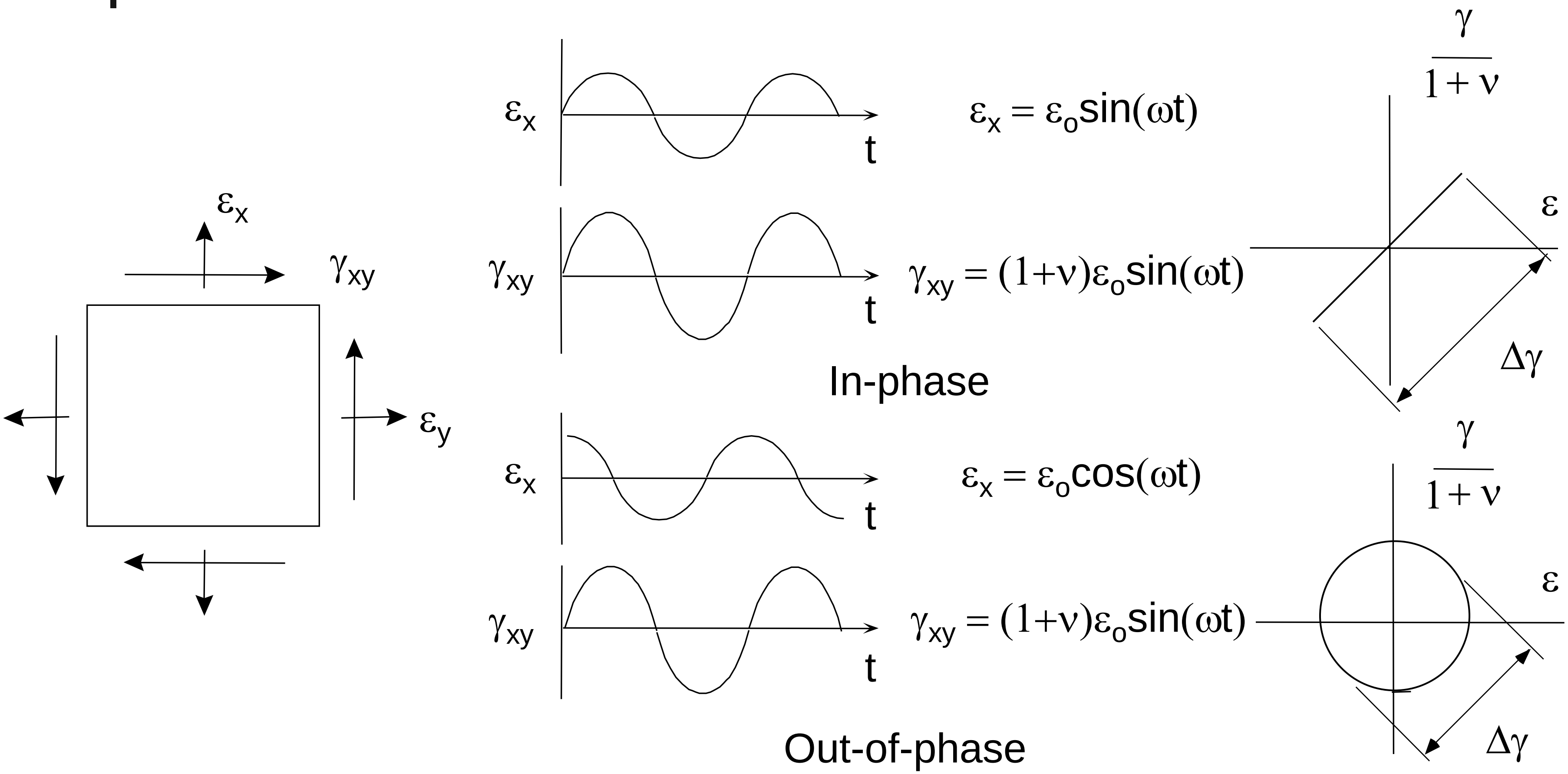
Outline

- State of Stress
- Stress-Strain Relationships
- Fatigue Mechanisms
- Multiaxial Testing
- Stress Based Models
- Strain Based Models
- Fracture Mechanics Models
- **Nonproportional Loading**
- Stress Concentrations

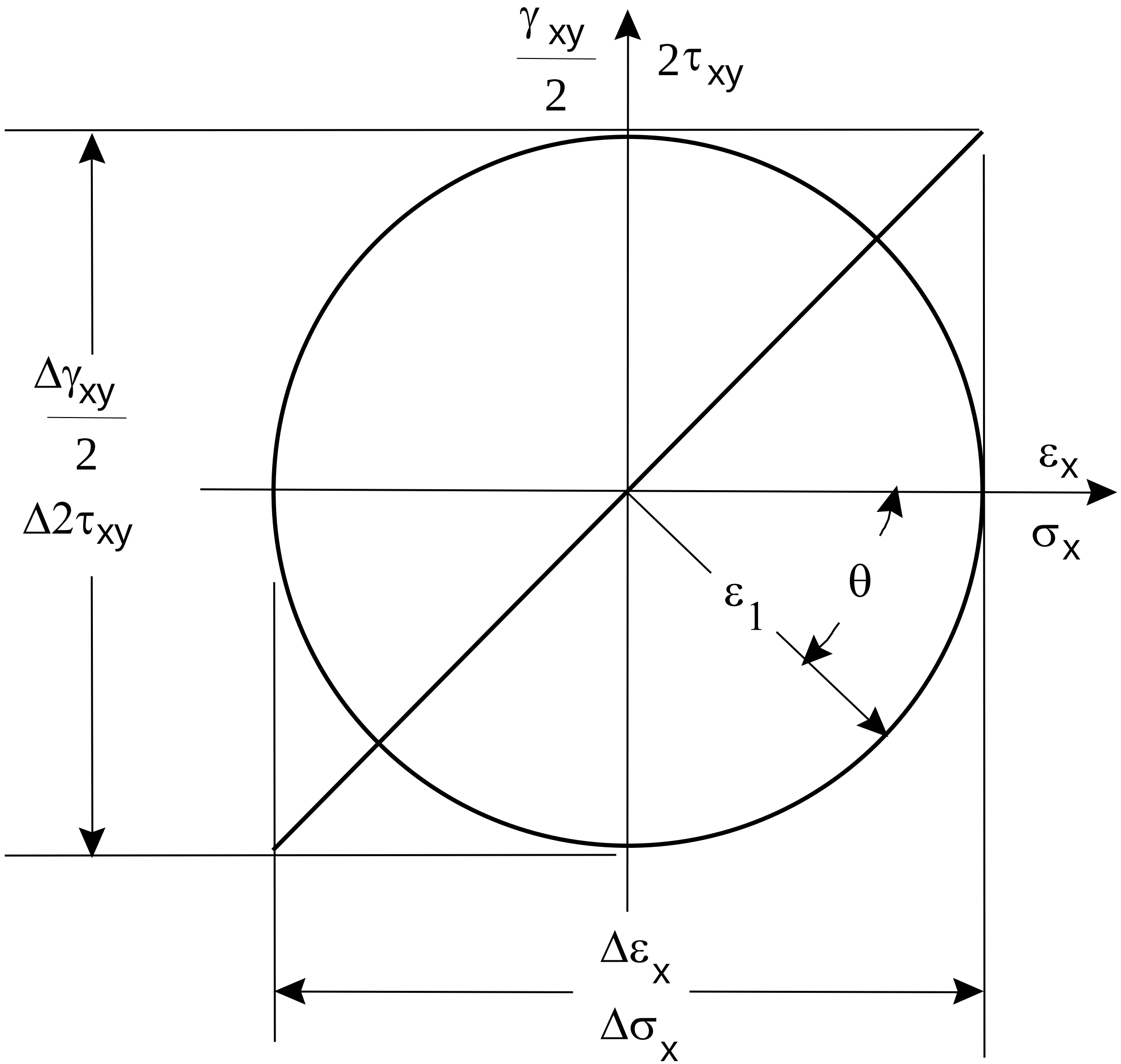
Nonproportional Loading

- In and Out-of-phase loading
- Nonproportional cyclic hardening
- Variable amplitude

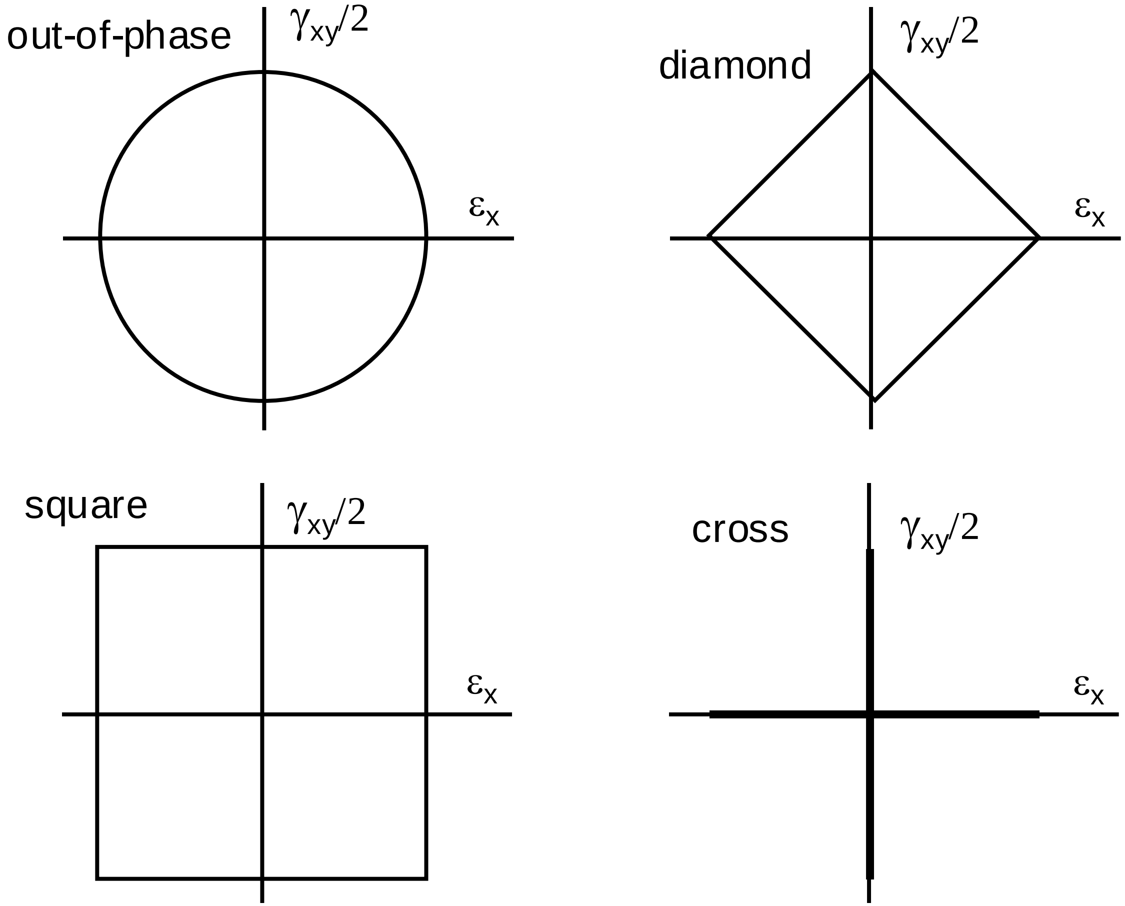
In and Out-of-Phase Loading



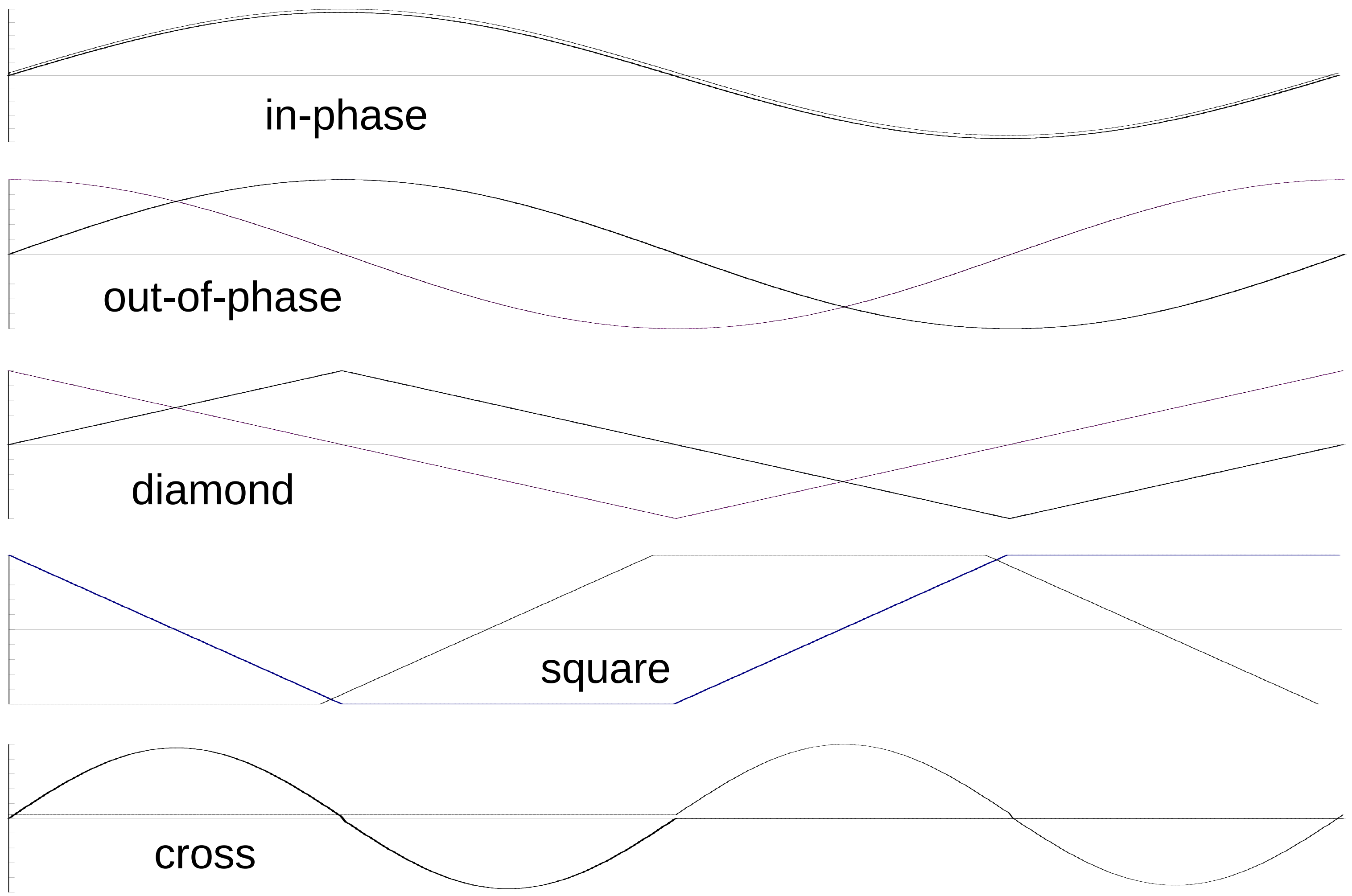
In-Phase and Out-of-Phase



Loading Histories

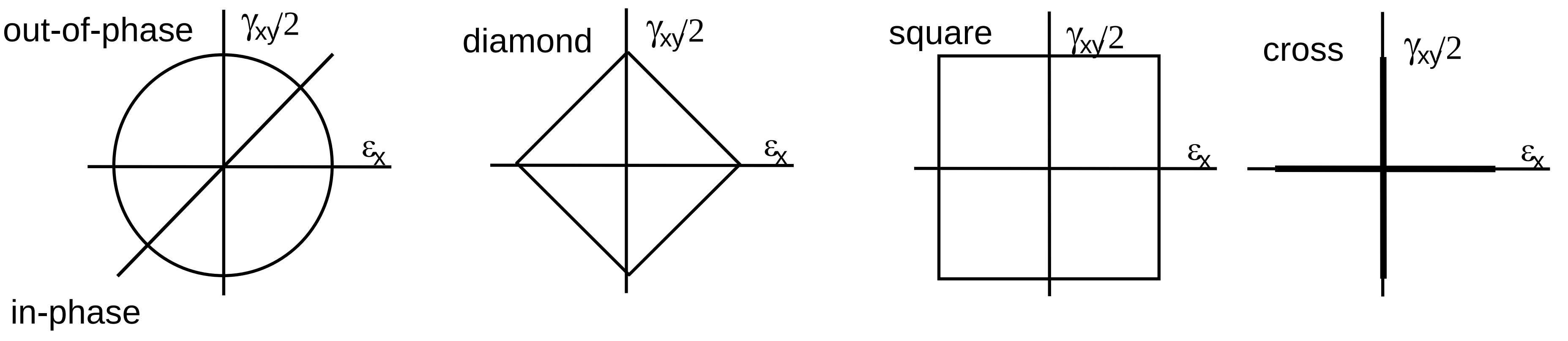


Loading Histories

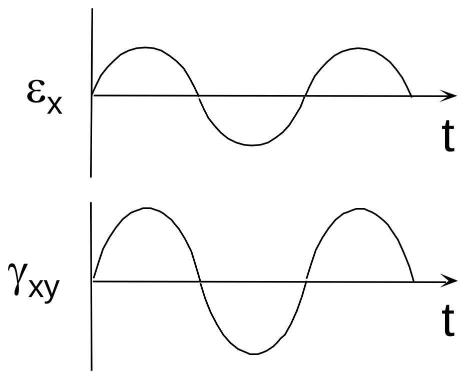


Findley Model Results

	$\Delta\tau/2$ MPa	$\sigma_{n,max}$ MPa	$\Delta\tau/2 + 0.3\ \sigma_{n,max}$	N/N_{ip}
in-phase	353	250	428	1.0
90° out-of-phase	250	500	400	2.0
diamond	250	500	400	2.0
square	353	603	534	0.11
cross - tension cycle	250	250	325	16
cross - torsion cycle	250	0	250	216



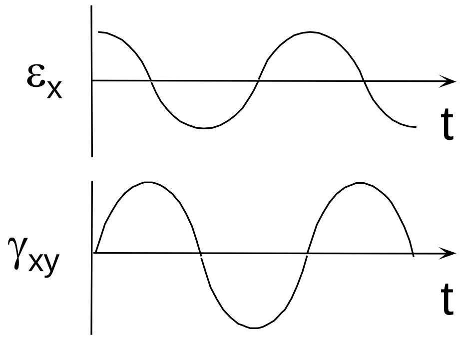
Nonproportional Hardening



$$\varepsilon_x = \varepsilon_o \sin(\omega t)$$

$$\gamma_{xy} = (1+\nu)\varepsilon_o \sin(\omega t)$$

In-phase

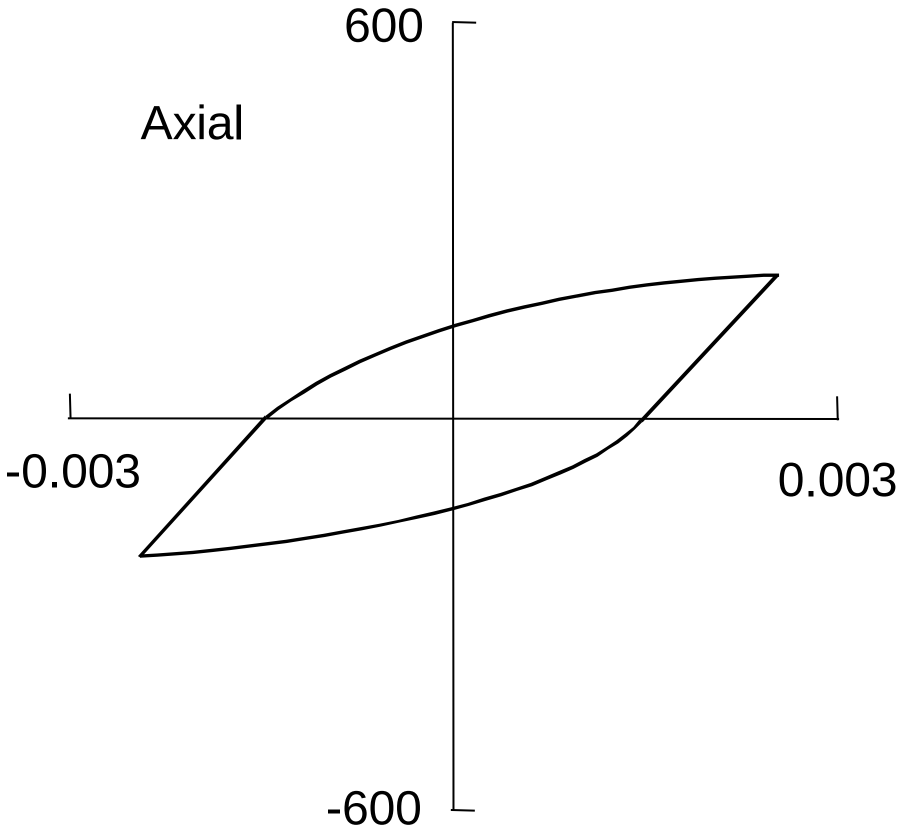


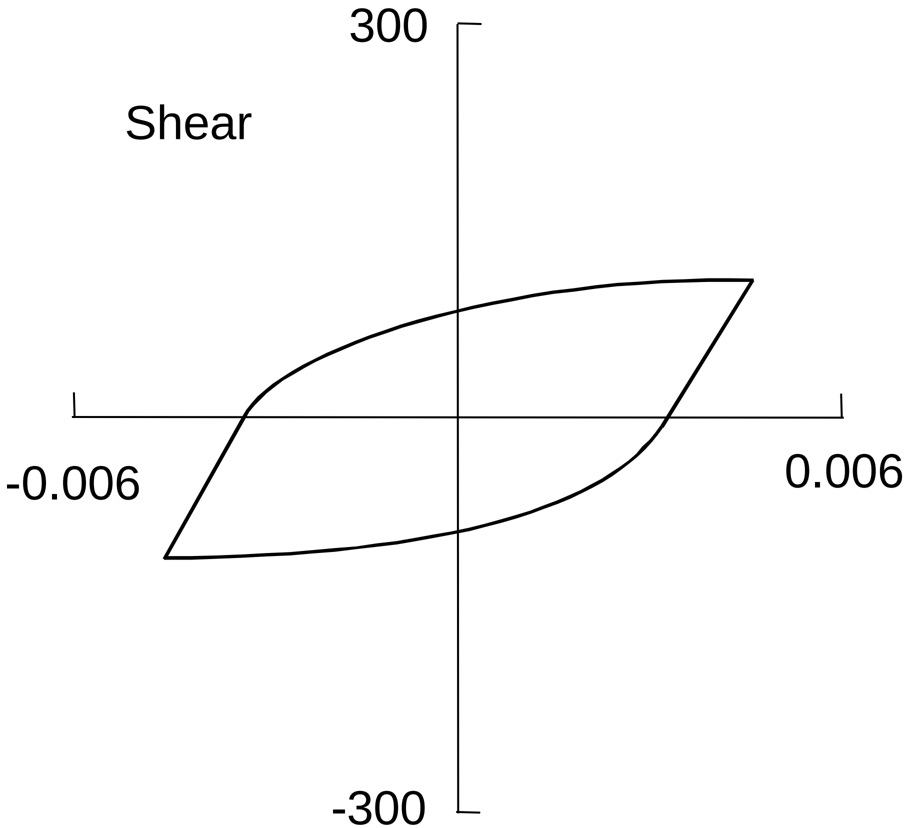
$$\varepsilon_x = \varepsilon_o \cos(\omega t)$$

$$\gamma_{xy} = (1+\nu)\varepsilon_o \sin(\omega t)$$

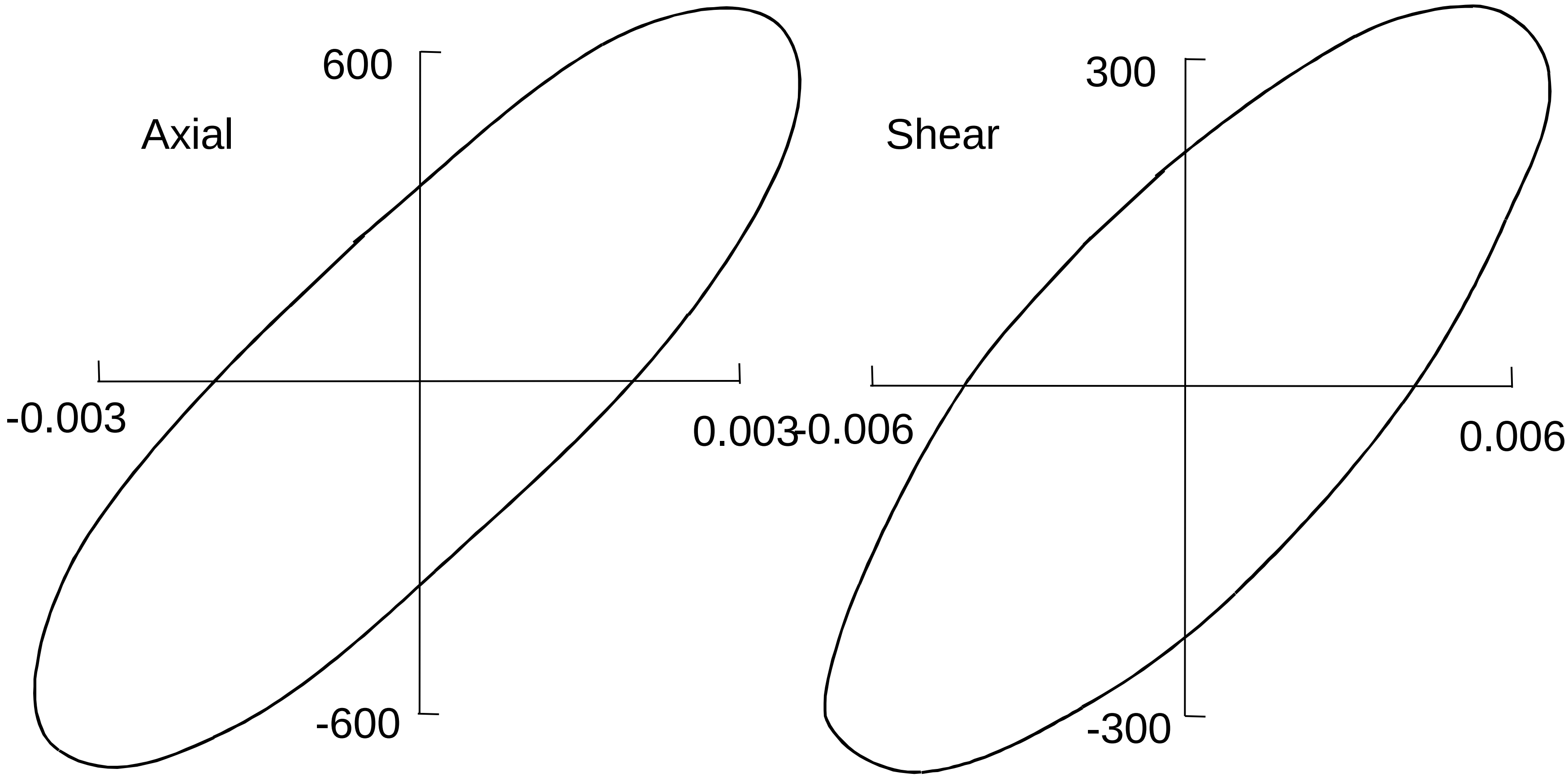
Out-of-phase

In-Phase

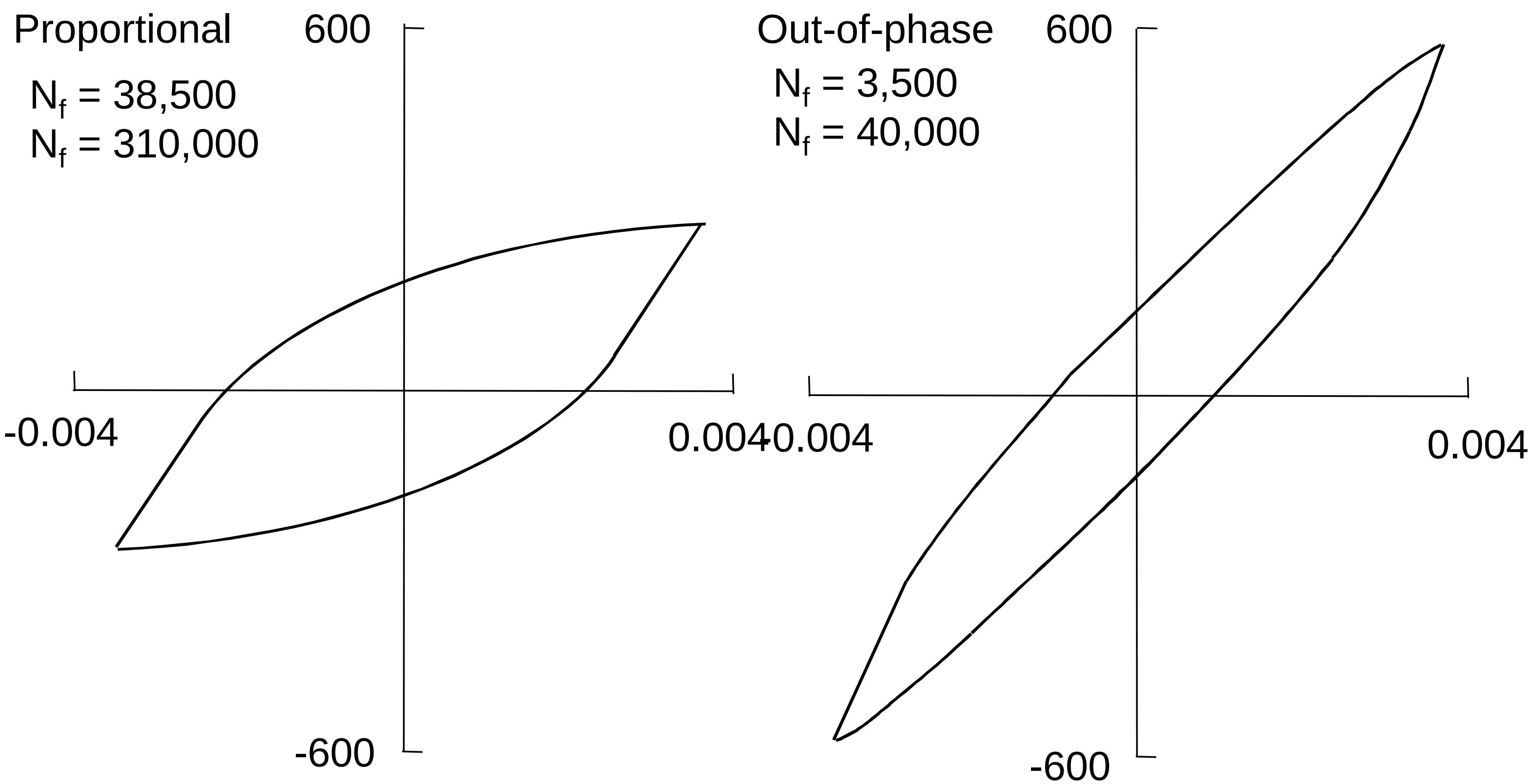




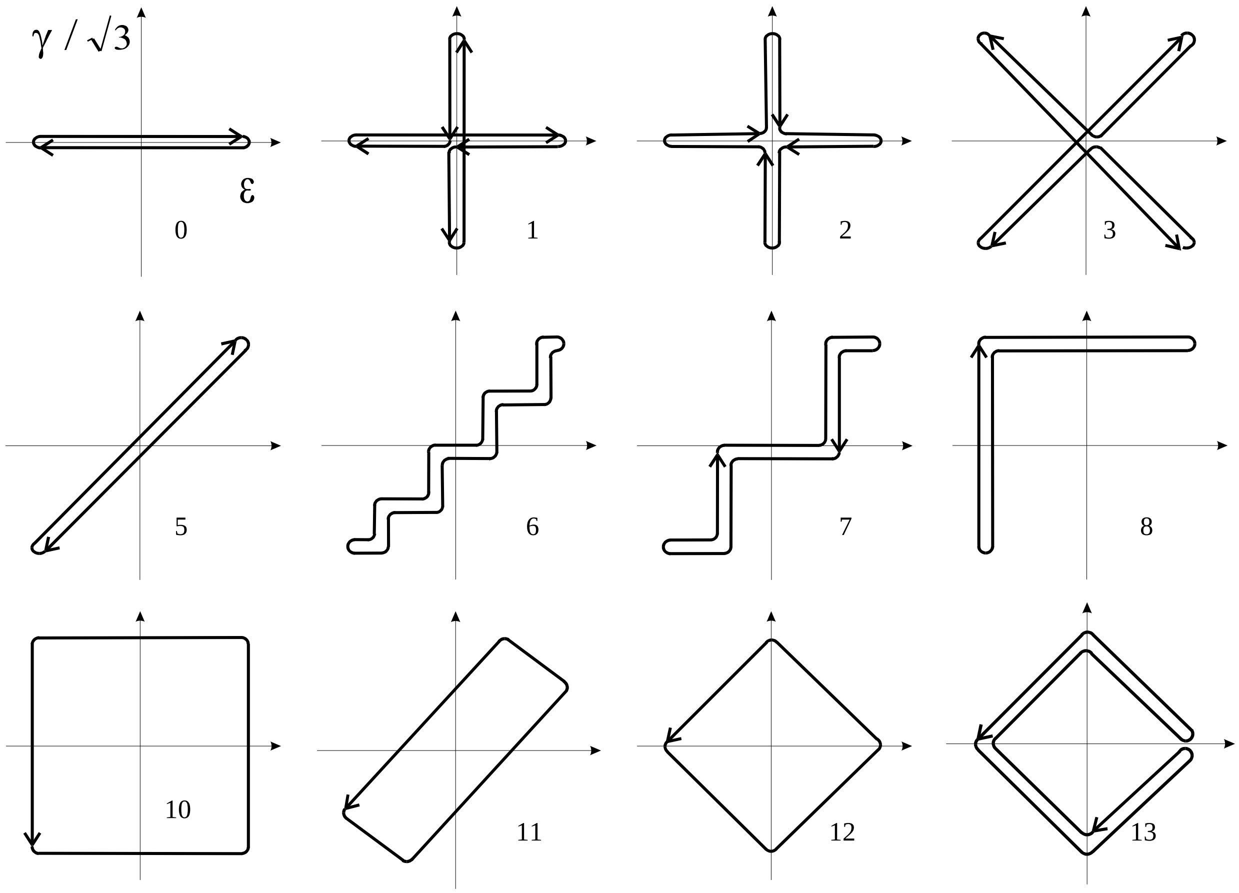
90° Out-of-Phase



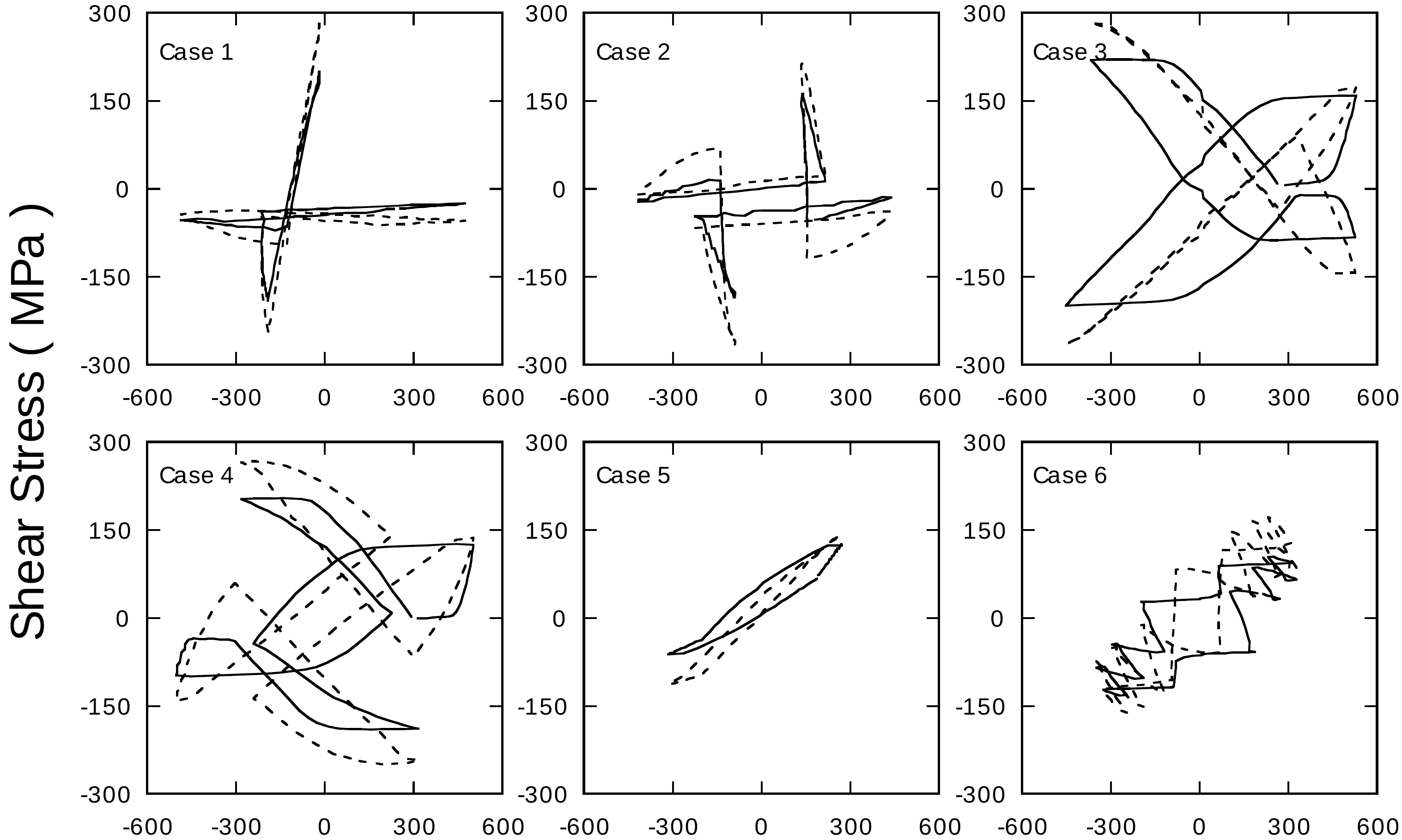
Critical Plane



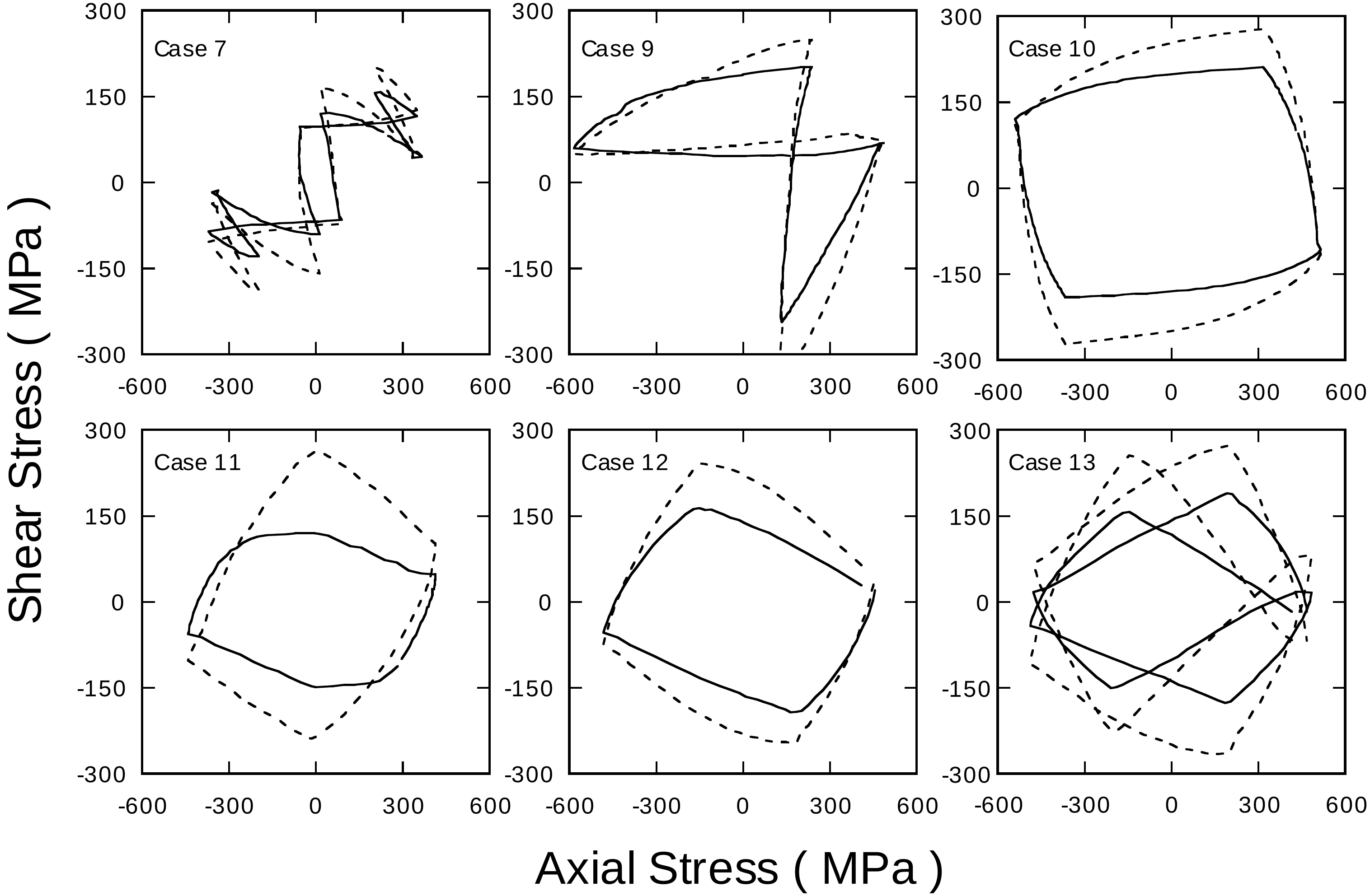
Loading Histories



Stress-Strain Response

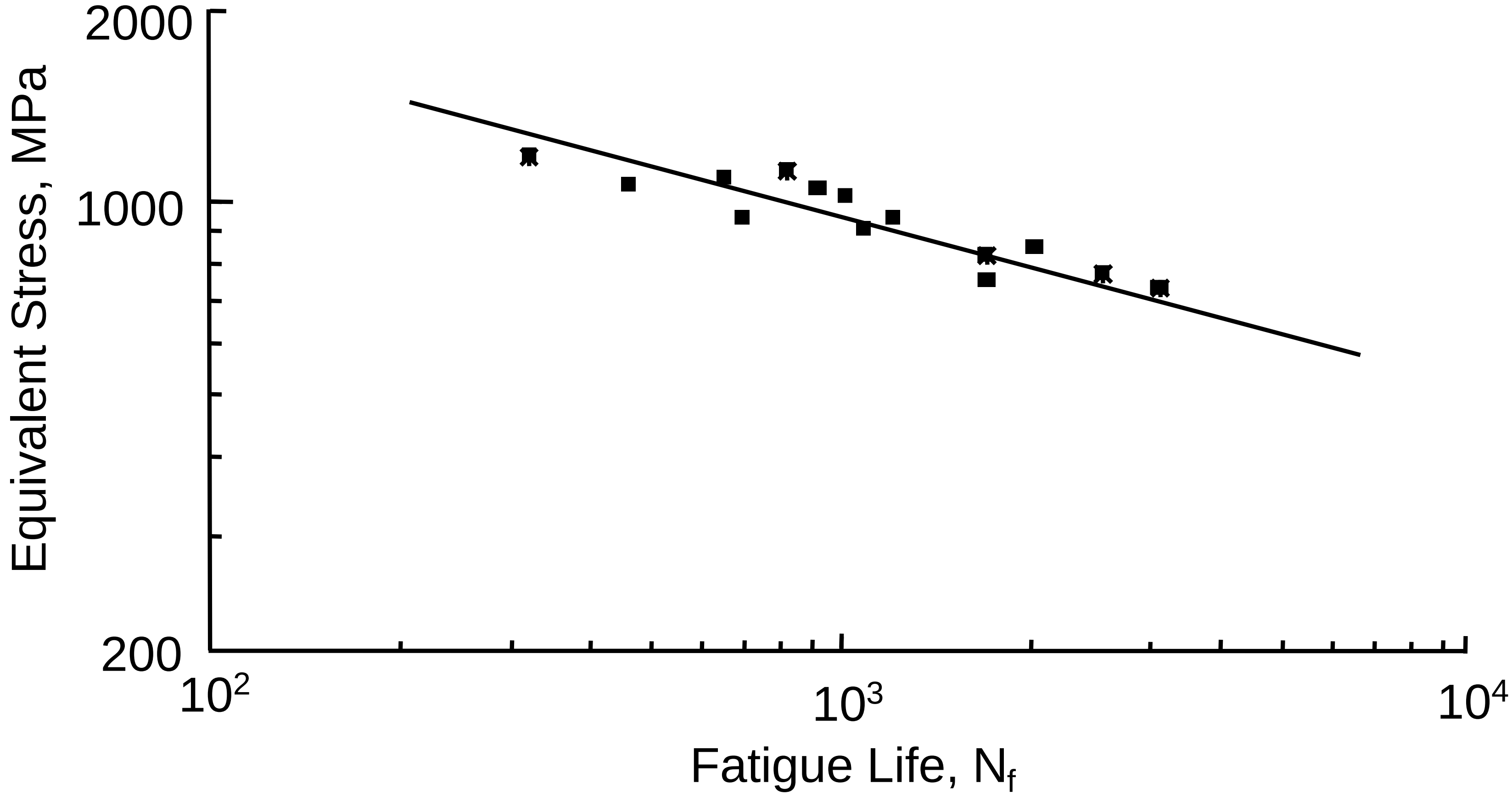


Stress-Strain Response (continued)



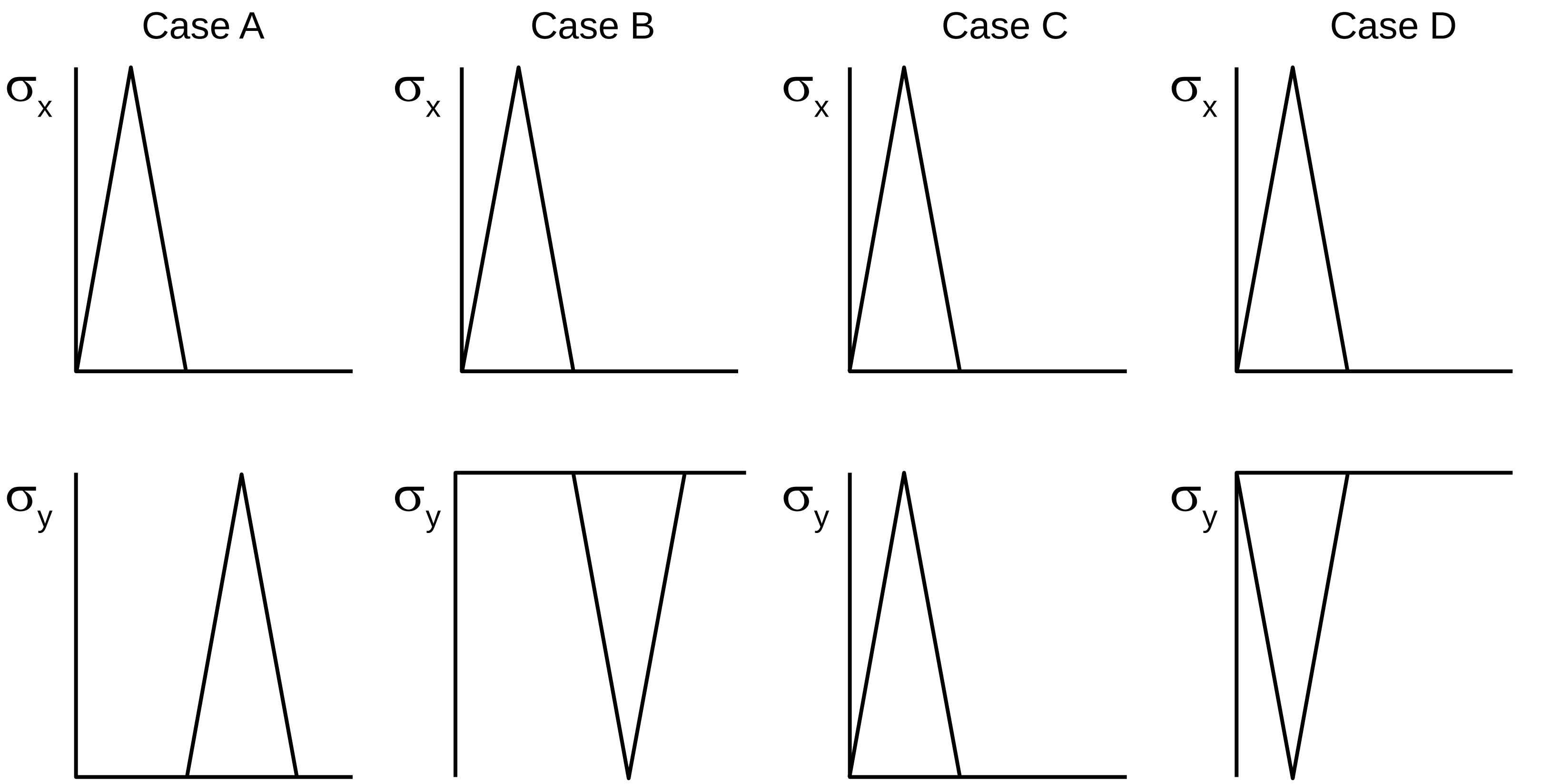
Maximum Stress

All tests have the same strain ranges

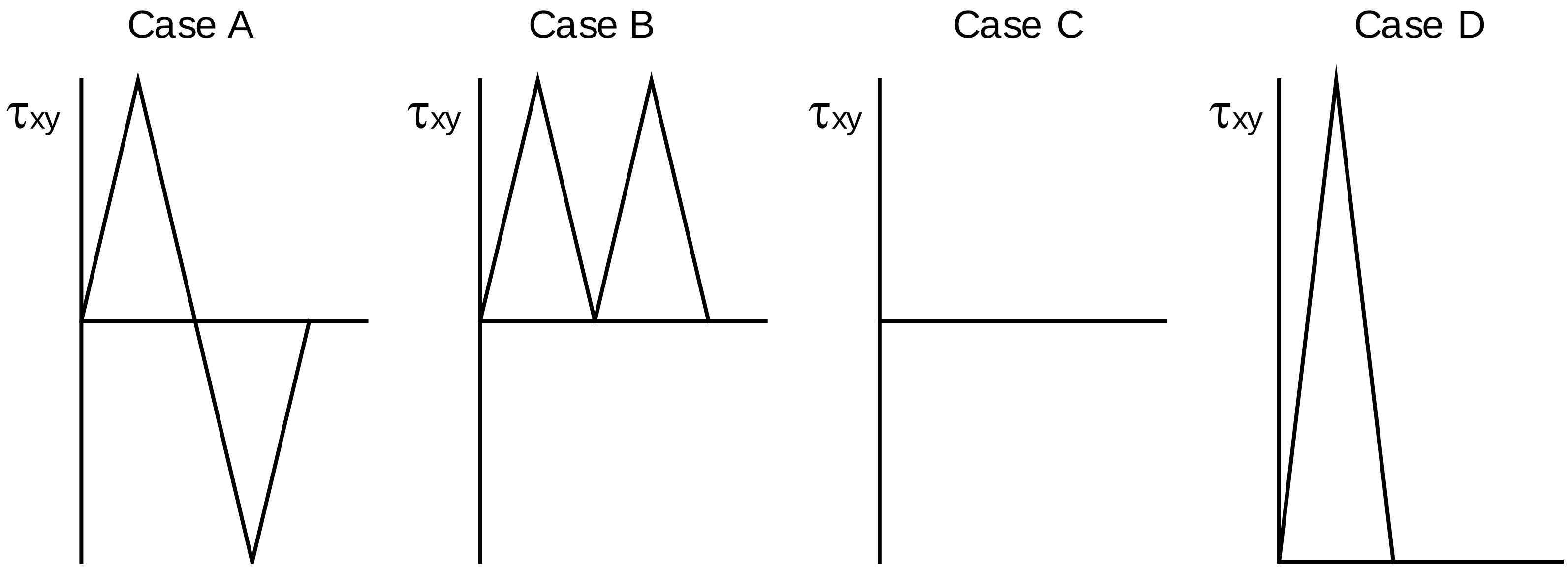


Nonproportional hardening results in lower fatigue lives

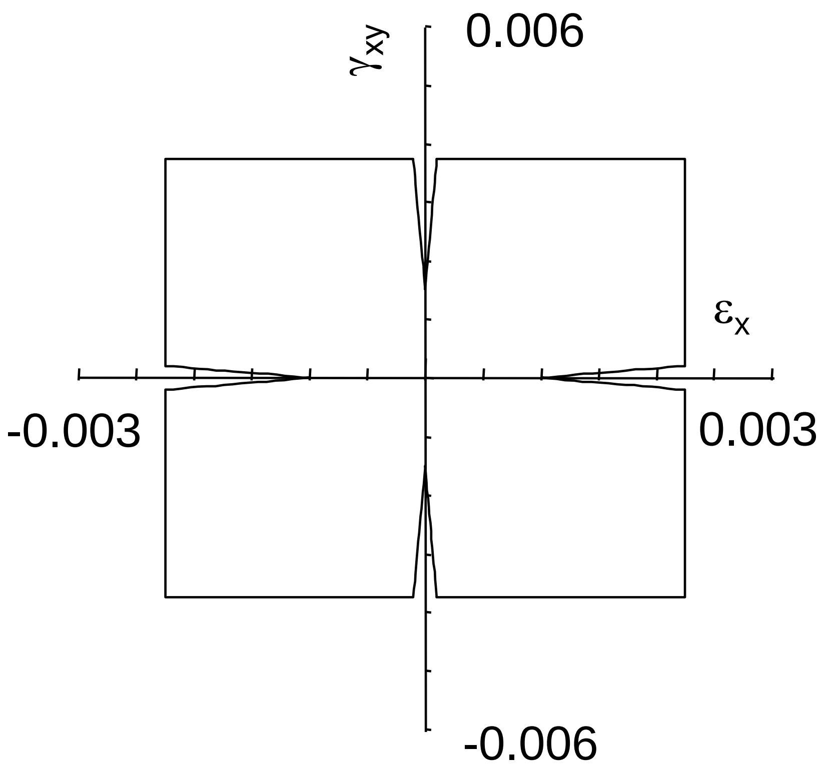
Nonproportional Example

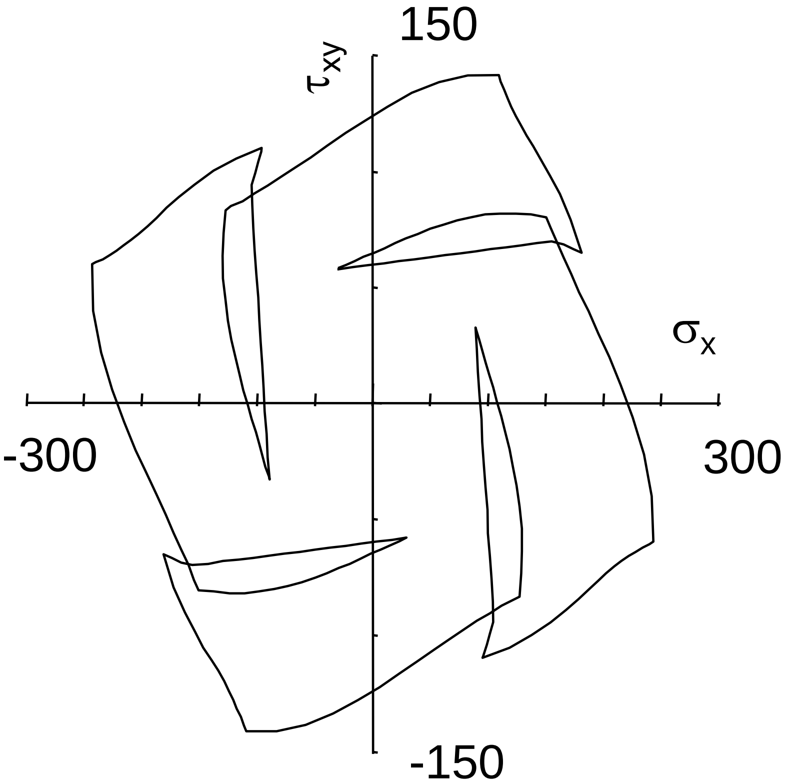


Shear Stresses

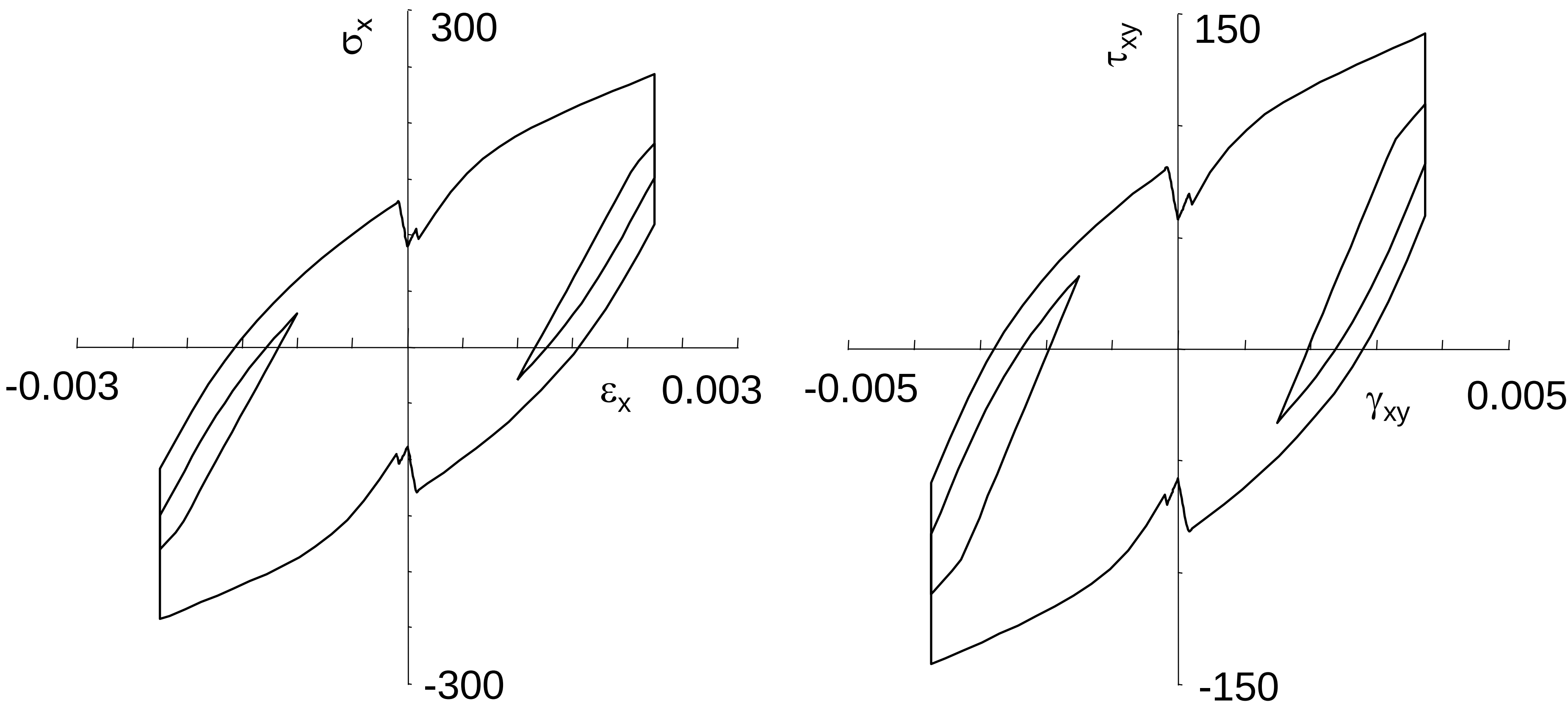


Simple Variable Amplitude History

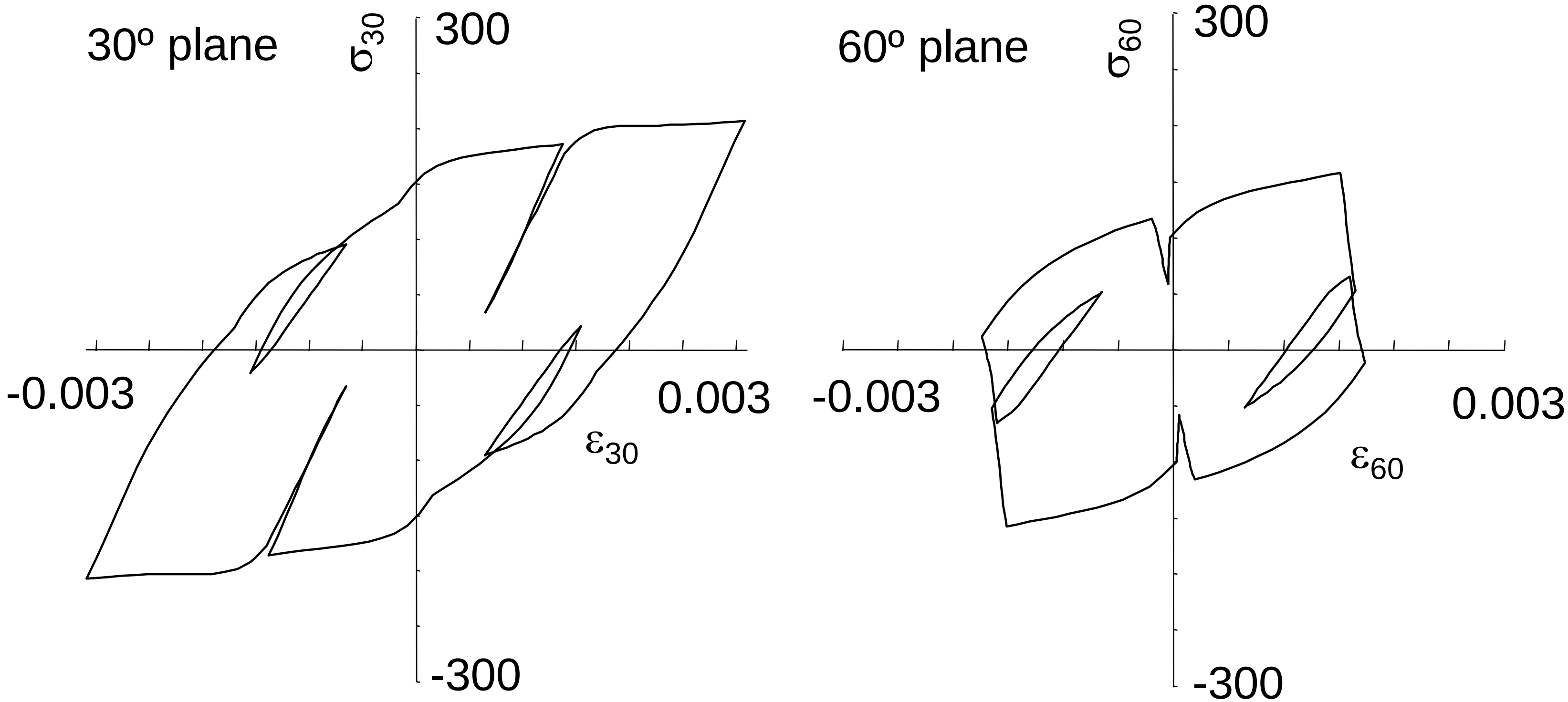




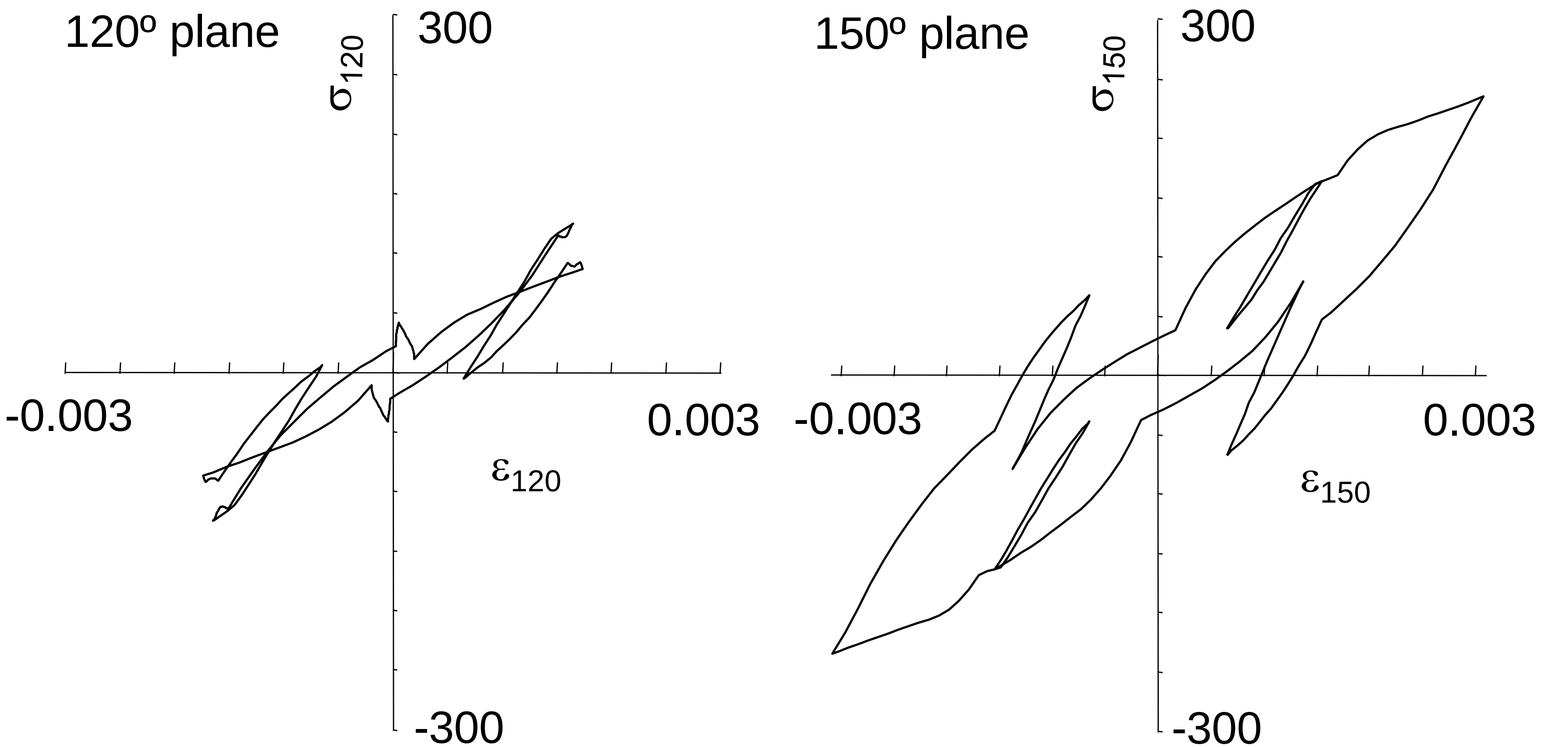
Stress-Strain on 0° Plane



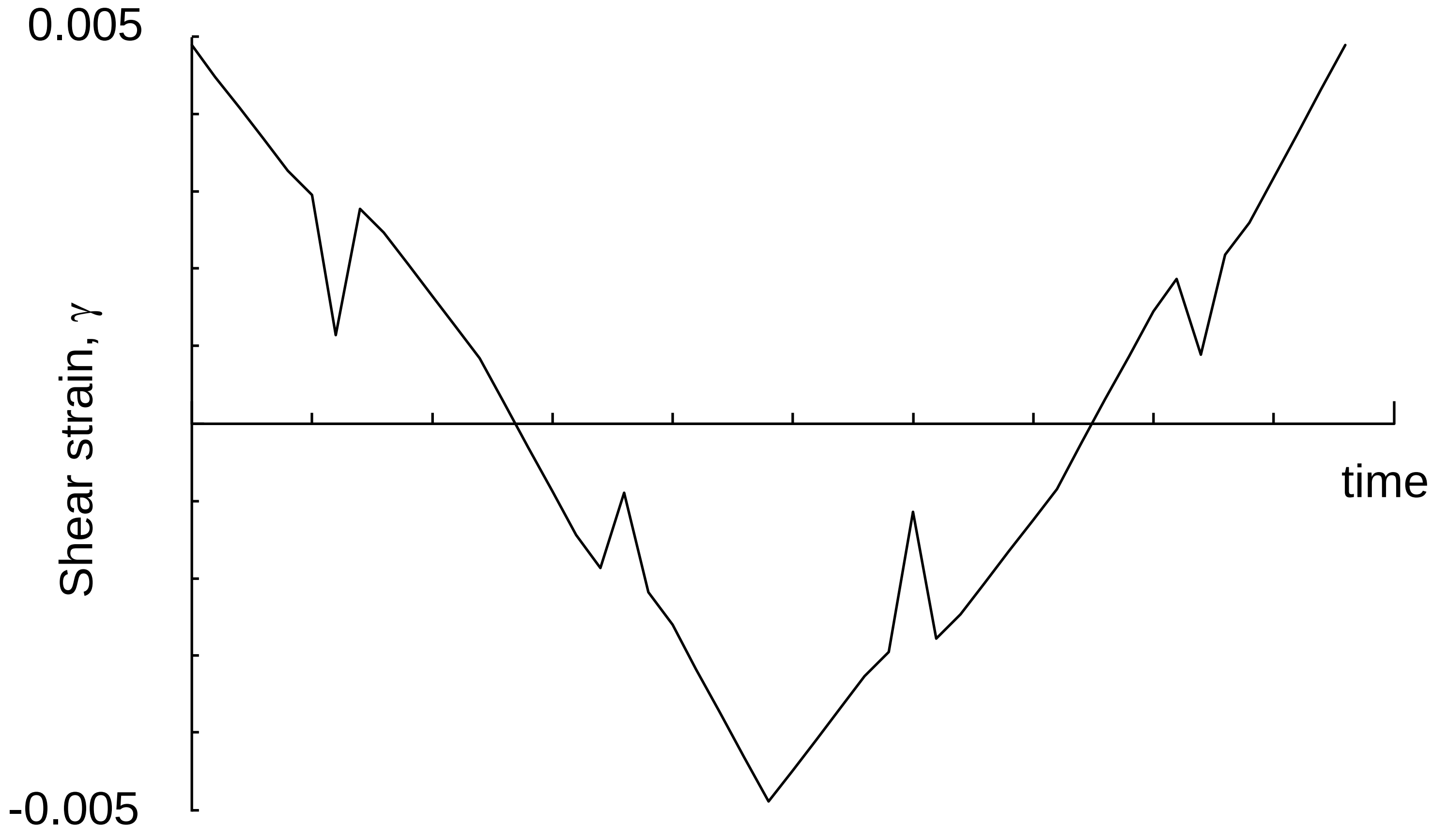
Stress-Strain on 30° and 60° Planes



Stress-Strain on 120° and 150° Planes



Shear Strain History on Critical Plane



Fatigue Calculations

Load or strain history

→ Cyclic plasticity model

→ Stress and strain tensor

→ Search for critical plane

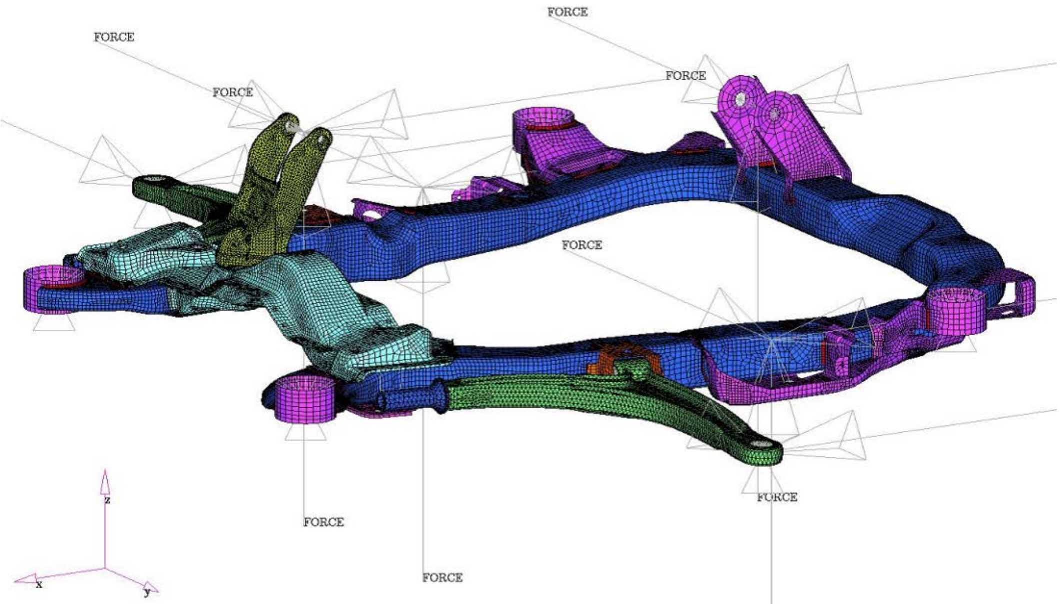
An Example

- Analysis model

  Single event

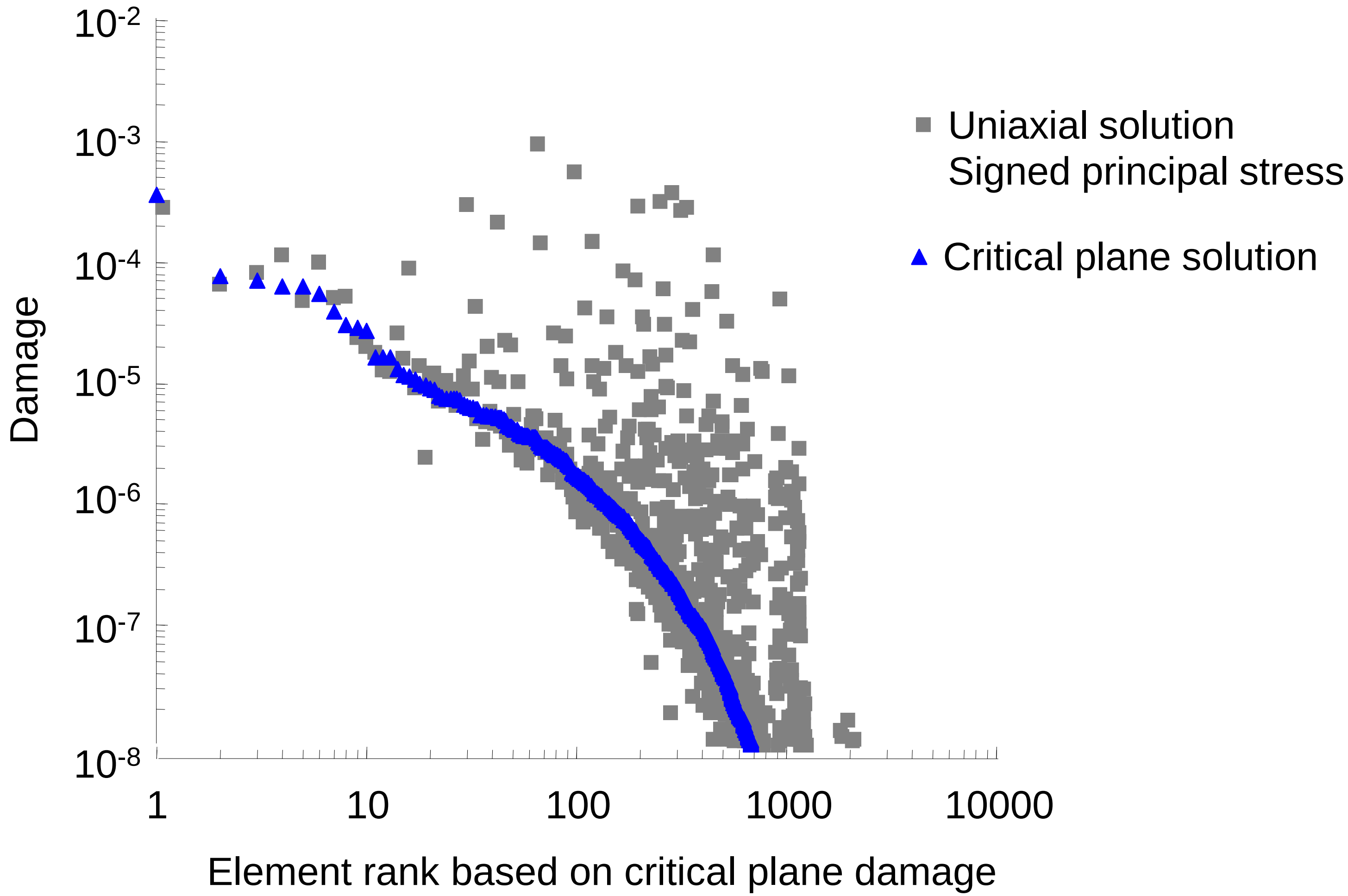
  16 input channels

  2240 elements



From Khosrovaneh, Pattu and Schnaidt “Discussion of Fatigue Analysis Techniques for Automotive Applications” Presented at SAE 2004.

Biaxial and Uniaxial Solution



Nonproportional Loading Summary

- Nonproportional cyclic hardening increases stress levels
- Critical plane models are used to assess fatigue damage

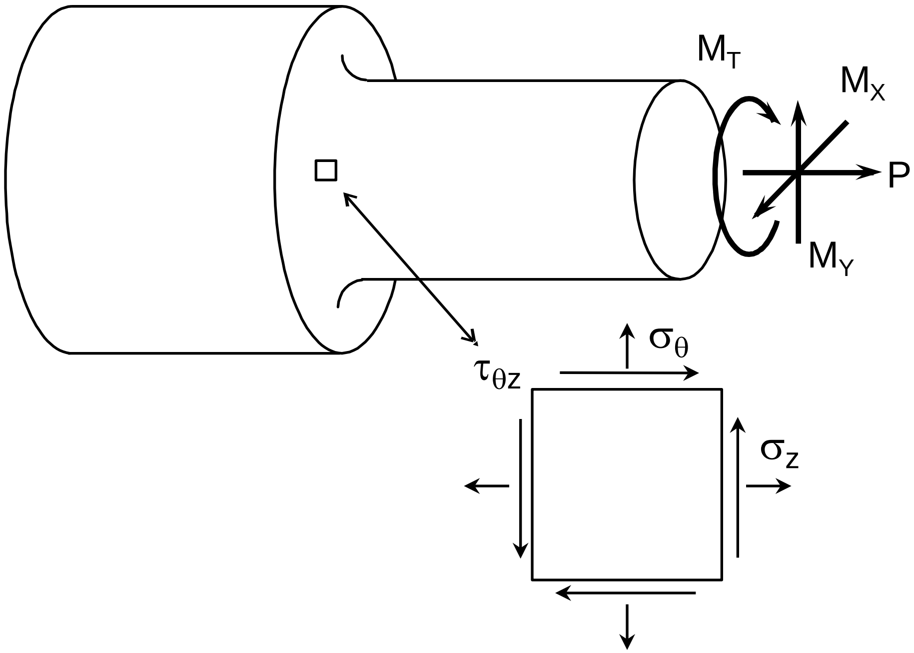
Outline

- State of Stress
- Stress-Strain Relationships
- Fatigue Mechanisms
- Multiaxial Testing
- Stress Based Models
- Strain Based Models
- Fracture Mechanics Models
- Nonproportional Loading
- **Stress Concentrations**

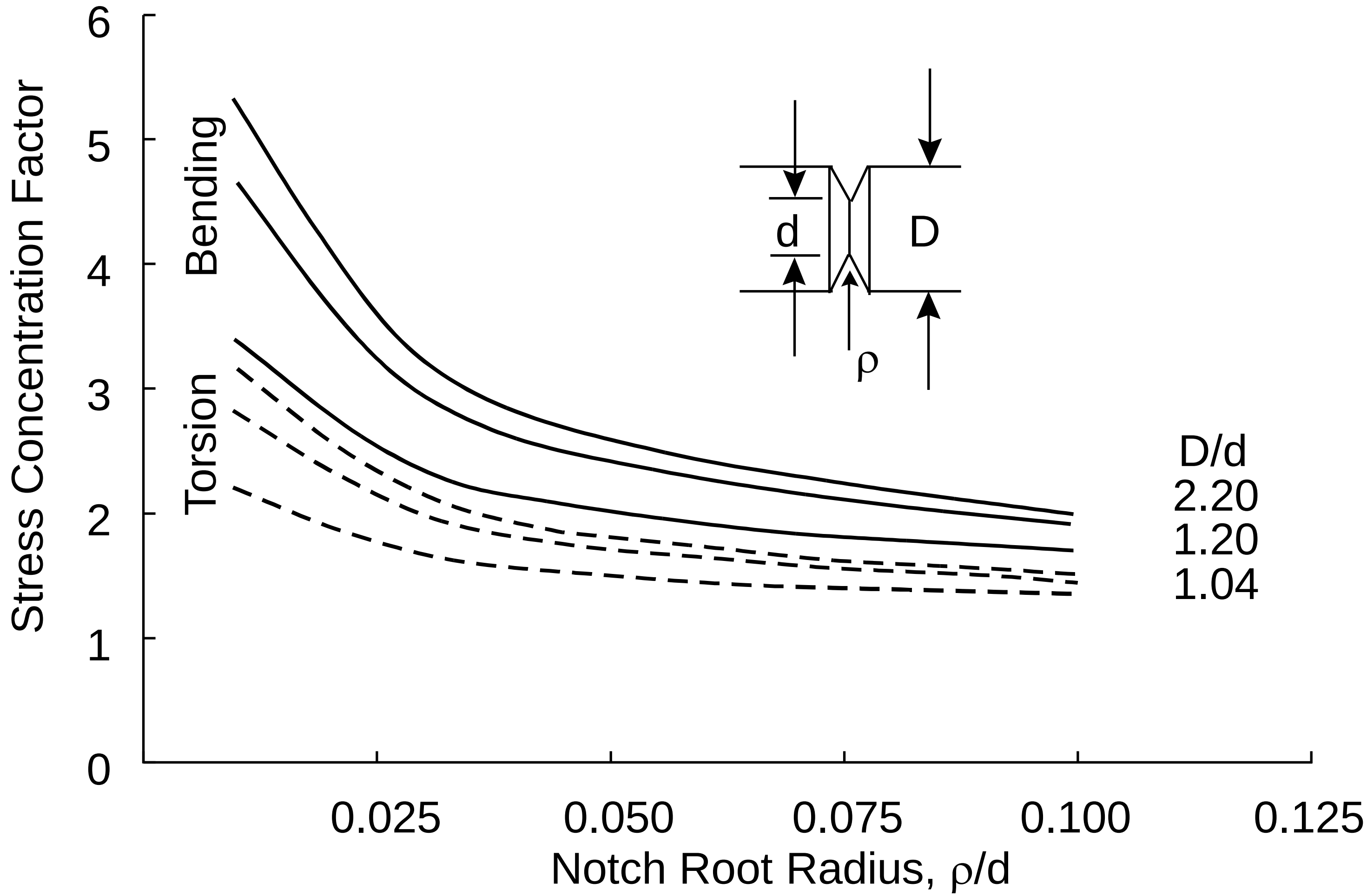
Notches

- Stress and strain concentrations
- Nonproportional loading and stressing
- Fatigue notch factors
- Cracks at notches

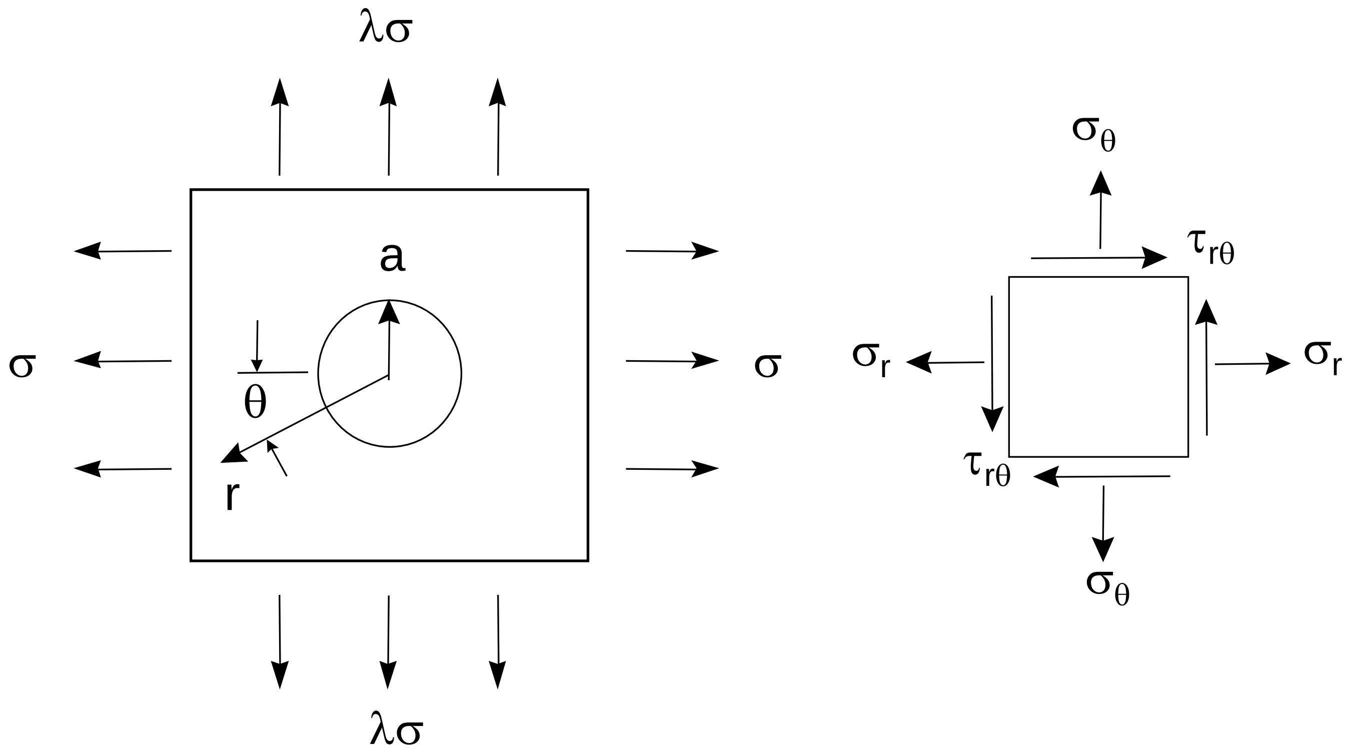
Notched Shaft Loading



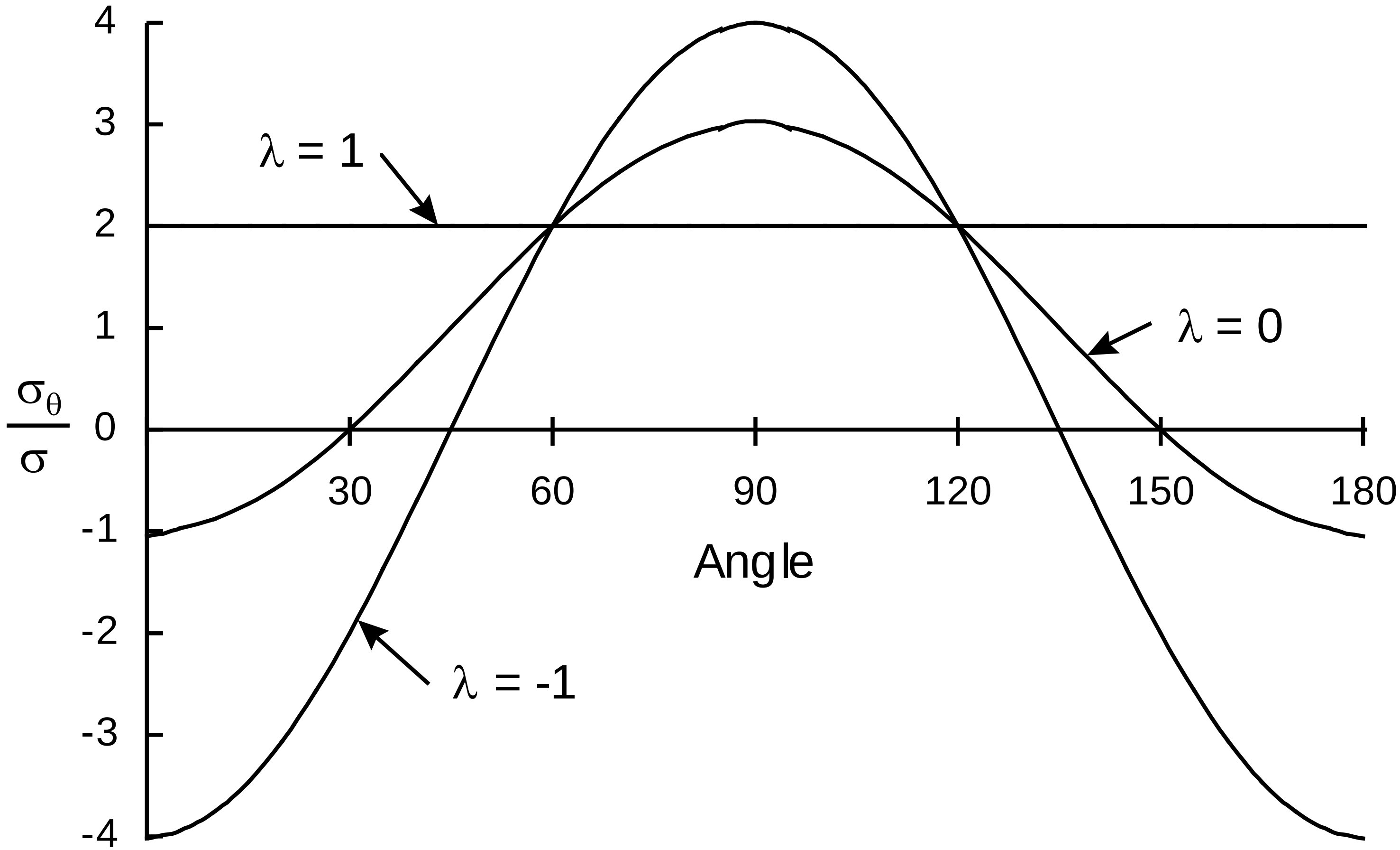
Stress Concentration Factors



Hole in a Plate

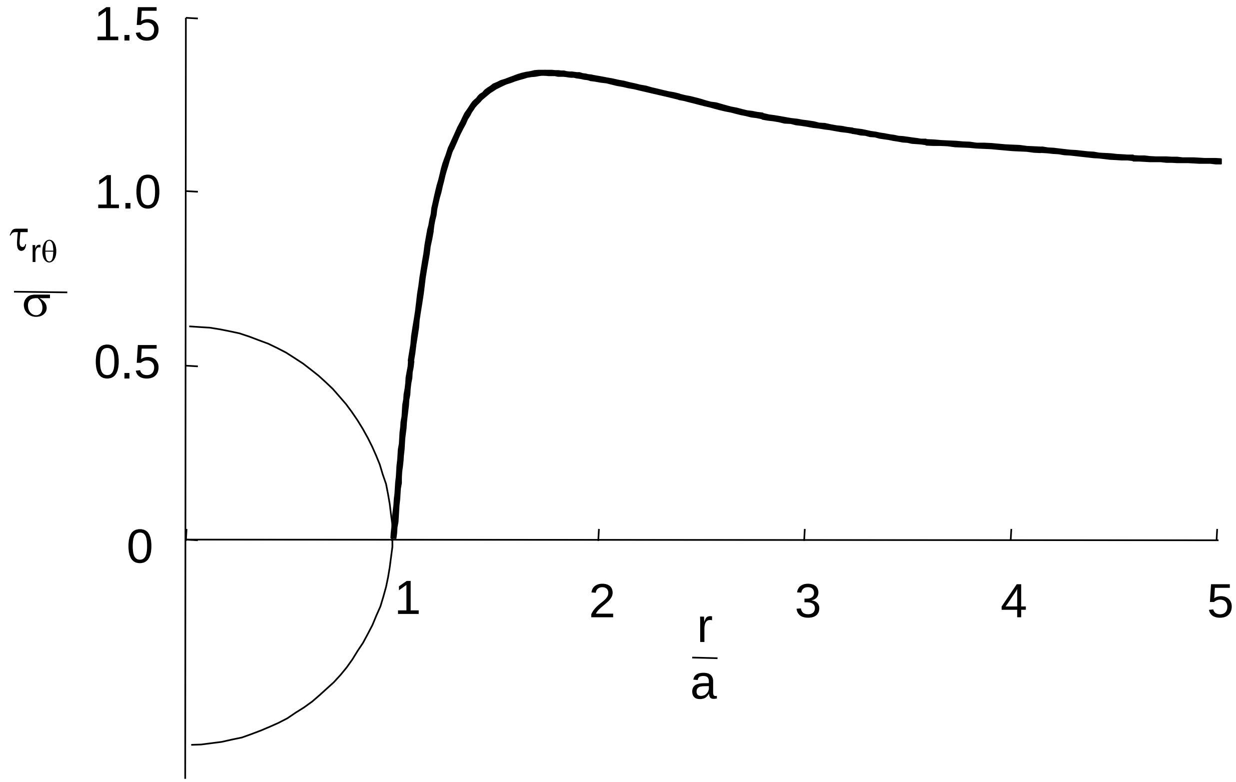


Stresses at the Hole

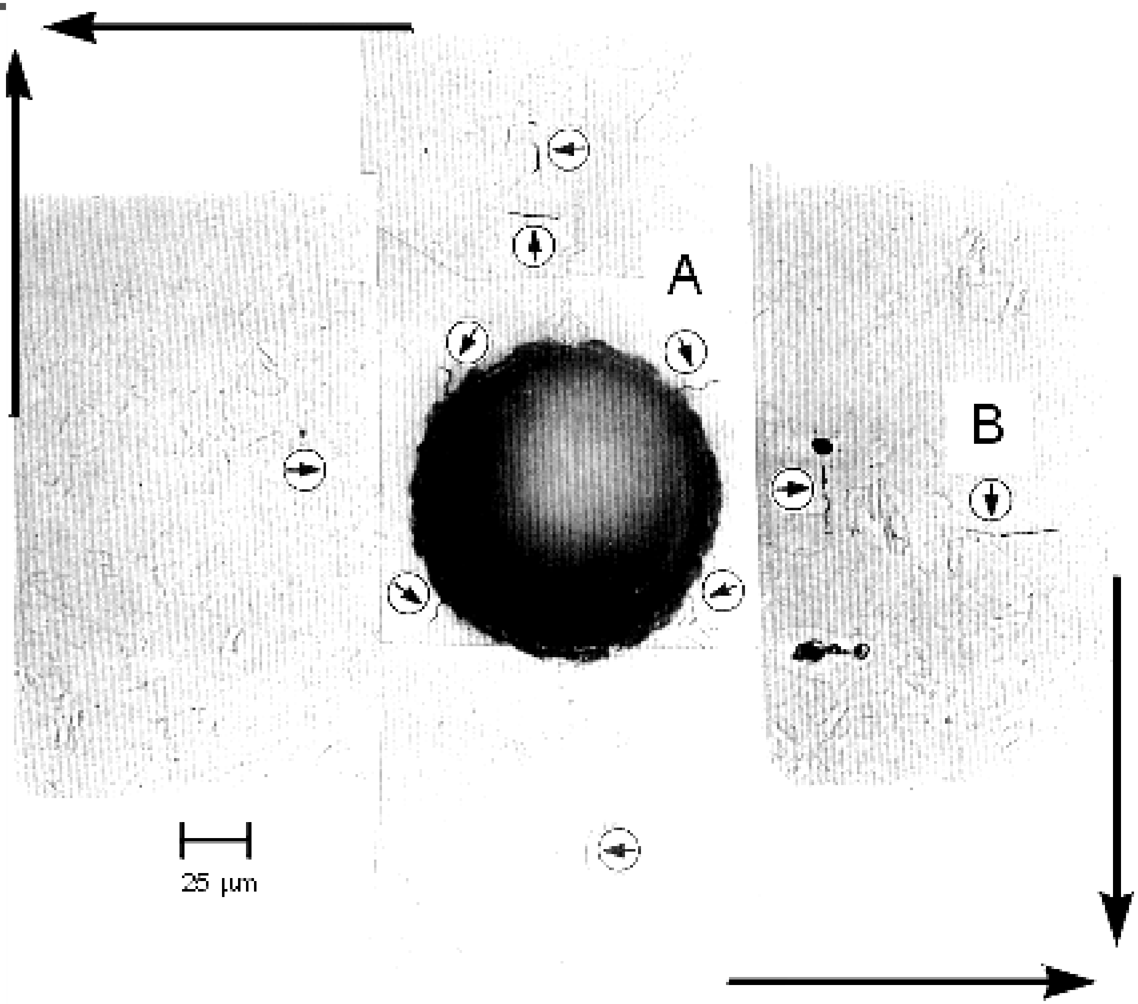


Stress concentration factor depends on type of loading

Shear Stresses during Torsion



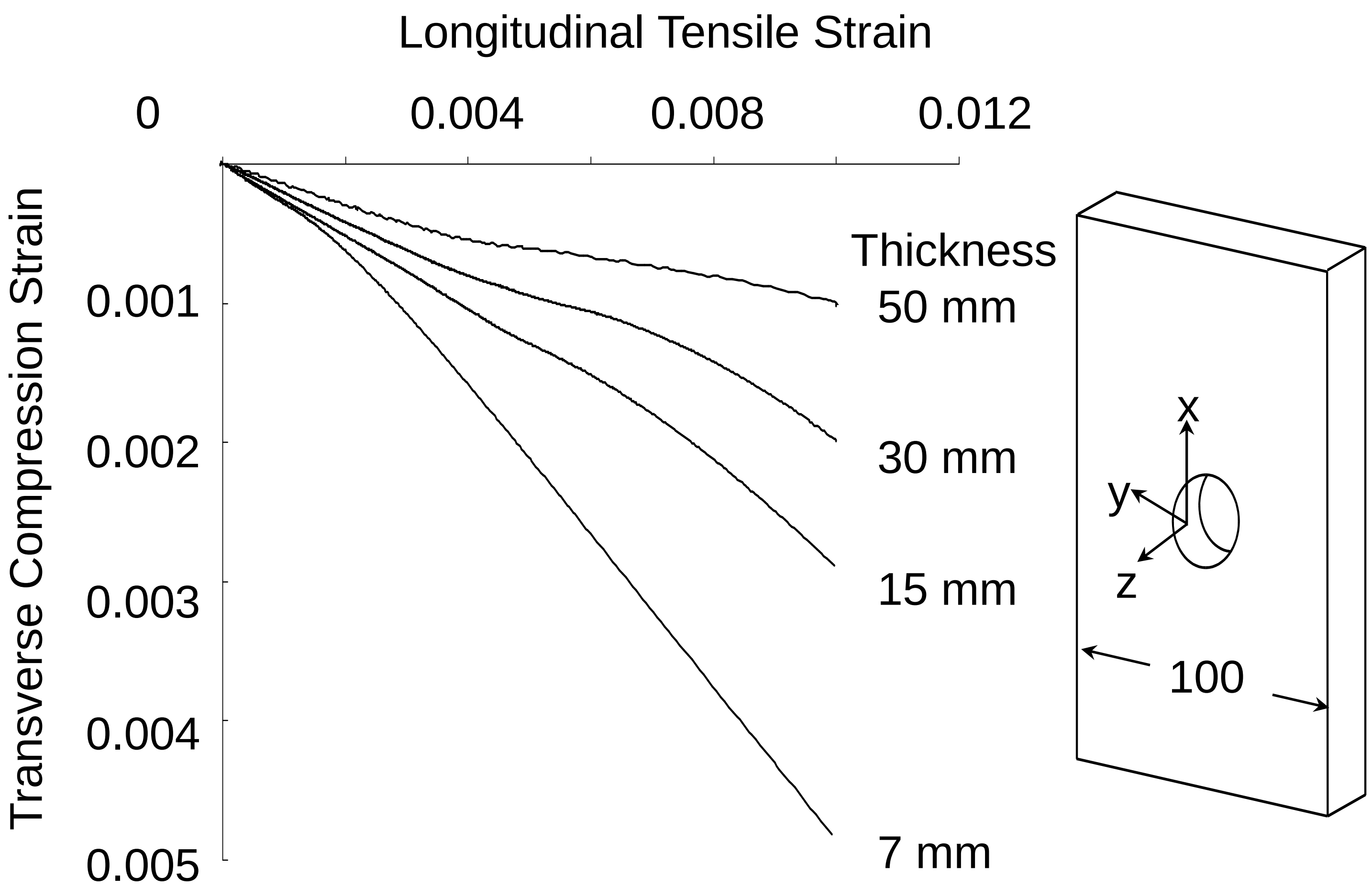
Torsion Experiments



Multiaxial Loading

- Uniaxial loading that produces multiaxial stresses at notches
- Multiaxial loading that produces uniaxial stresses at notches
- Multiaxial loading that produces multiaxial stresses at notches

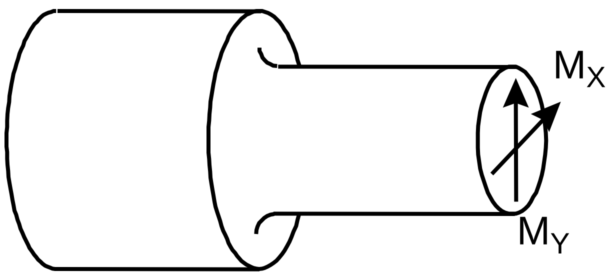
Thickness Effects

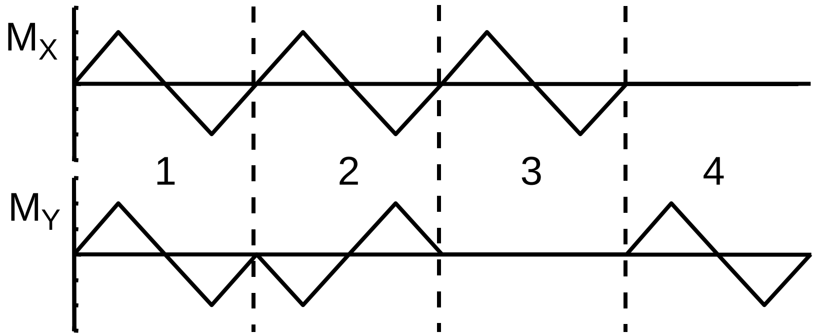


Multiaxial Loading

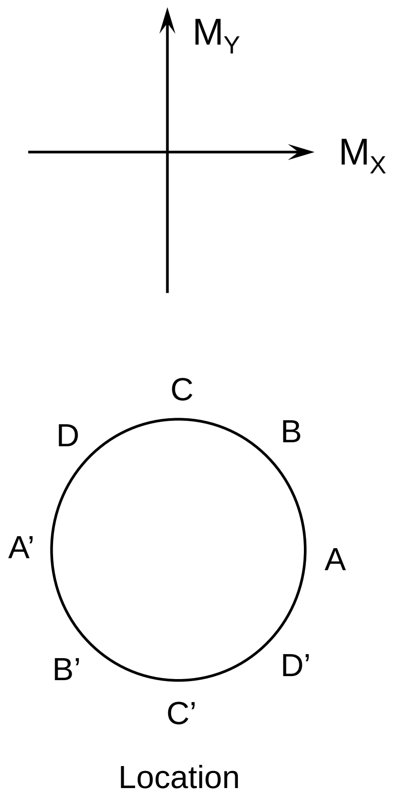
- Uniaxial loading that produces multiaxial stresses at notches
- Multiaxial loading that produces uniaxial stresses at notches
- Multiaxial loading that produces multiaxial stresses at notches

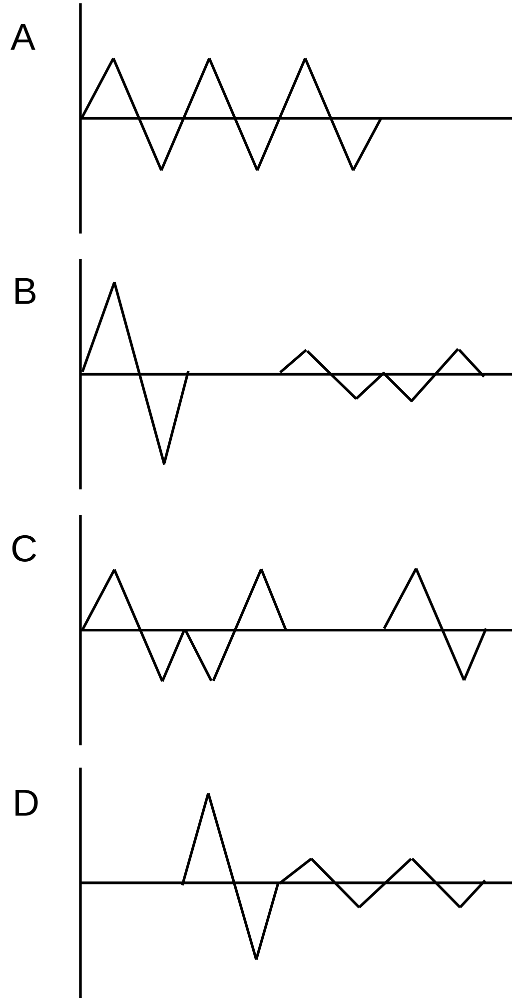
Applied Bending Moments





Bending Moments on the Shaft





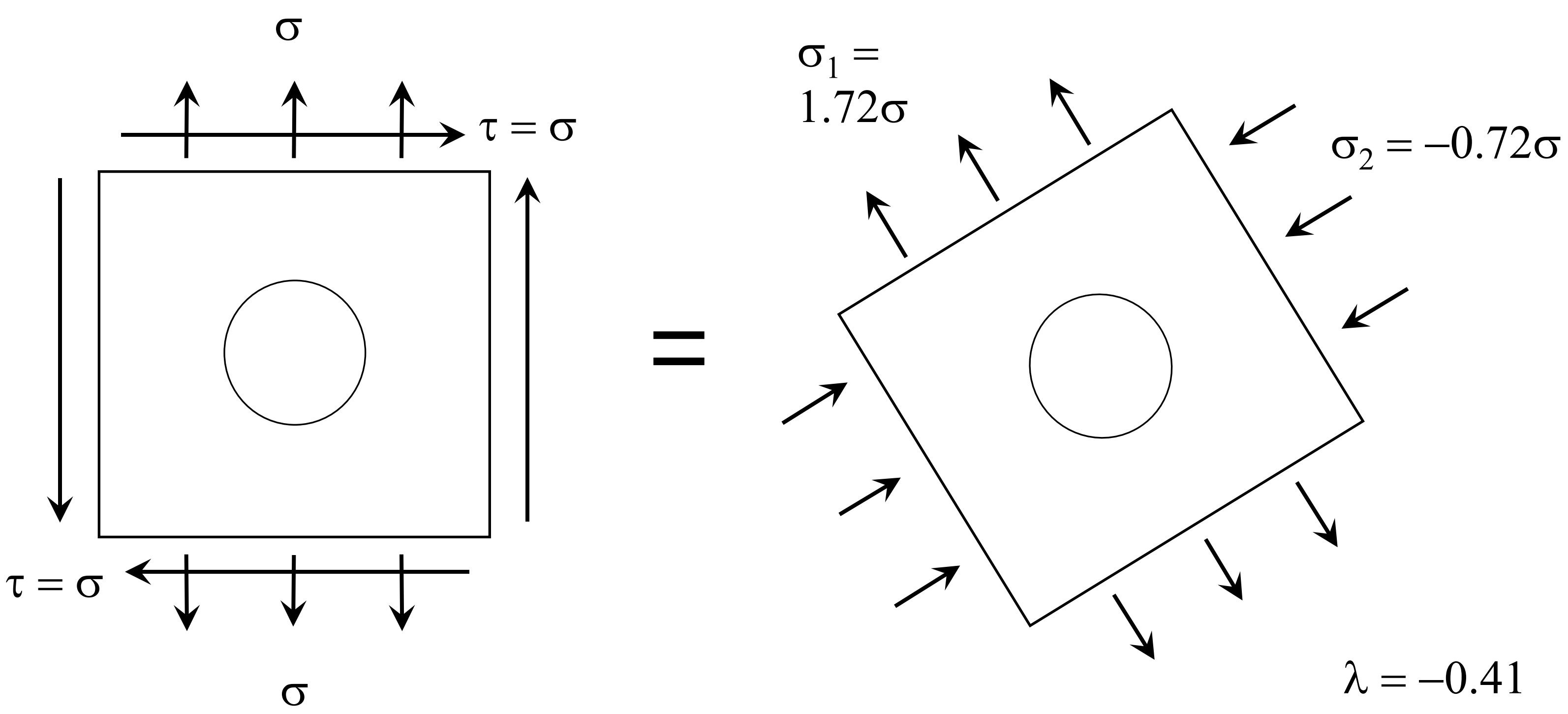
Bending Moments

ΔM	A	B	C	D
2.82		1		1
2.00	3		2	
1.41		2		1
1.00			2	
0.71				2

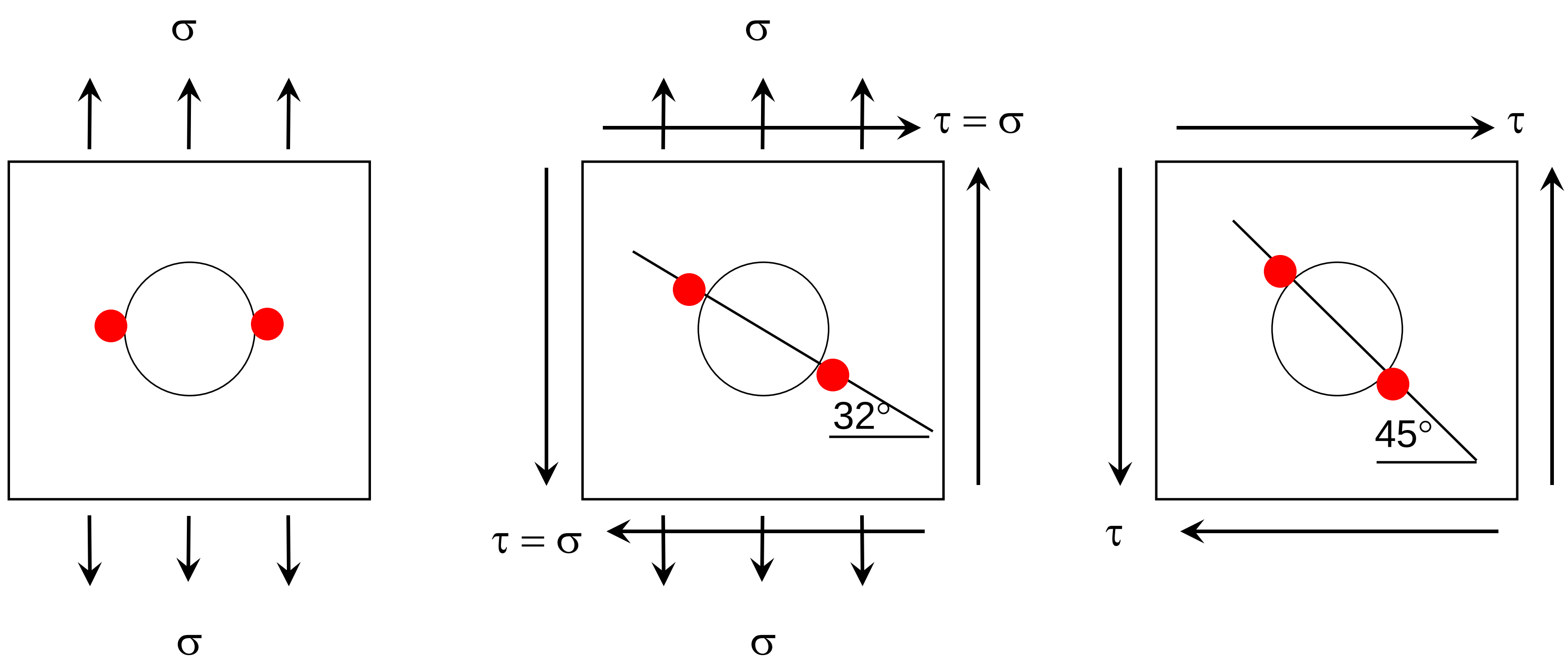
$$\Delta\overline{M} = \sqrt[5]{\sum \Delta M^5}$$

	A	B	C	D
$\Delta\overline{M}$	2.49	2.85	2.31	2.84

Combined Loading



Maximum Tensile Stress Location



$K_t = 3$

$\sigma_1 = \sigma$

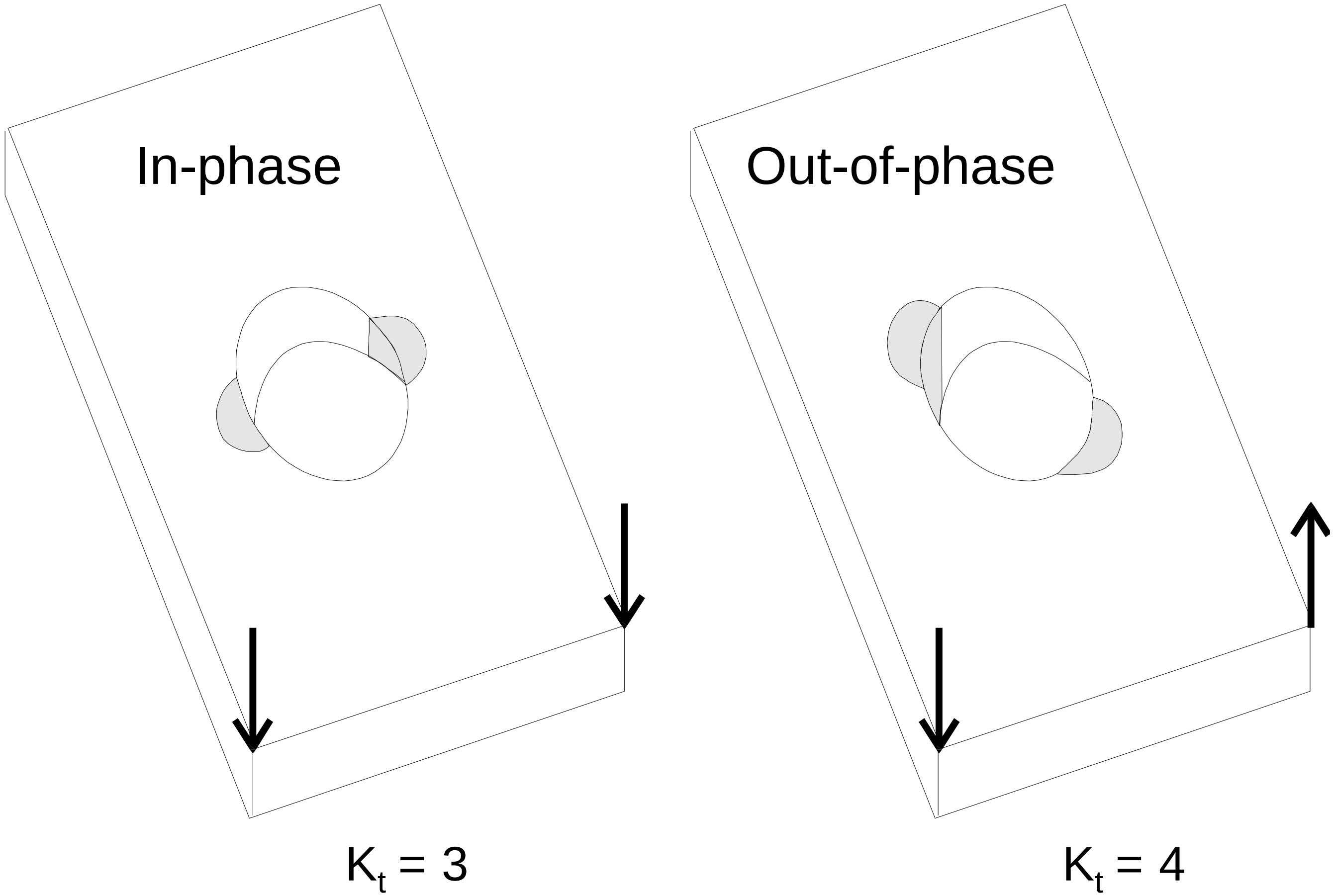
$K_t = 3.41$

$\sigma_1 = 1.72\sigma$

$K_t = 4$

$\sigma_1 = \tau$

In and Out of Phase Loading

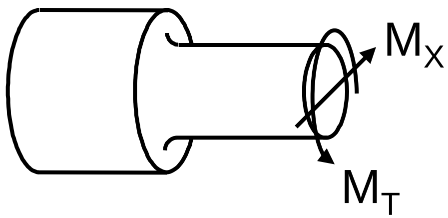


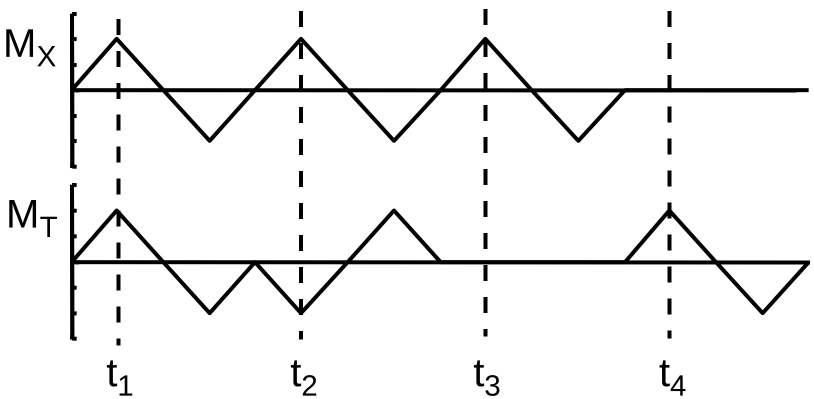
Damage location changes with load phasing

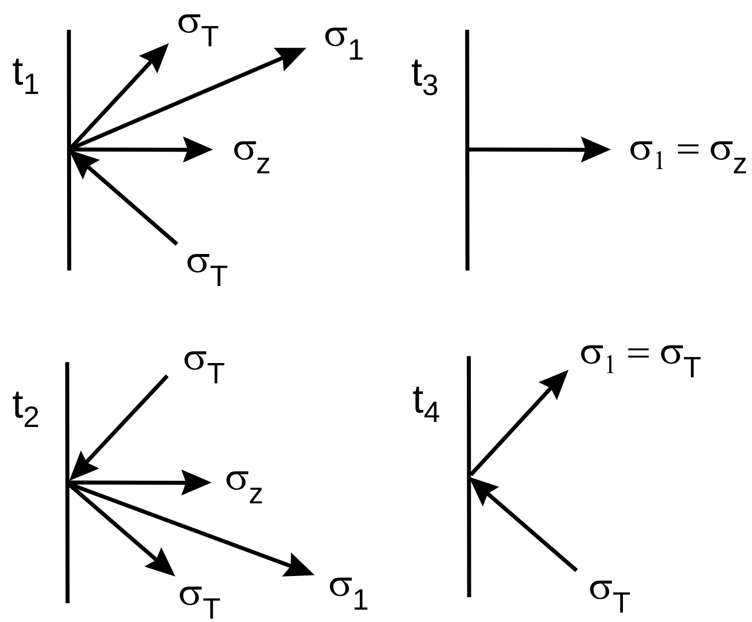
Multiaxial Loading

- Uniaxial loading that produces multiaxial stresses at notches
- Multiaxial loading that produces uniaxial stresses at notches
- Multiaxial loading that produces multiaxial stresses at notches

Torsion Loading

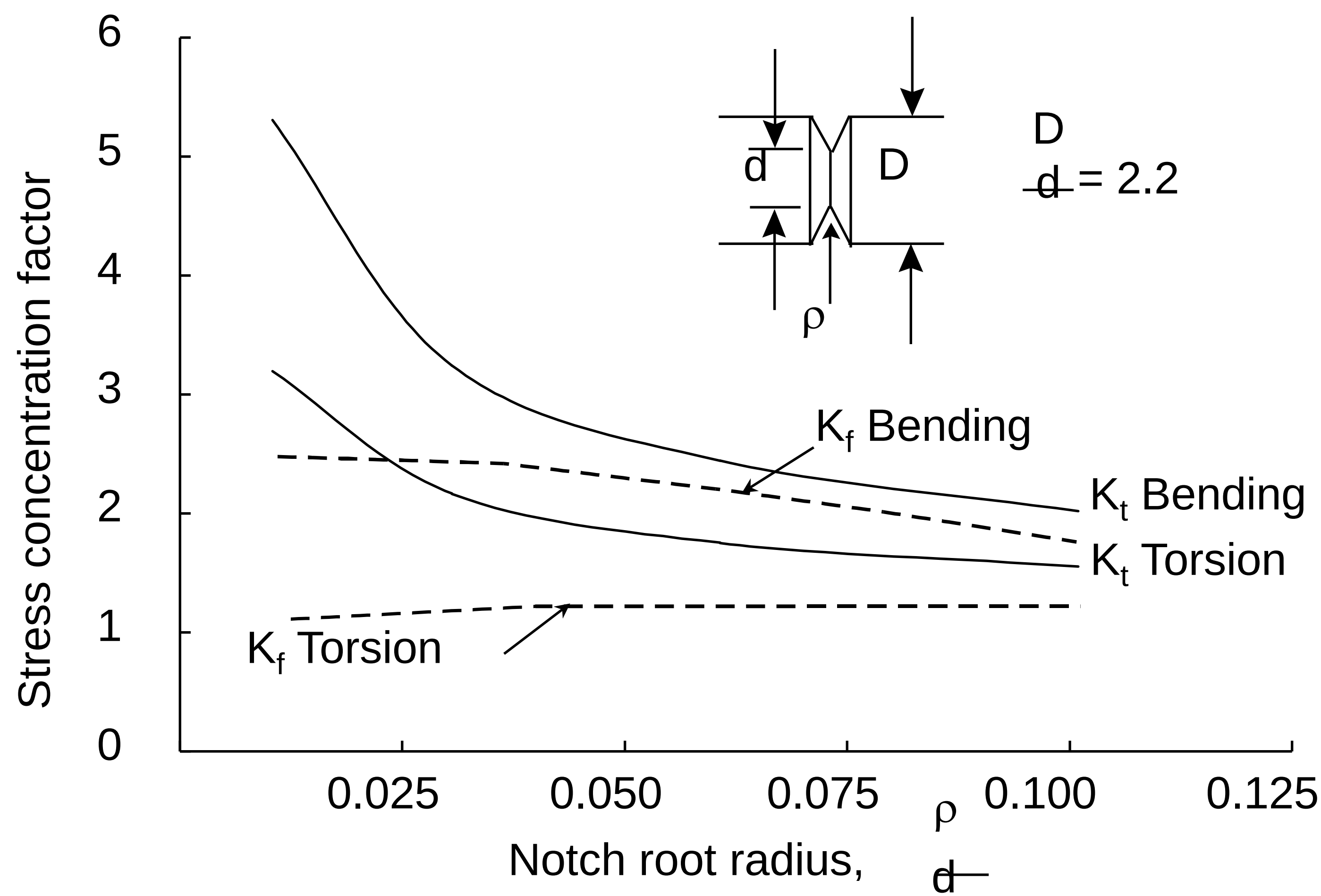




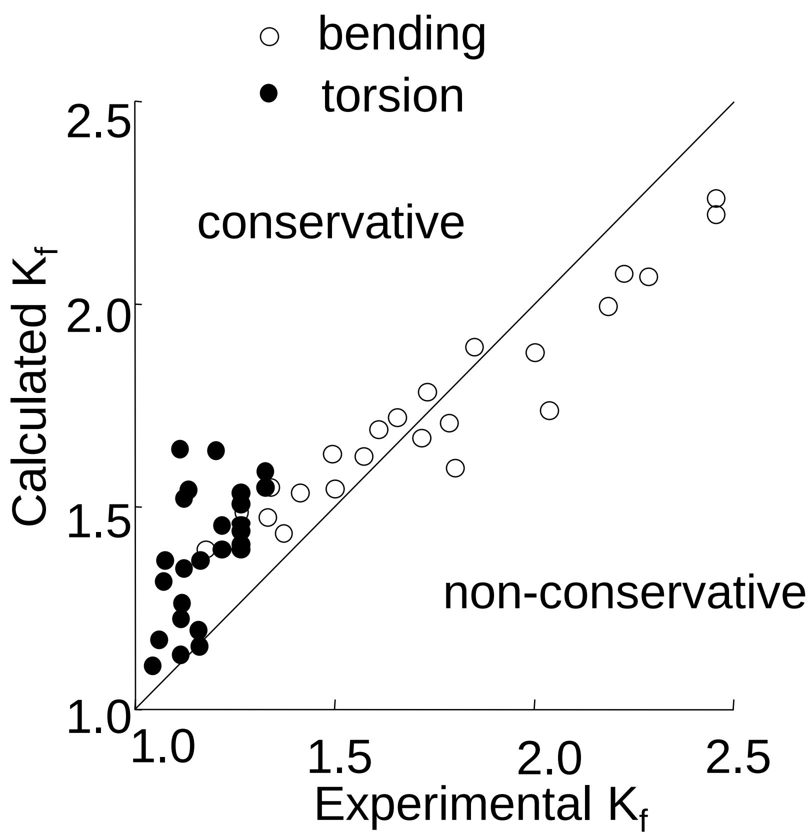


Out-of-phase shear loading is needed to produce nonproportional stressing

Fatigue Notch Factors



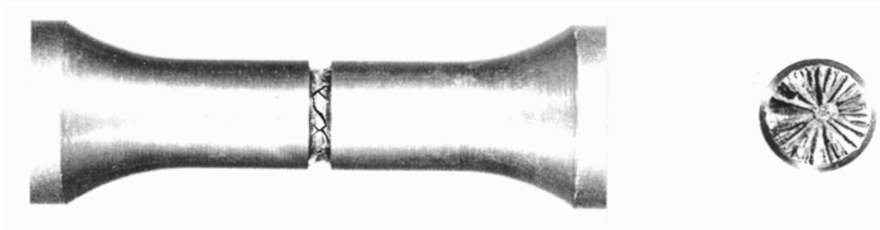
Fatigue Notch Factors (continued)



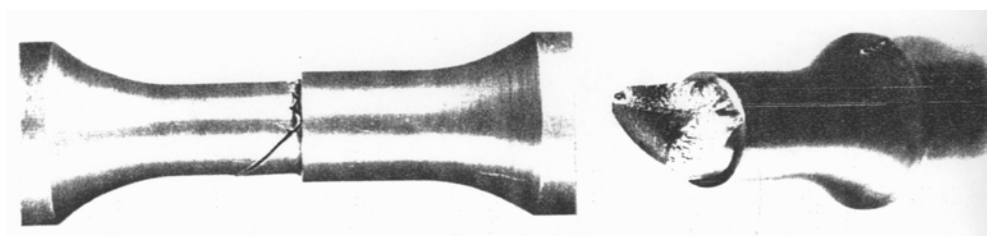
Peterson's Equation

$$K_f = 1 + \frac{K_T - 1}{1 + \frac{a}{r}}$$

Fracture Surfaces in Torsion

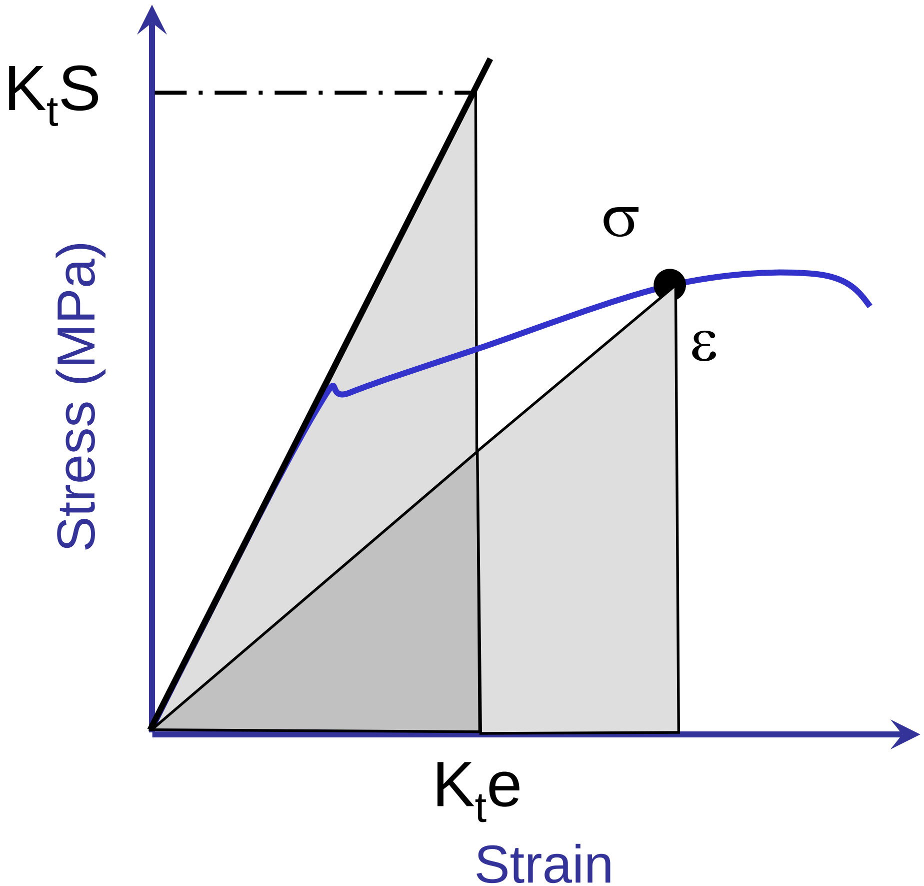


Circumferencial Notch



Shoulder Fillet

Neuber's Rule



Actual stress

$$\underbrace{K_t S}\, K_t e = \sigma \varepsilon$$

Stress calculated with elastic assumptions

$$^{e}S\ ^{e}e = \sigma \varepsilon$$

For cyclic loading

$$\Delta\,^{e}S^{2} = E \Delta\sigma \Delta\varepsilon$$

Multiaxial Neuber's Rule

Define Neuber's rule in equivalent variables

$$\Delta^e \overline{S}^2 = E \Delta\overline{\sigma}\Delta\overline{\varepsilon}$$

Stress strain curve

$$\Delta\overline{\varepsilon} = \frac{\Delta\overline{\sigma}}{E} + \left(\frac{\Delta\overline{\sigma}}{K'}\right)^{\frac{1}{n'}}$$

Constitutive equation

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} f(E,K',n') \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Five equations and six unknowns

Ignore Plasticity Theory

$$\varepsilon_2 = \frac{^e e_2}{^e e_1} \varepsilon_1$$

$$\varepsilon_3 = \frac{^e e_3}{^e e_1} \varepsilon_1$$

$$\sigma_2 = \frac{^e S_2}{^e S_1} \sigma_1$$

$$\sigma_3 = \frac{^e S_3}{^e S_1} \sigma_1$$

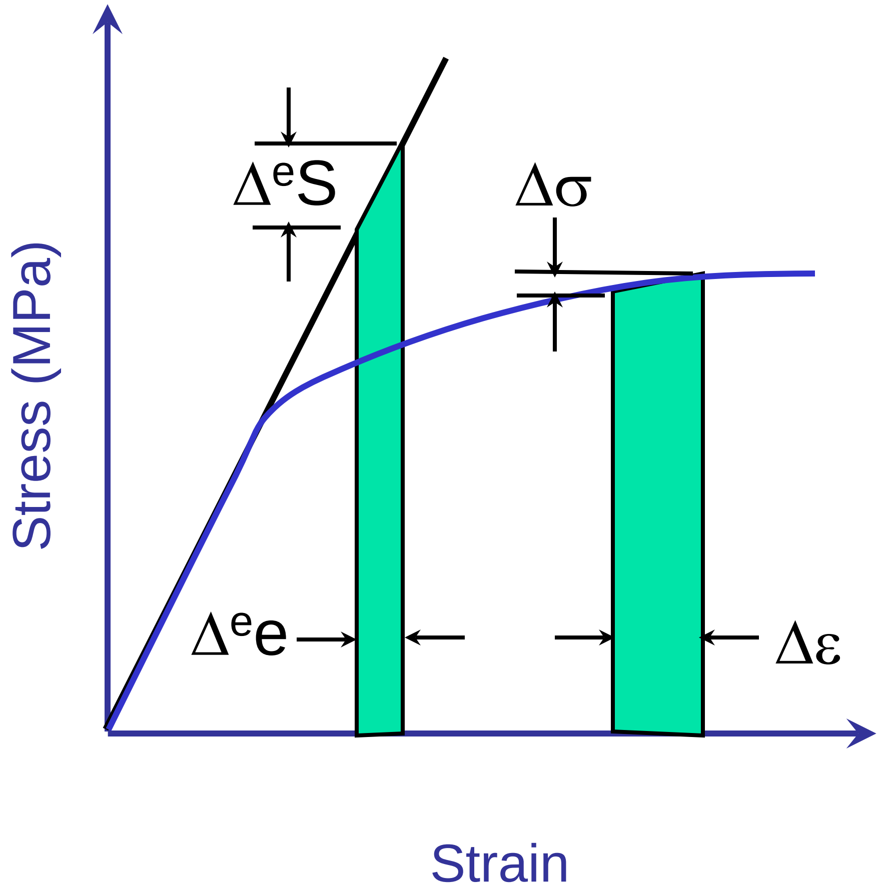
Hoffman and Seeger

$$\frac{\sigma_2}{\sigma_1} = \frac{{}^{e}S_2}{{}^{e}S_1}$$

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{{}^{e}e_2}{{}^{e}e_1}$$

Glinka

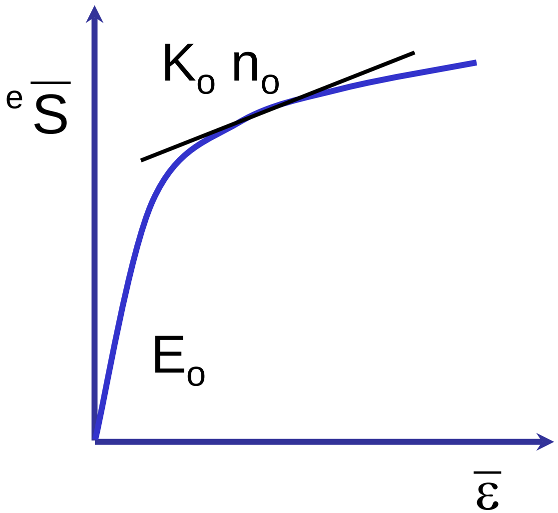
Strain energy density



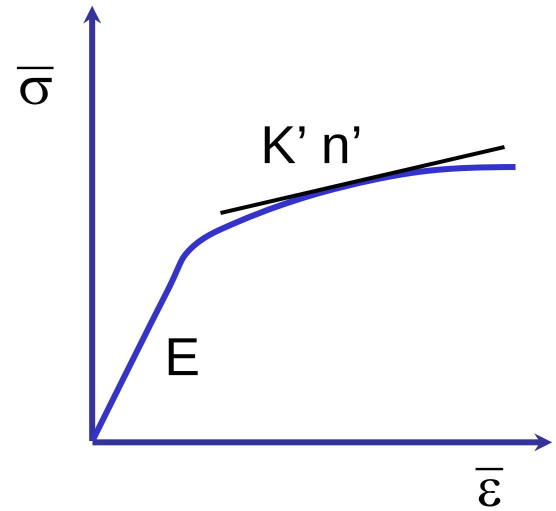
$$\frac{\Delta\sigma_{ij}\Delta\varepsilon_{ij}}{\sum \Delta\sigma_{ij}\Delta\varepsilon_{ij}} = \frac{\Delta^{e} S_{ij} \Delta^{e} e_{ij}}{\sum \Delta^{e} S_{ij} \Delta^{e} e_{ij}}$$

Koettgen-Barkey-Socie

Structural Yield Surface



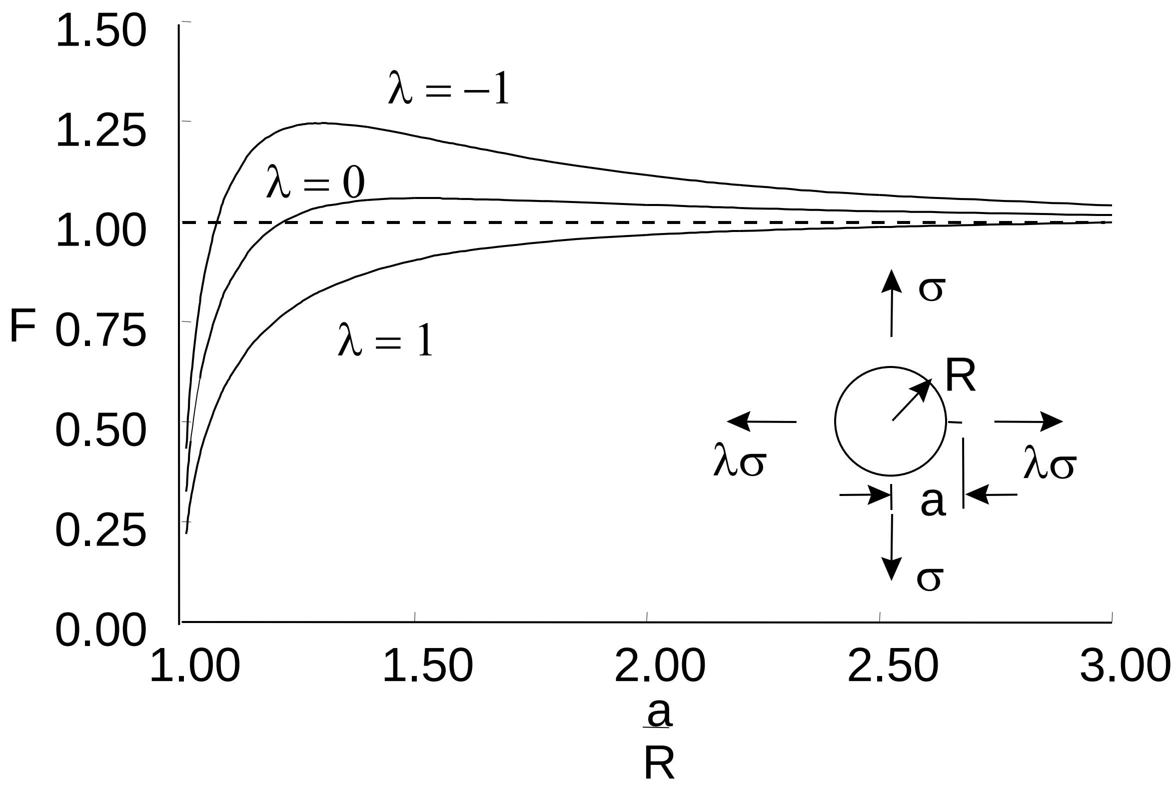
Material Yield Surface



$$\begin{bmatrix} {}^eS_x \\ {}^eS_y \\ {}^eT_{xy} \end{bmatrix} = \begin{bmatrix} f_o(E_o,K_o,n_o) \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

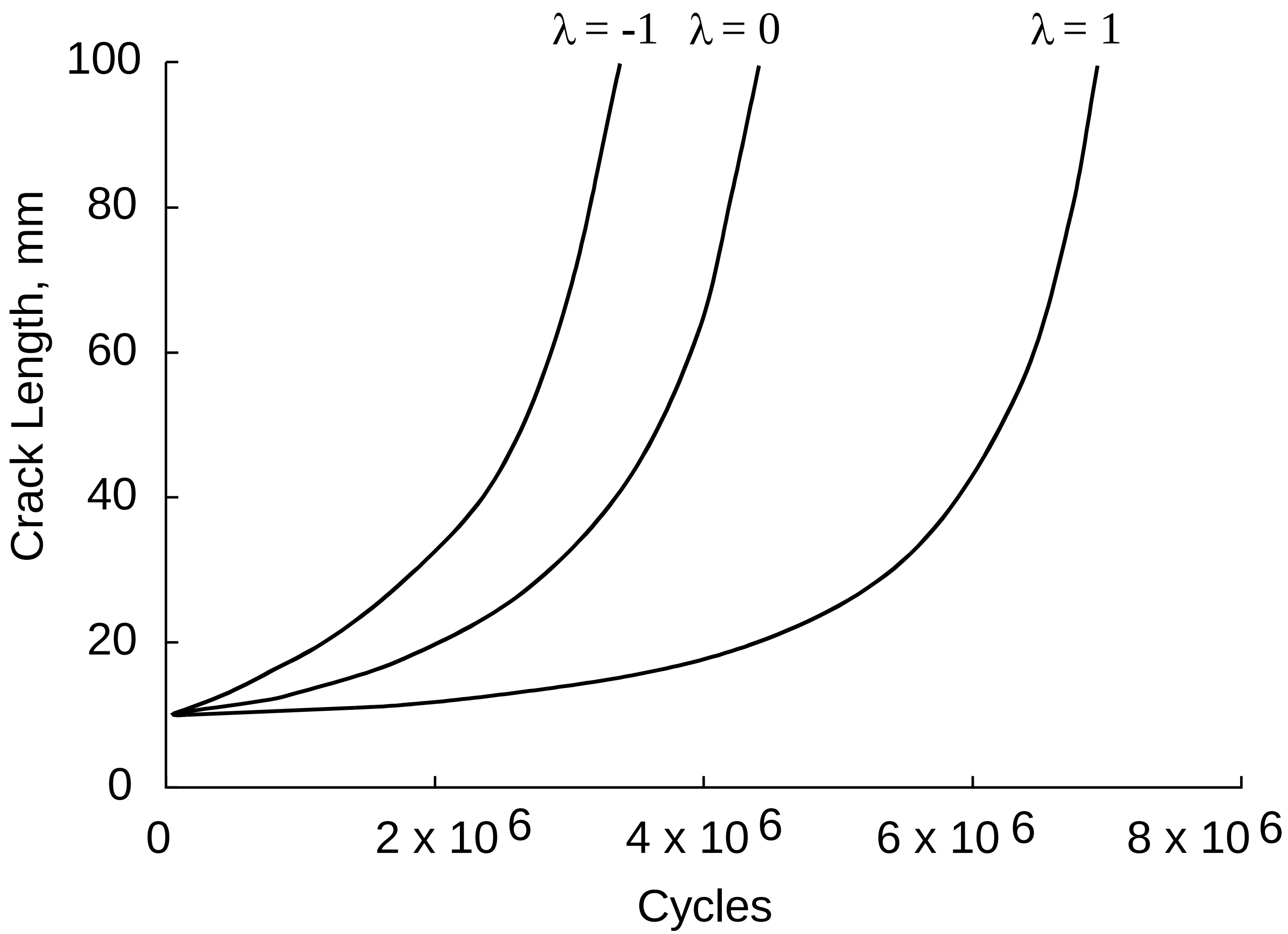
$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} f(E,K',n') \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Stress Intensity Factors



$$\frac{da}{dN} = C\left(\Delta K_{eq}\right)^m \qquad K_I = F\,\Delta\sigma\sqrt{\pi a}$$

Crack Growth From a Hole



Notches Summary

- Uniaxial loading can produce multiaxial stresses at notches
- Multiaxial loading can produce uniaxial stresses at notches
- Multiaxial stresses are not very important in thin plate and shell structures
- Multiaxial stresses are not very important in crack growth

Multiaxial Fatigue