The Local Stress-Strain Fatigue Method (ε-N)

Introduction

Although most engineering structures and machine components are designed such that the nominal stress remains elastic ($S_n < \sigma_{ys}$) stress concentrations often cause plastic strains to develop in the vicinity of notches where the stress is elevated due to the stress concentration effect. Due to the constraint imposed by the elastically stresses material surrounding the notch-tip plastic zone deformation at the notch root is considered strain controlled.

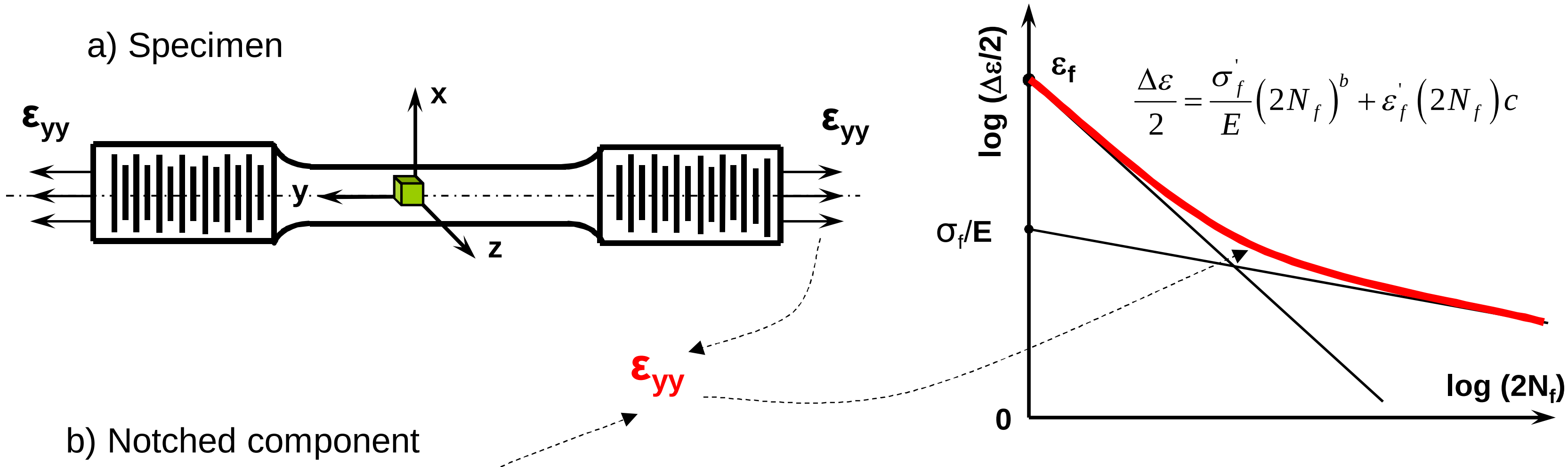
The basic assumption of the strain-life fatigue analysis approach is that the fatigue damage accumulation and the fatigue life to crack initiation at the notch tip are the same as in a smooth material specimen (see the Figure) if the stress-strain states in the notch and in the specimen are the same. In other words:

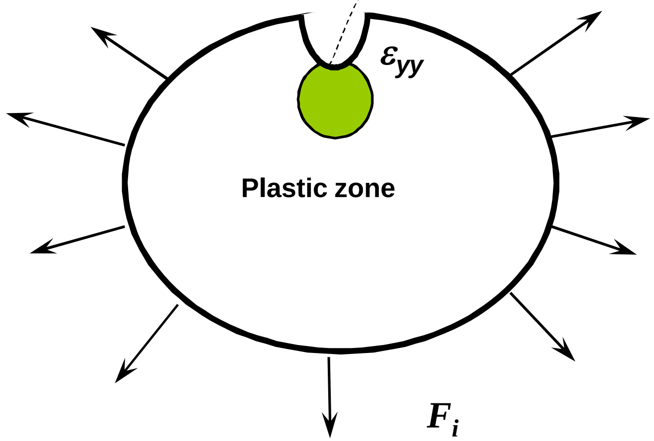
The local strain approach relates deformation occurring in the immediate vicinity of a stress concentration to the remote or local pseudo-elastic stresses and strains using the constitutive response determined from fatigue tests on simple laboratory specimens (i.e. the cyclic stress-strain curve and the strain-life curve.

From knowledge of the geometry and imposed loads on notched components, the local stress-strain histories at the tip of the notch must be determined (Neuber or ESED method).

Fatigue damage must be calculated for each cycle of the local stress-strain history (hysteresis loops, linear damage summation)

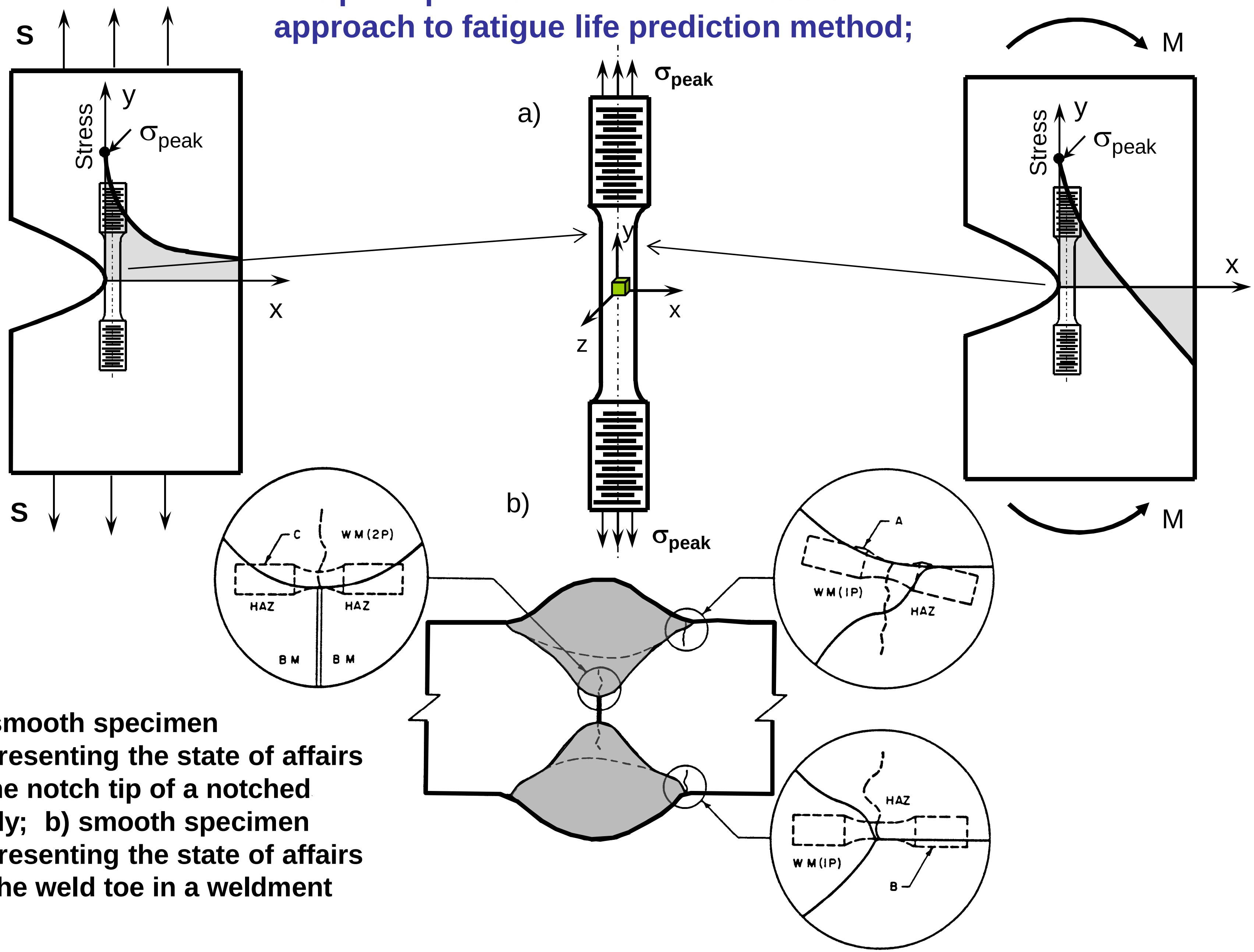
The Basic Concept of the ε-N Method





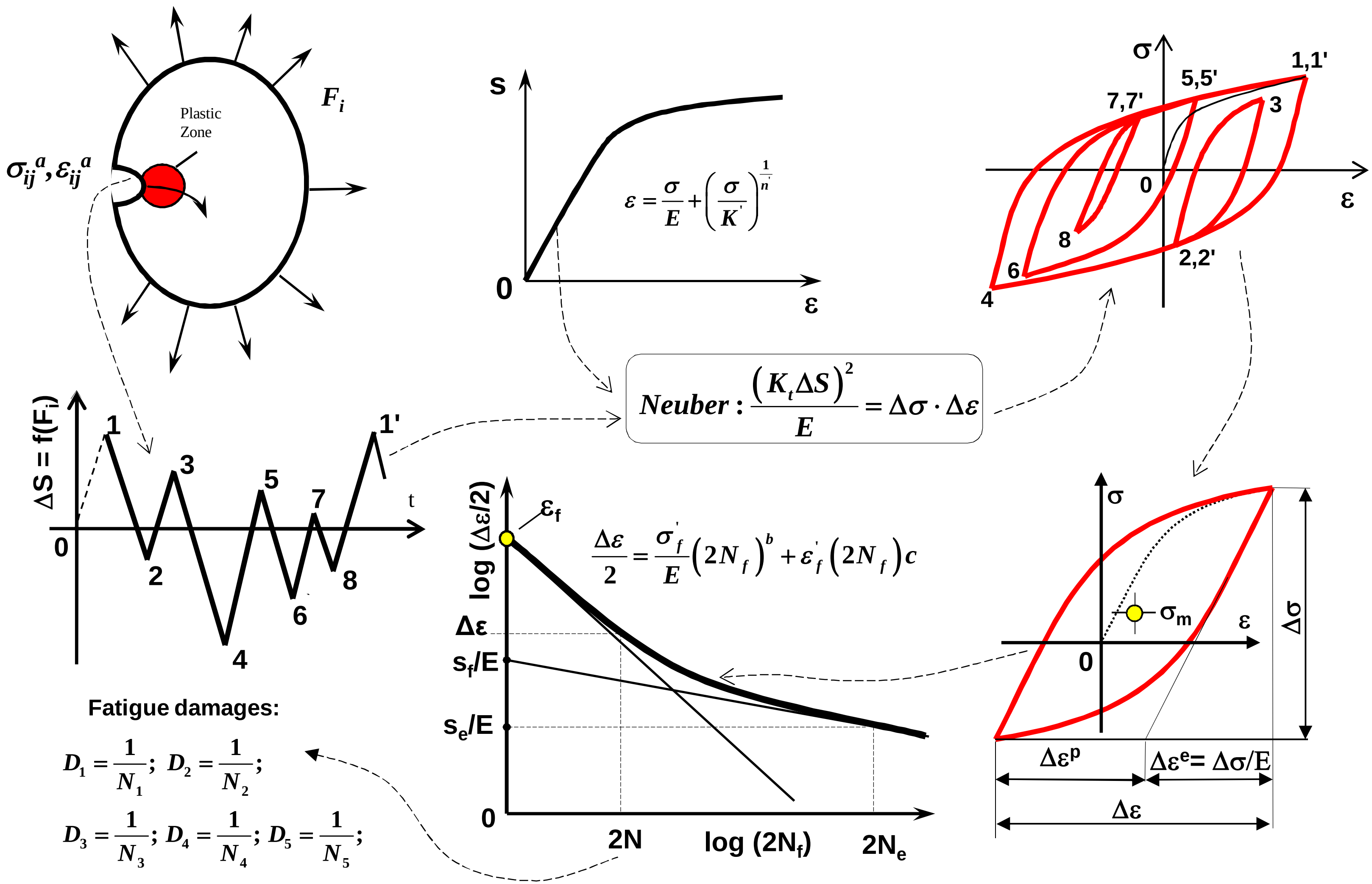
The **Similitude Concept** states that if the local notch-tip strain history in the notch tip and the strain history in the test specimen are the same, then the fatigue response in the notch tip region and in the specimen will also be the same and can be described by the material strain-life (ε-N) curve.

The principal idea of the local stress strain approach to fatigue life prediction method;



a) smooth specimen representing the state of affairs at he notch tip of a notched body; b) smooth specimen representing the state of affairs at the weld toe in a weldment

Main Steps in the Strain-Life Fatigue Analysis of Notched Bodies



Fatigue damages:

$$D_1 = \frac{1}{N_1};\ D_2 = \frac{1}{N_2};$$

$$D_3 = \frac{1}{N_3};\ D_4 = \frac{1}{N_4};\ D_5 = \frac{1}{N_5};$$

Total damage: $D = D_1 + D_2 + D_3 + D_4 + D_5;$

Fatigue life: $N_T = N_{blck} = 1/D$

The stepwise ε - N procedure for estimating fatigue life (*can be summarised as follows - see the Figure below).*

• Analysis of external forces acting on the structure and the component in question (a),

• Analysis of internal loads in chosen cross section of a component (b),

• Selection of critical locations (stress concentration points) in the structure (c),

• Calculation of the elastic local stress, σ_{peak}, at the critical point (usually the notch tip, d)

• Assembling of the local stress history in form of the form of peak and valley sequence (f),

• Determination of the elastic-plastic response at the critical location (h),

• Identification (extraction) of cycles represented by closed stress-strain hysteresis loops (h, i),

• Calculation of fatigue damage (k),

• Fatigue damage summation (Miner- Palmgren hypothesis, l),

• Determination of fatigue life (m) in terms of number of stress history repetitions, N_{blck}, (No. of blocks) or the number of cycles to fatigue crack initiation, N.

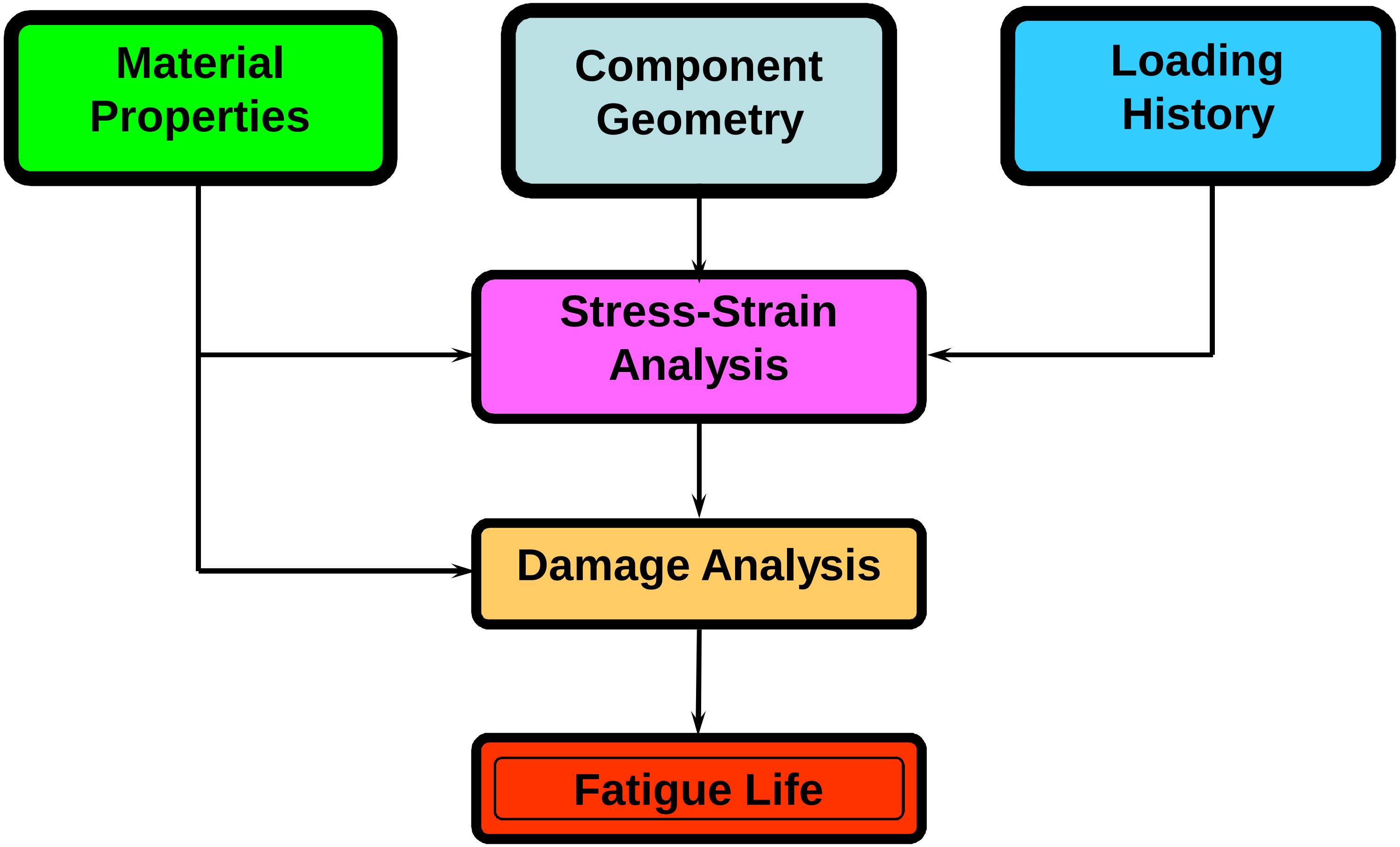
The details concerning many other aspects of that methodology are discussed below.

Because the crack initiation period occupies major part of a fatigue life of a smooth specimen the life of the specimen is assumed to be equal to the fatigue crack initiation life. Therefore, only the fatigue crack initiation life at the notch tip can be estimated from the fatigue data obtained form a smooth specimen subjected to the same stress-strain history as that one occurring in the notch tip. The same history means the same magnitudes of all stress and strain components. If such conditions are satisfied the equality of one stress or strain component in the notch and the smooth specimen assures that the other components are the same as well. Therefore, it is possible to use in such a case only one strain or one stress component as a parameter for fatigue damage calculation and fatigue life estimation. It means that one component characterizes in those cases the entire stress-strain state.

However, if the stress-strain state in the notch tip and in the specimen are not the same calculations based on only one stress or strain component might be inaccurate.

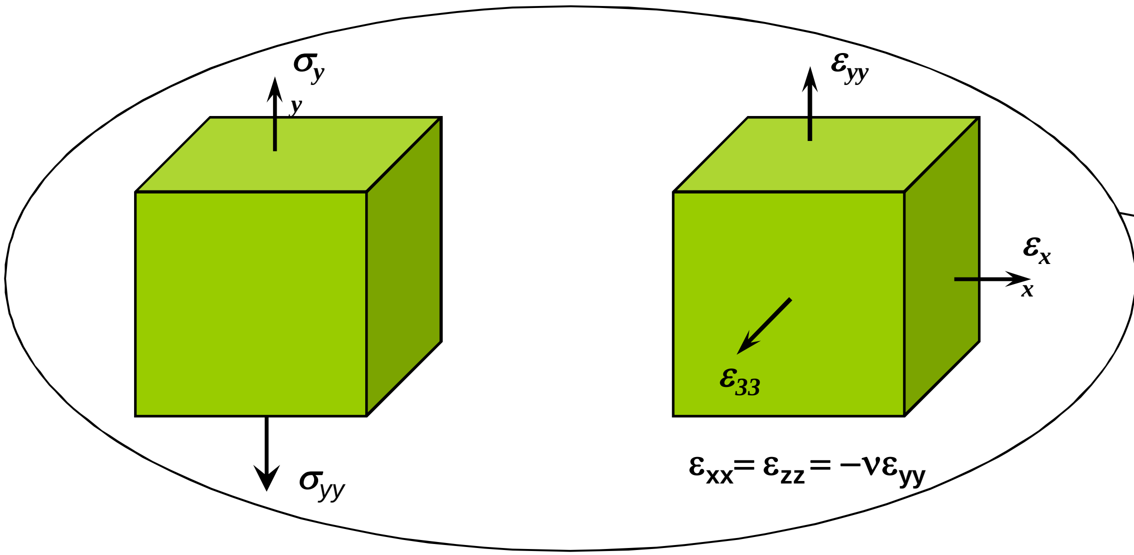
Therefore, it seems important to review the elastic plastic stress-strain behavior of materials and their mathematical models used in fatigue applications. It is also important to know the modifications, which should be applied before the uni-axial strain-life (**ε–N**) properties can be used if the stress-strain state in the notch tip is not the same as that one in the material specimen used for obtaining relevant material properties.

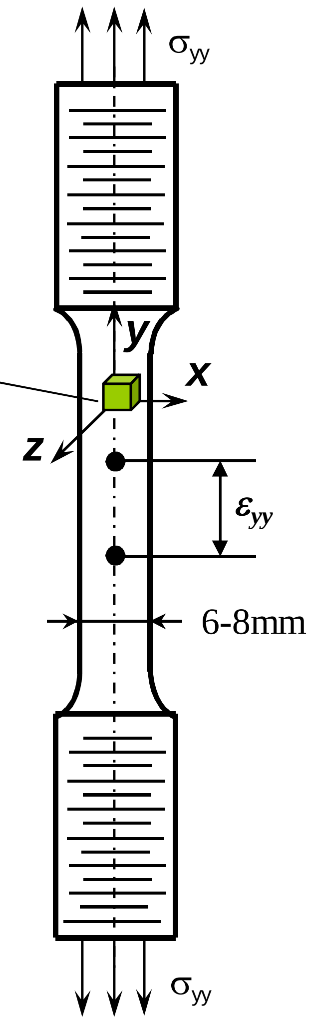
Information Path for Strength and Fatigue Life Analysis



Smooth Laboratory Specimens Used for the Determination of the σ - ε Curve under Monotonic and Cyclic Loading

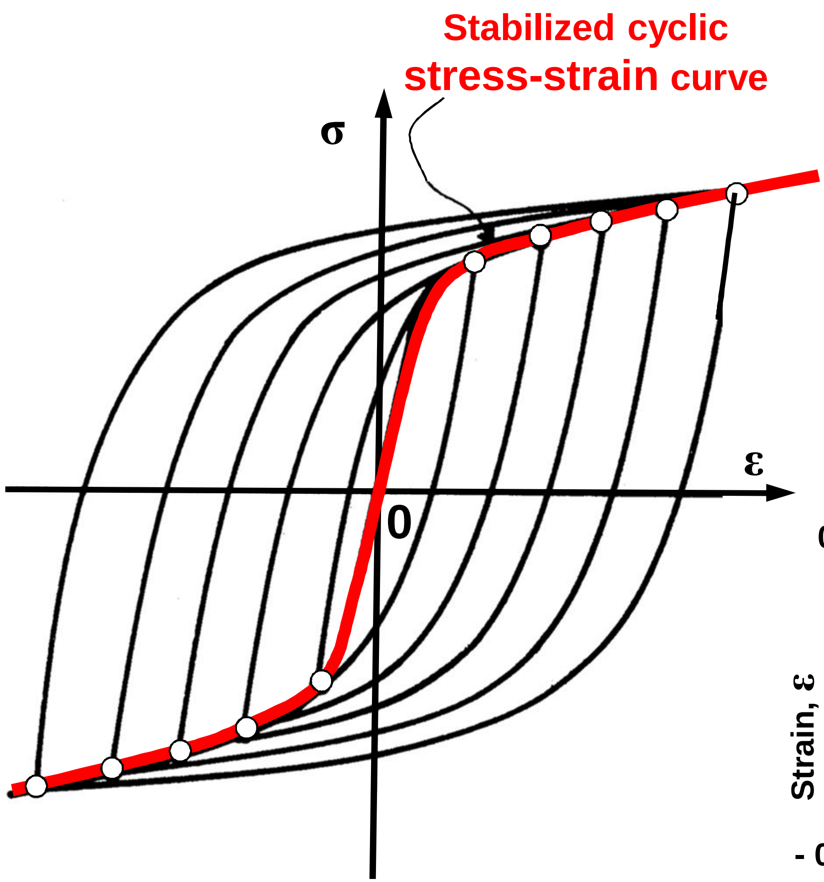
Stress and strain state in specimens used for determination of material properties

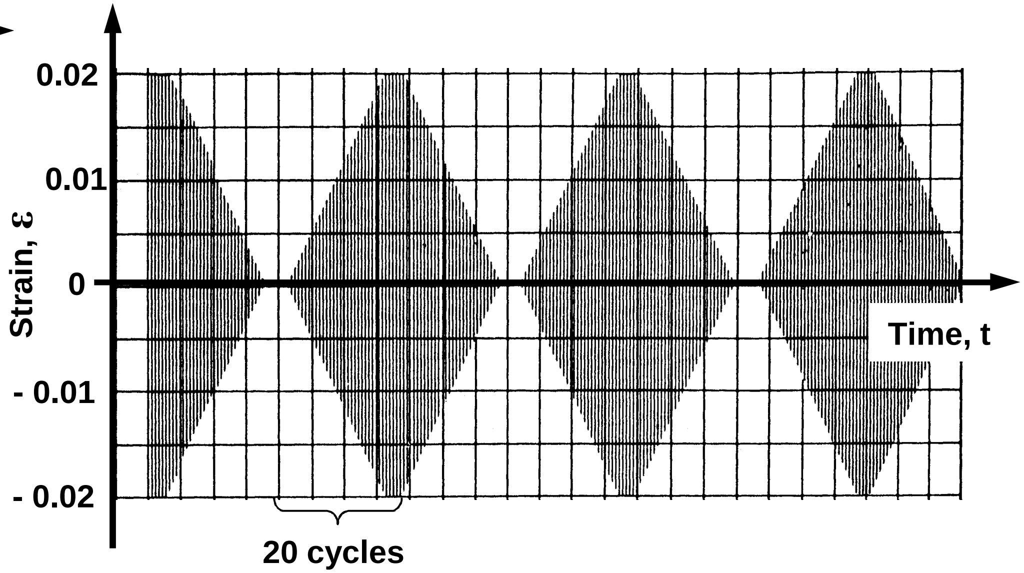




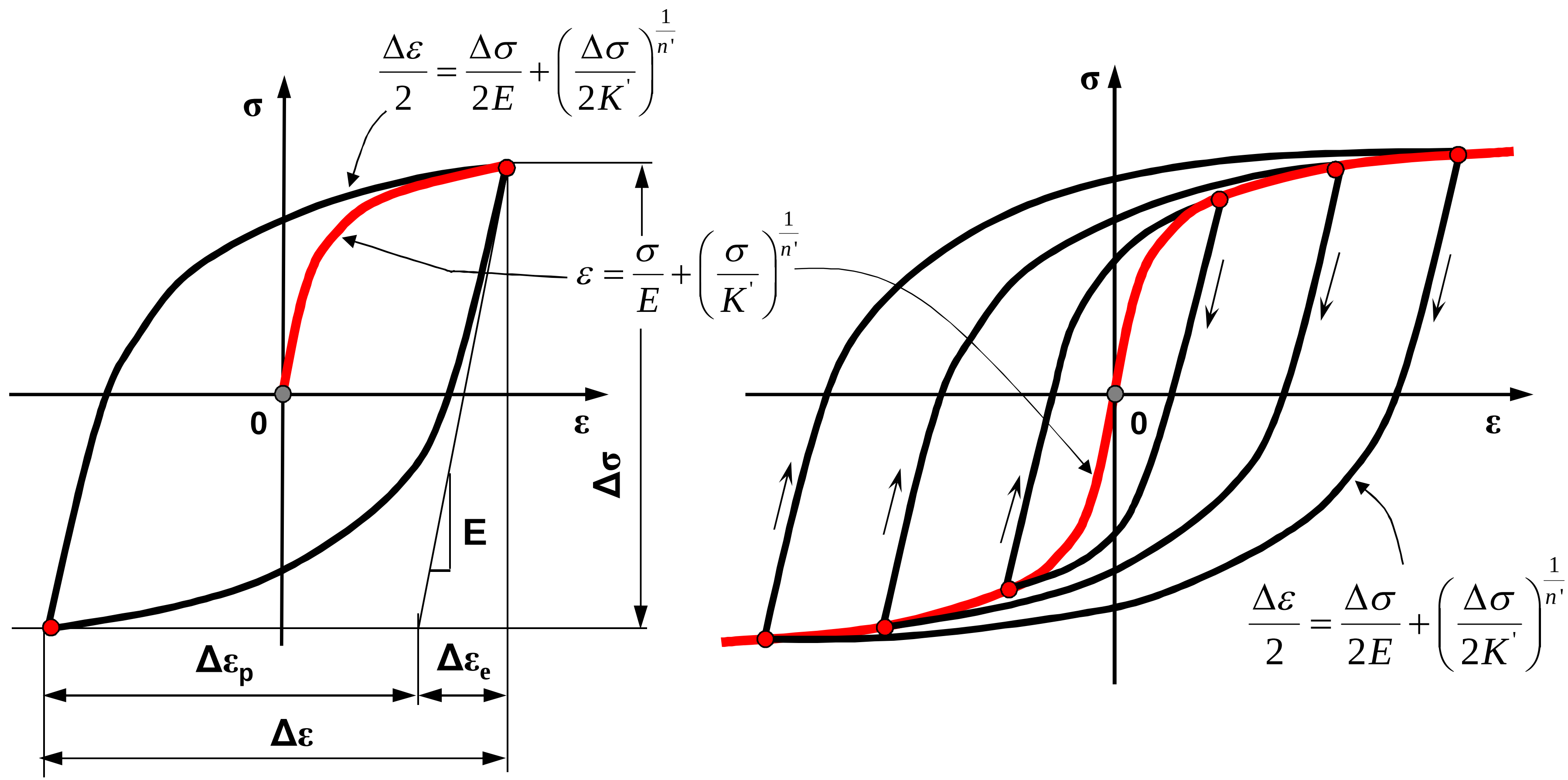
$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$

Determination of the Stabilized Cyclic Stress-Strain Curve





Mathematical Expressions Describing the Stress-Strain Curve and the Shape of the Hysteresis Lop



Equation of the cyclic stress-strain curve

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}}$$

Equation of the hysteresis loop branch

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}}$$

The Massing Hypothesis

Massing's hypothesis states that the stabilised hysteresis loop branch may be obtained by doubling the basic material stress-strain curve.

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K^{'}} \right)^{1/n^1}$$

-cyclic stress – strain curve (amplitudes)

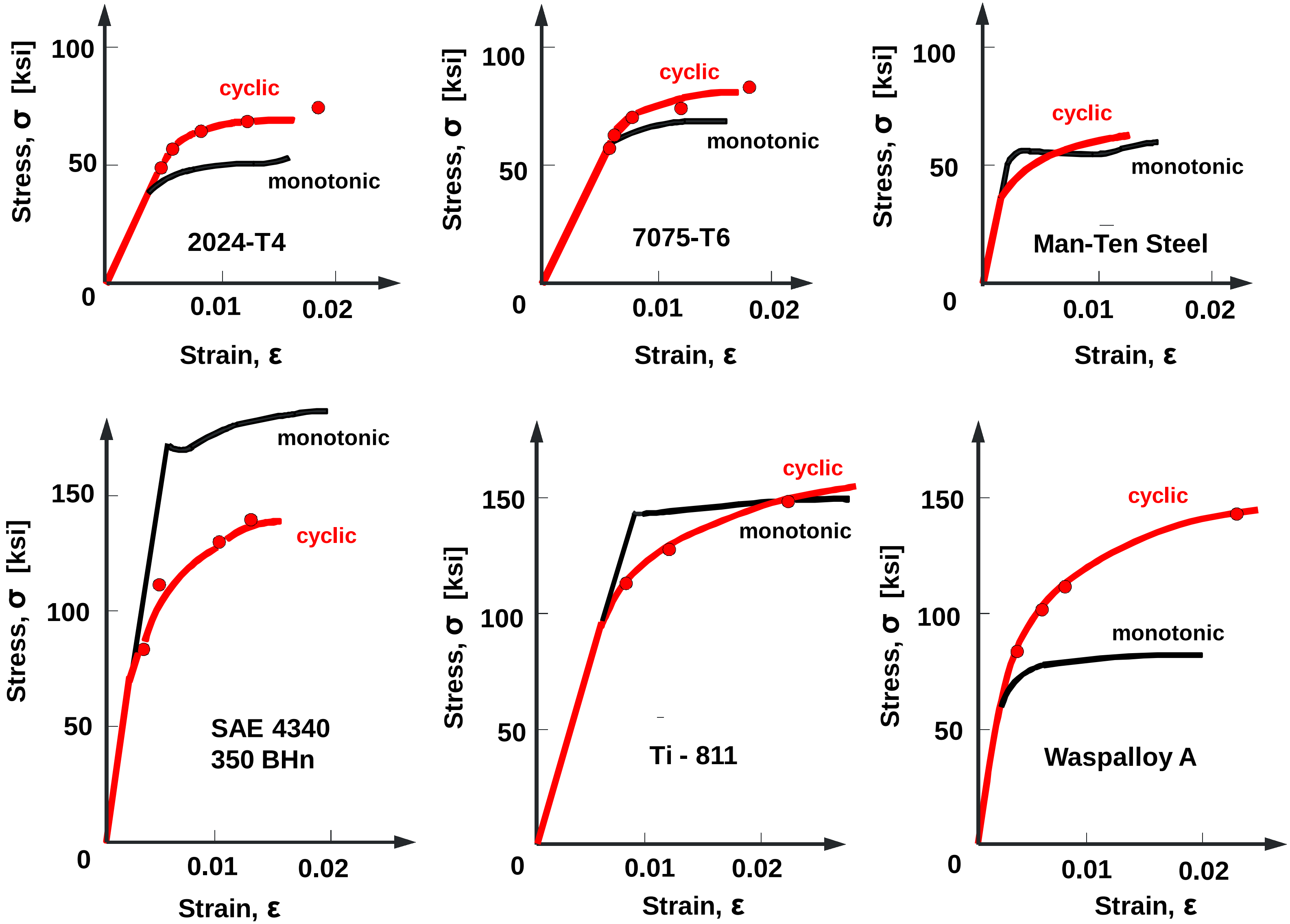
$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K^{'}} \right)^{1/n^1}$$

- doubled stress – strain curve (ranges)

or

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K^{'}} \right)^{1/n^1}$$

Monotonic and cyclic stress-strain curves for various metallic materials



The stress-strain response of metals is often drastically altered due to repeated loading. The material may:

- *Cyclically harden*
- *Cyclically soften*
- *Be cyclically stable*
- *Have mixed behaviour (soften or harden depending on $\Delta\varepsilon$)*

- **The reason materials soften or harden appears to be related to the nature and stability of the dislocation substructure of the material.**
- **For a soft material, initially the dislocation density is low. The density rapidly increases due to cyclic plastic straining contributing to significant cyclic strain hardening.**
- **For a hard material subsequent strain cycling causes a rearrangement of dislocations, which offers less resistance to deformation and the material cyclically softens**

If $\frac{\sigma_{ult}}{\sigma_y} > 1.4$ **the material will cyclically harden**

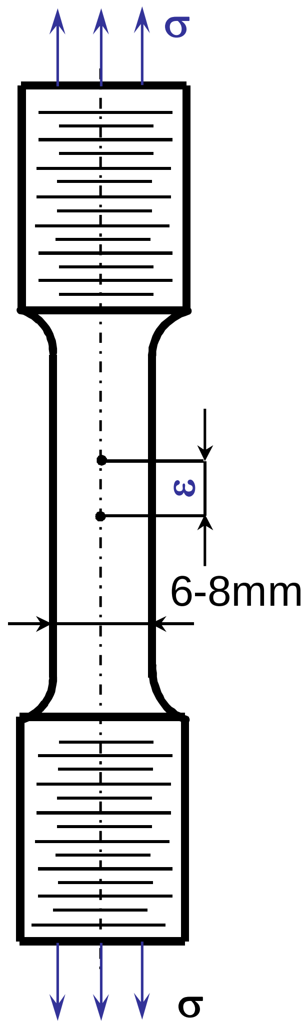
If $\frac{\sigma_{ult}}{\sigma_y} < 1.2$ **the material will cyclically soften**

Determination of the fatigue strain-life curve

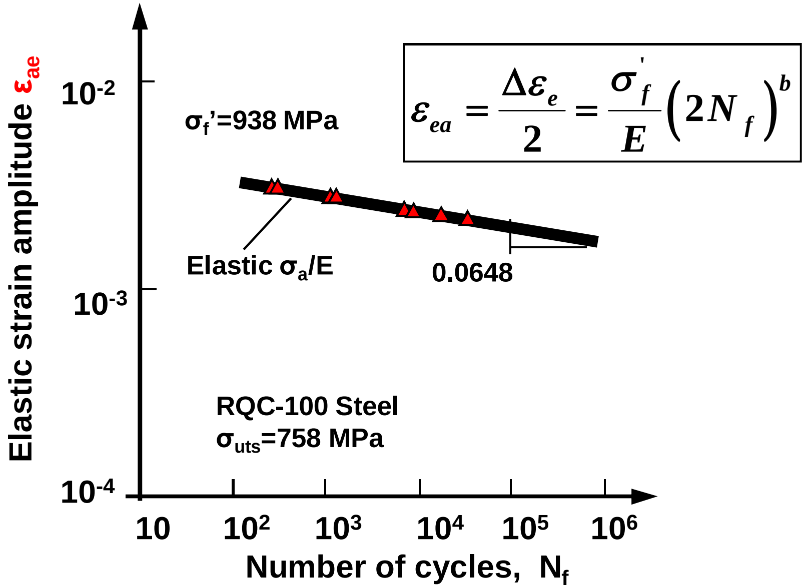
1. Smooth laboratory specimens are used for the determination of the σ - ε and ε - N curves.

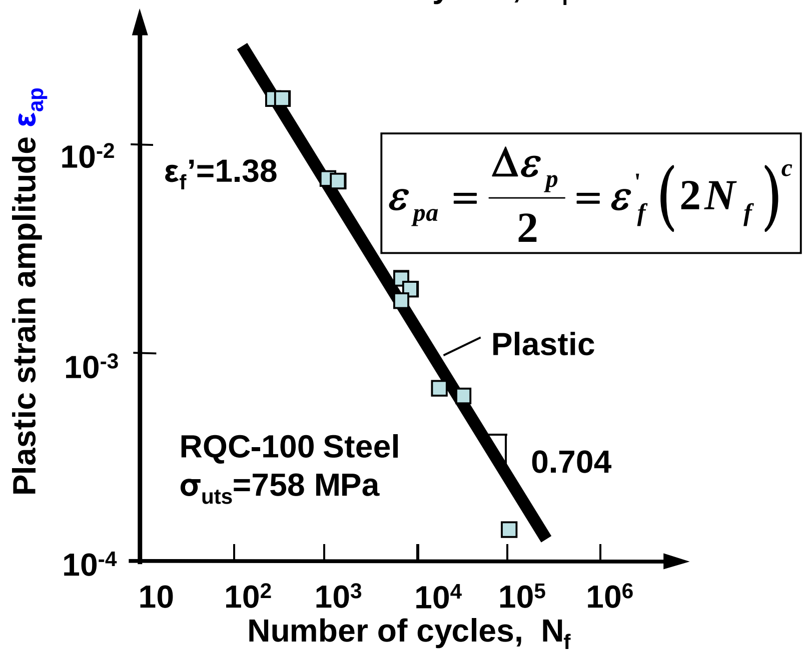
2. The σ - ε data points are obtained at half life of each specimen to assure that the material is stabilized.

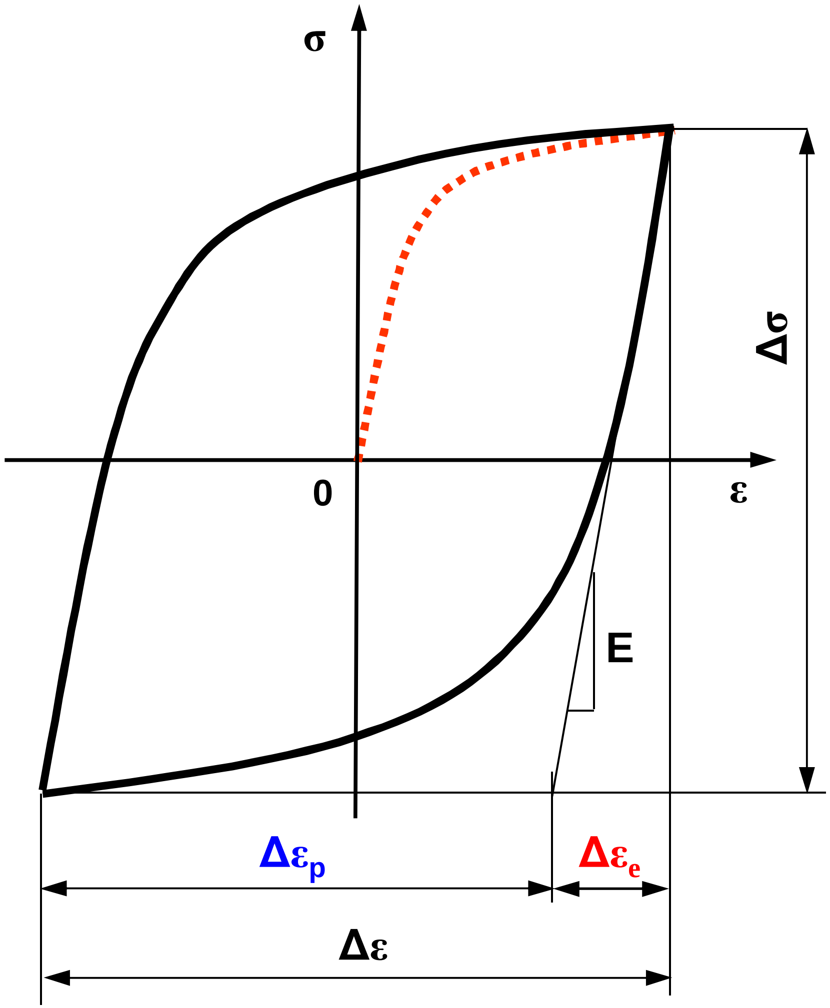
3. **80% -95%** of the specimen life spent to create a crack up to **0.5 -1 mm deep**.
diameter: **6 - 8 mm**



Determination of the Fatigue Strain-Life Curve







Fatigue Strain – Life Properties

In 1910, Basquin observed that stress-life (S-N) data could be plotted linearly on a log-log scale.

$$\frac{\Delta\sigma}{2} = \sigma_f^{'}(2N_f)^b$$

where: $\Delta\sigma/2$ - true stress amplitude; $2N_f$ - reversals to failure (1 rev = ½ cycle);

$\sigma_f^{'}$ - fatigue strength coefficient, b - fatigue strength exponent (Basquin's exponent)

Parameters $\sigma_f^{'}$ and b are fatigue properties of the material. The fatigue strength coefficient, $\sigma_f^{'}$, is approximately equal to the true fracture strength at fracture σ_f. The fatigue strength exponent, b, varies in the range of 0.05 and –0.12.

Manson and Coffin, working independently (1950), found that plastic strain-life data (ε_p-N) could be linearized in log-log co-ordinates.

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon_f^{'}(2N_f)^c$$

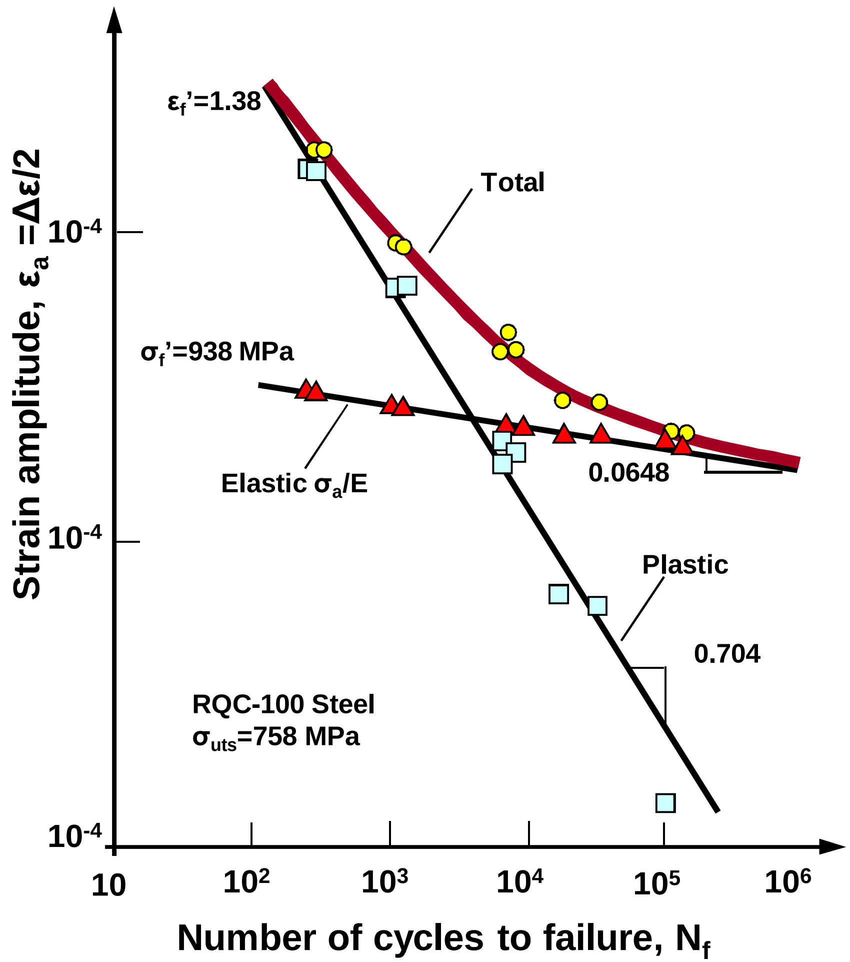
where: $\frac{\Delta\varepsilon_p}{2}$ - plastic strain amplitude; $2N_f$ - reversals to failure; $\varepsilon_f^{'}$ - fatigue ductility coefficient

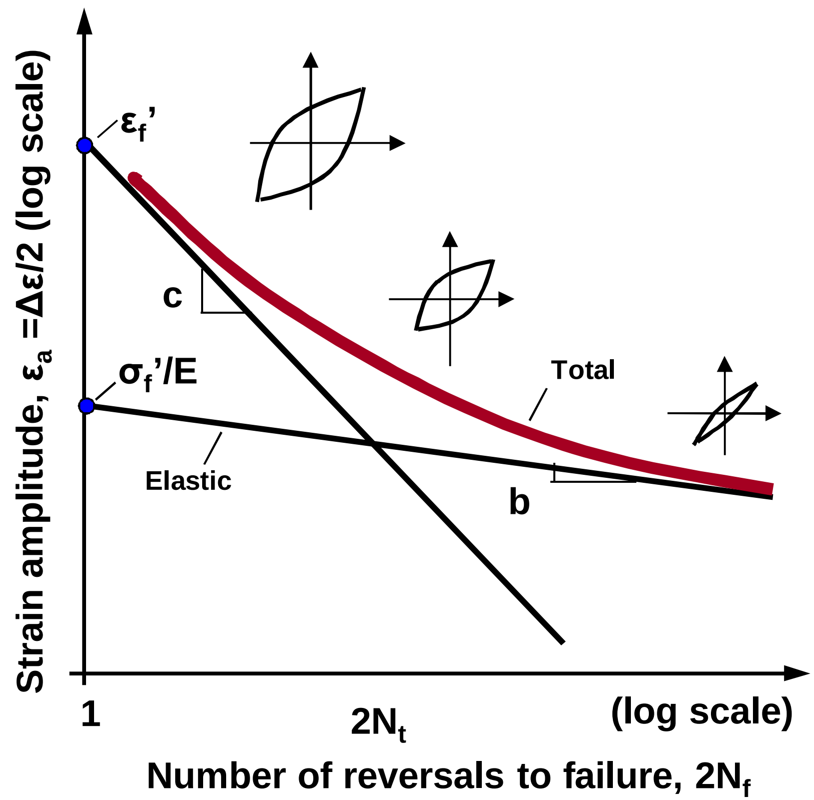
c - fatigue ductility exponent

Parameters $\varepsilon_f^{'}$ and c are fatigue properties of the material. The fatigue ductility coefficient, $\varepsilon_f^{'}$, is approximately equal to true fracture ductility (true strain at fracture), $\varepsilon_f^{'}$. The fatigue ductility exponent, c, varies in the range of –0.5 and –0.7.

Fatigue Strain-Life Curve

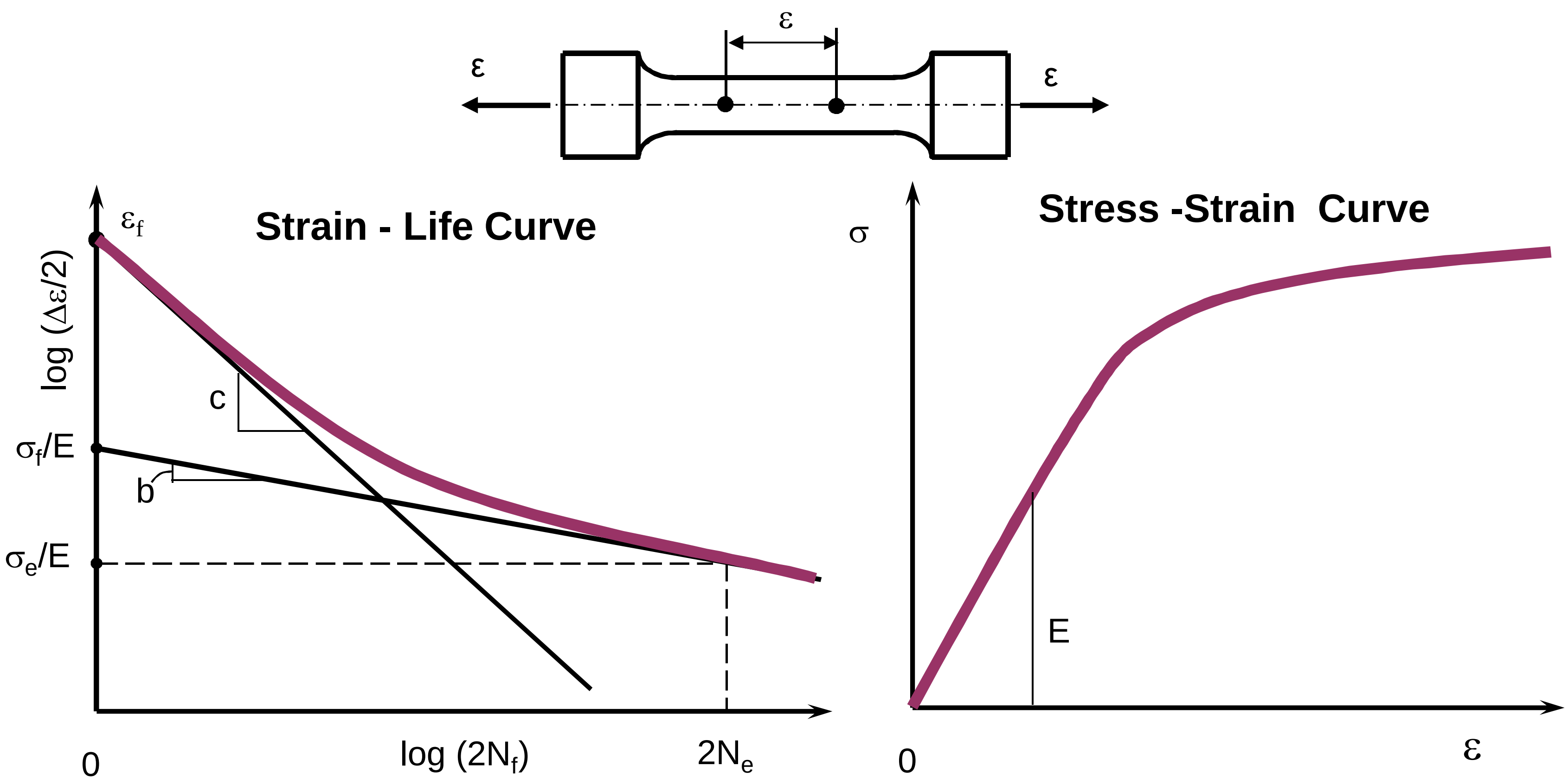
$$\varepsilon_a = \frac{\Delta\varepsilon}{2} = \frac{\sigma_f^{'}}{E}\left(2N_f\right)^b + \varepsilon_f^{'}\left(2N_f\right)^c$$





The Strain-life and the Cyclic Stress-Strain Curve Obtained from Smooth Cylindrical Specimens Tested Under Strain Control

(Uni-axial Stress State)



$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}}$$

CYCLIC PROPERTIES

K' - cyclic strength coefficient
n' - cyclic strain hardening exponent
σ'_{ys} - cyclic yield strength
E - modulus of elasticity

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'}$$

The Ramberg-Osgood curve

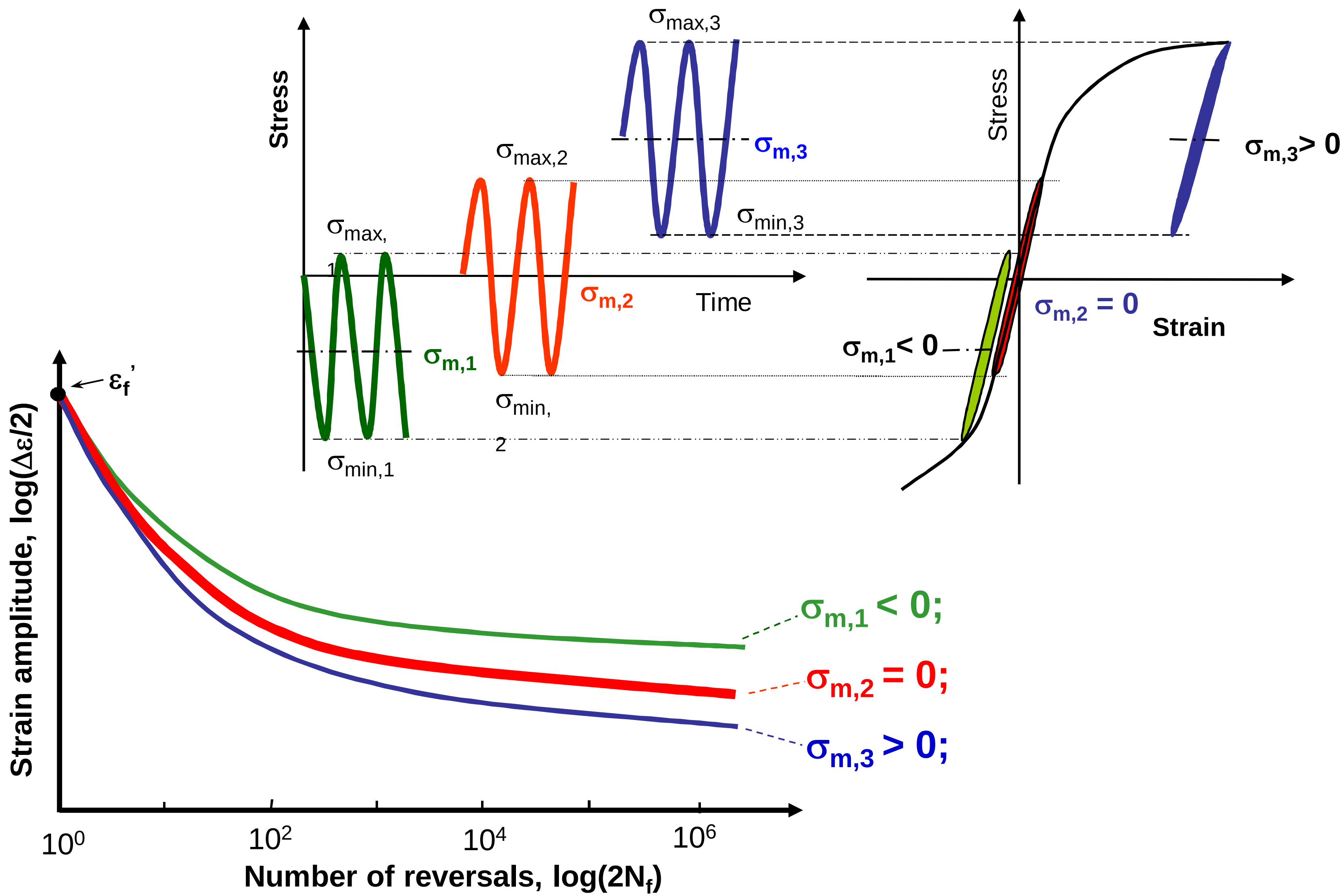
FATIGUE PROPERTIES

ε_f' - fatigue ductility coefficient
c - fatigue ductility exponent
σ_f' - fatigue strength coefficient
b - fatigue strength exponent

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c$$

The Manson-Coffin curve

The Mean Stress Effect



Mean Stress Effect Correction Models

Morrow

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f^{'} - \sigma_m}{E} \cdot \left(2N_f\right)^b + \varepsilon_f^{'} \cdot \left(2N_f\right)^c$$

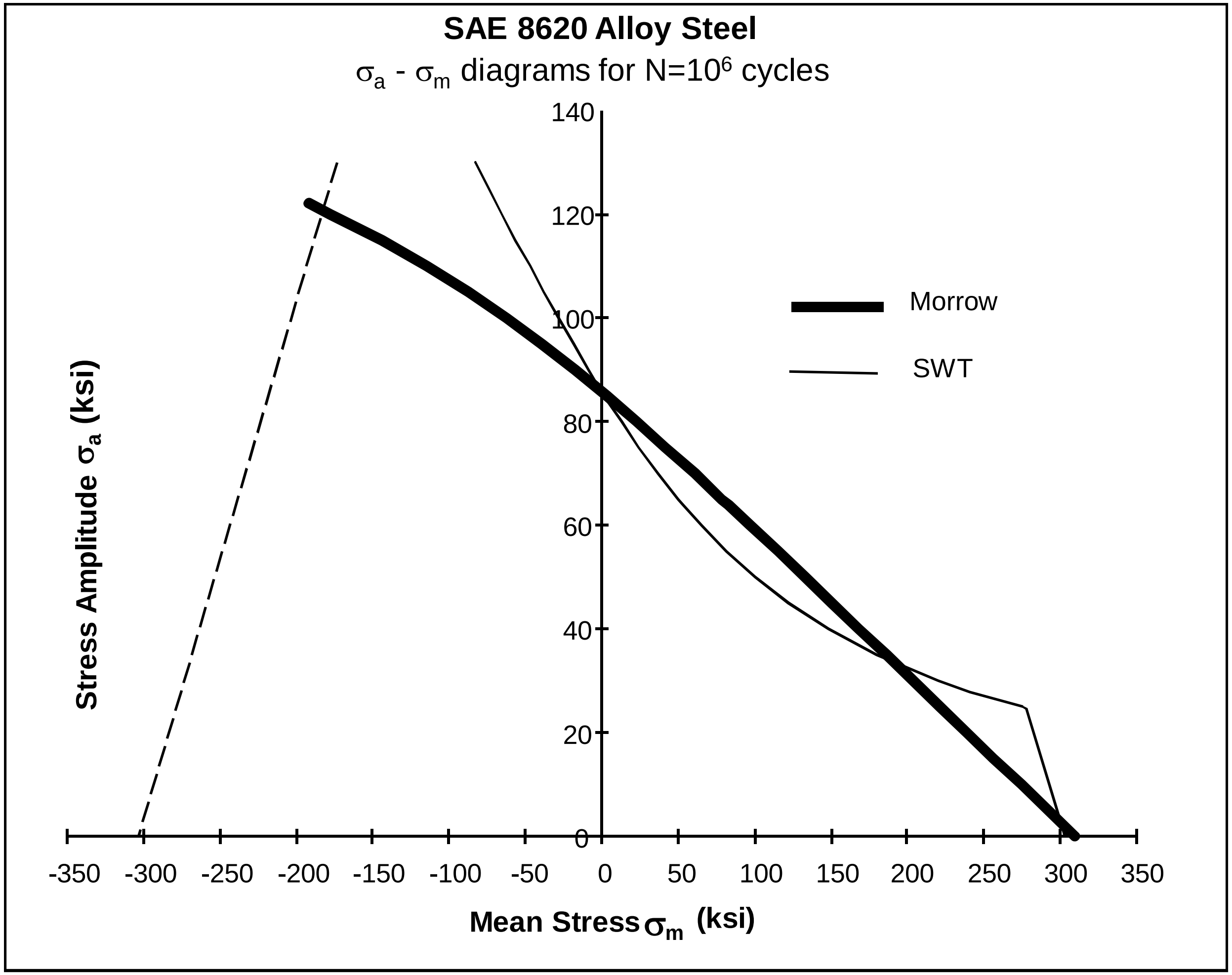
Manson-Halford

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f^{'} - \sigma_m}{E} \cdot \left(2N_f\right)^b + \varepsilon_f^{'} \cdot \left(\frac{\sigma_f^{'} - \sigma_m}{\sigma_f^{'}}\right)^{c/b} \cdot \left(2N_f\right)^c$$

***Smith-Watson-Topper** (SWT)*

$$\sigma_{\max} \frac{\Delta\varepsilon}{2} = \frac{\left(\sigma_f^{'}\right)^2}{E} \cdot \left(2N_f\right)^{2b} + \varepsilon_f^{'} \sigma_f^{'} \cdot \left(2N_f\right)^{b+c}$$

Comparison of Constant Life σ_a - σ_m Curves According to Morrow's and the SWT Mean Stress Correction Model; SAE 8620 Alloy Steel; at $N_f = 10^6$ cycles



Limitations and Physical Interpretation of Mean Stress Correction Models

Morrow's model

- The predictions made with Morrow's mean stress correction model are consistent with the observations that mean stress effects are significant at low values of plastic strain, where the elastic strain dominates. The correction also reflects the trend that mean stresses have little effect at shorter lives, where plastic strains are large.
- However Morrow,s mean stress model incorrectly predicts that the ratio of elastic to plastic strain range is dependent on mean stress. This is clearly not true, because the shape of the stress-strain hysteresis loop does not depend on the mean stress.
- Although Morrow's mean stress correction model violets the constitutive relationship, it generally correctly predicts mean stress effects.

Limitations and Physical Interpretation of Mean Stress Correction Models

Manson and Halford model

- Manson and Halford modified both the elastic and plastic terms of the strain-life equation to maintain the independence of the elastic-plastic strain ratio from mean stress.

- This equation tends to predict too much mean stress effect at short lives or where plastic strains dominate. At high plastic strains, mean stress relaxation occurs.

Smith, Watson, and Topper (SWT) model

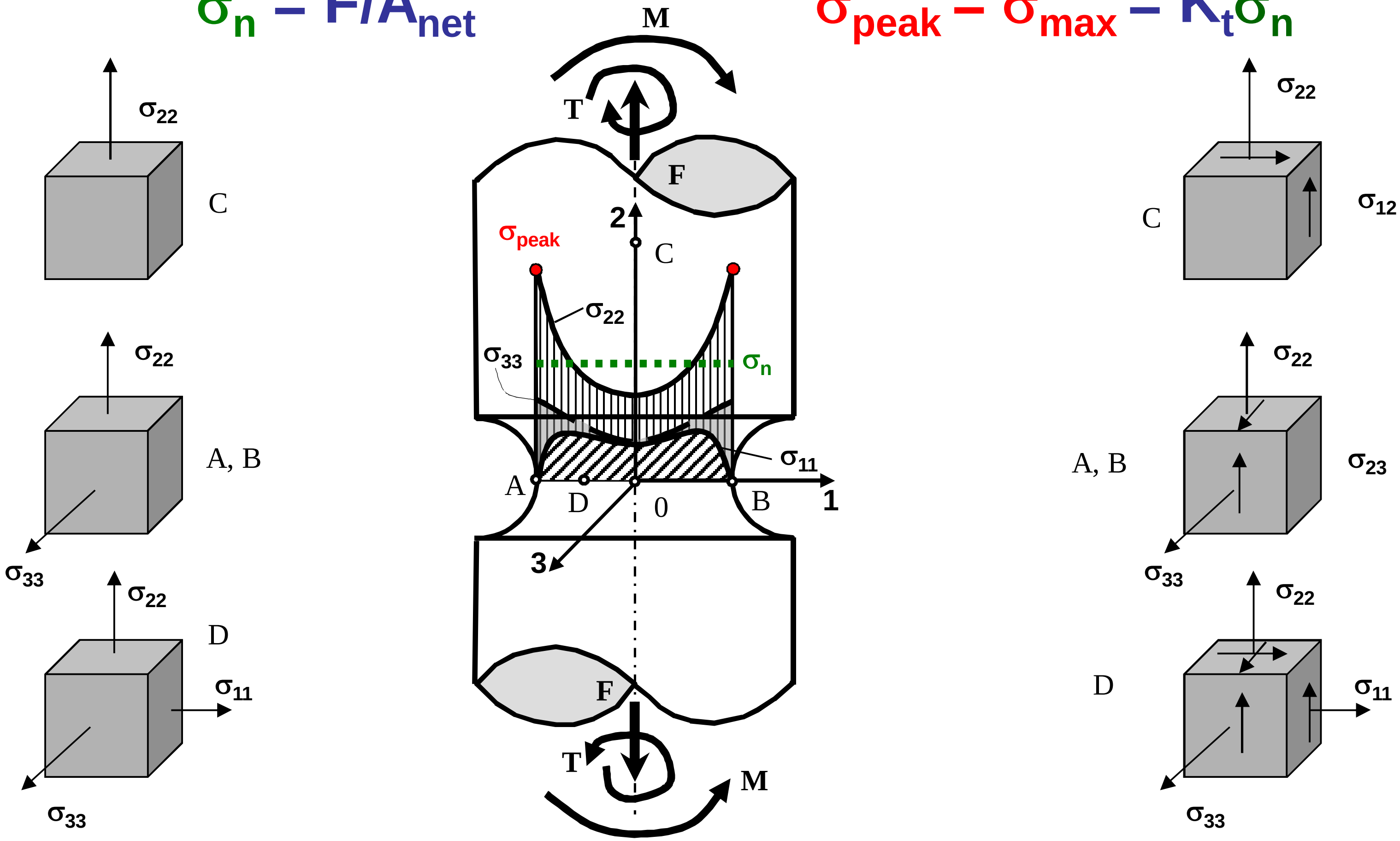
- Since SWT parameter is in the general form of

$$\sigma_{\max}\Delta\varepsilon = f\left(N_f\right) > 0$$

it becomes undefined when σ_{max} is negative ($\sigma_{max} < 0$). The physical interpretation of this approach assumes that no fatigue damage occurs when the maximum stress is compressive.

Stress States in a Notched Body

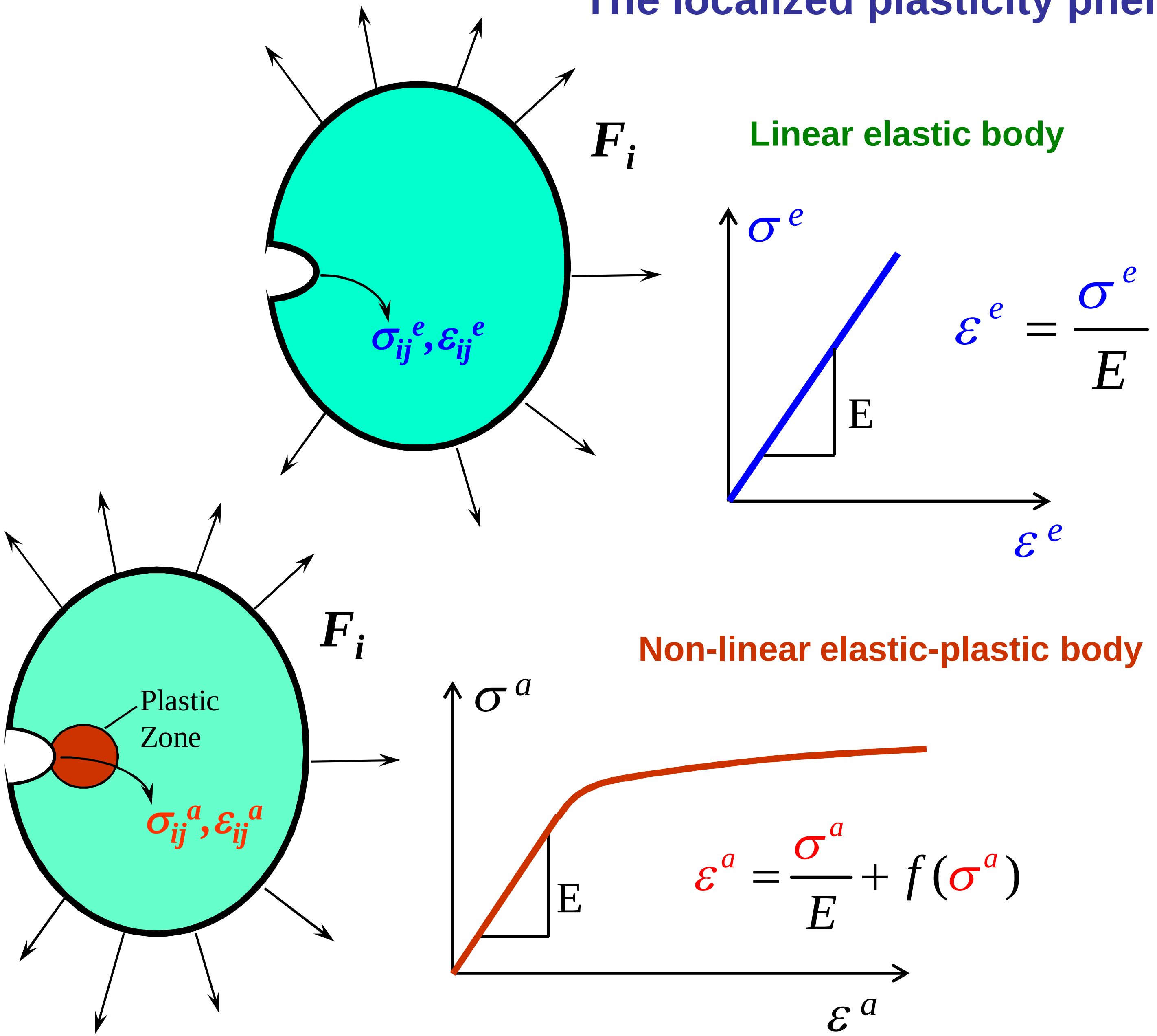
$\sigma_n = F/A_{net}$ $\sigma_{peak} = \sigma_{max} = K_t\sigma_n$



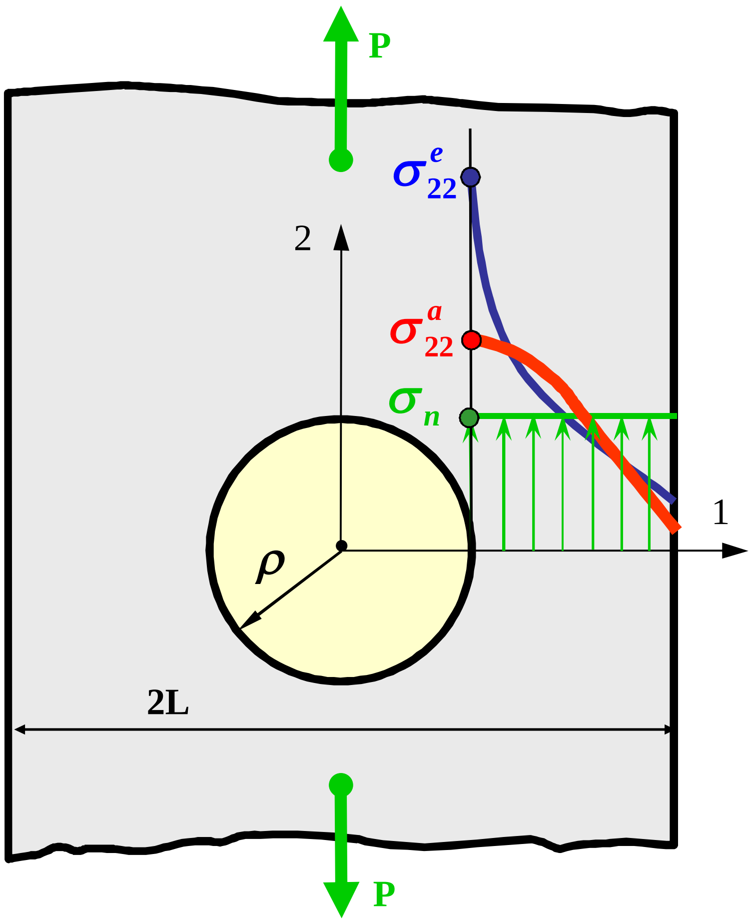
Stresses in axisymmetric notched body under axial loading

Stresses in axisymmetric notched body under axial, bending and torsion loading

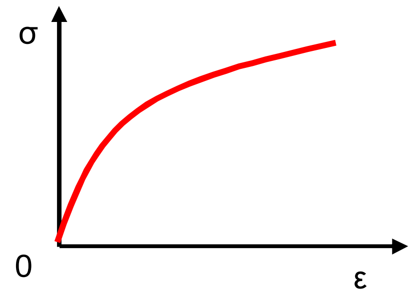
The localized plasticity phenomenon



The Neuber Rule and material stress-strain curve

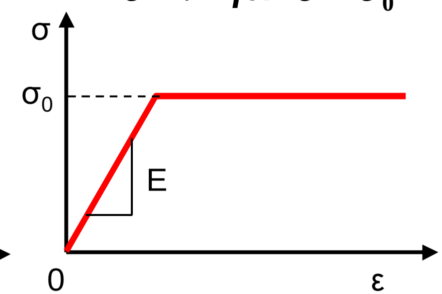


$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}}$$



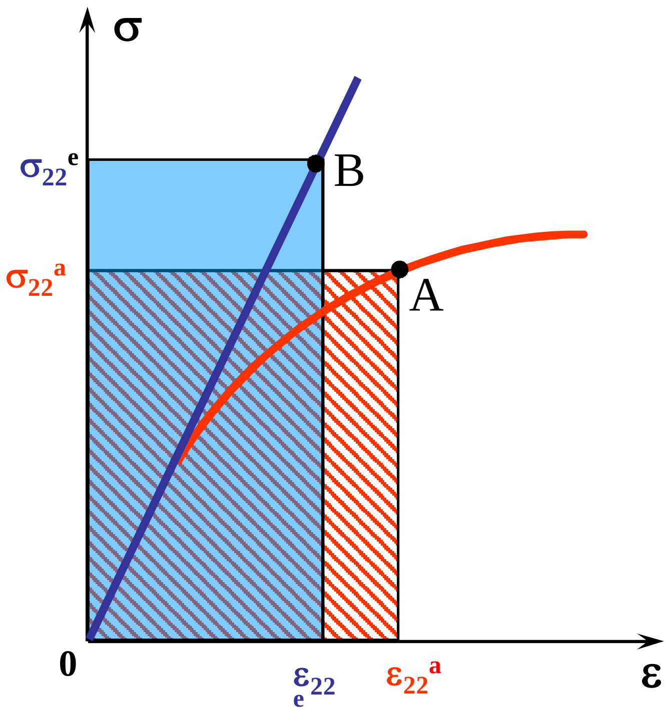
$$\varepsilon = \frac{\sigma}{E} \quad for \ \ \sigma < \sigma_0$$

$$\varepsilon = ? \quad for \ \ \sigma = \sigma_0$$



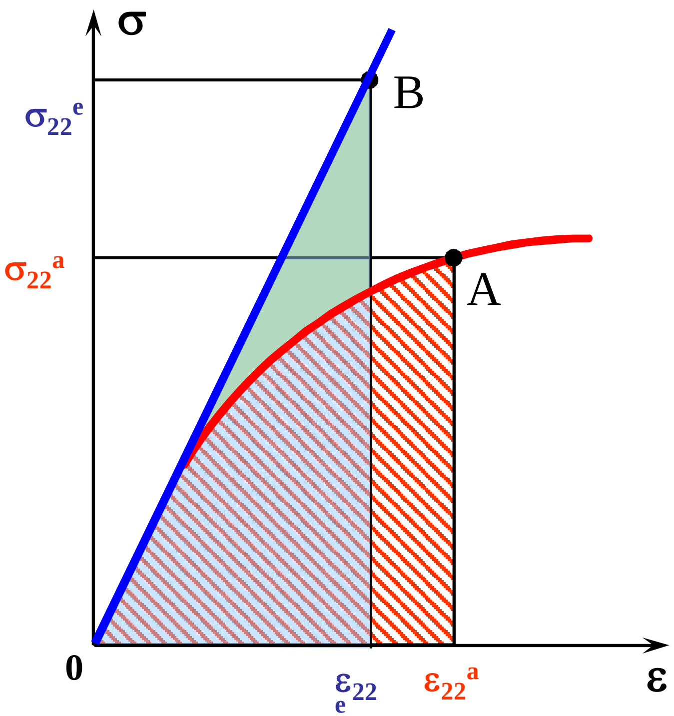
$$\begin{cases} \dfrac{\left(K_t \sigma_n\right)^2}{E} = \sigma_{22}^{a} \varepsilon_{22}^{a} \quad or \quad \sigma_{22}^{e} \varepsilon_{22}^{e} = \sigma_{22}^{a} \varepsilon_{22}^{a} \\ \varepsilon_{22}^{a} = \dfrac{\sigma_{22}^{a}}{E} + f\left(\sigma_{22}^{a}\right) \end{cases}$$

Neuber's Rule



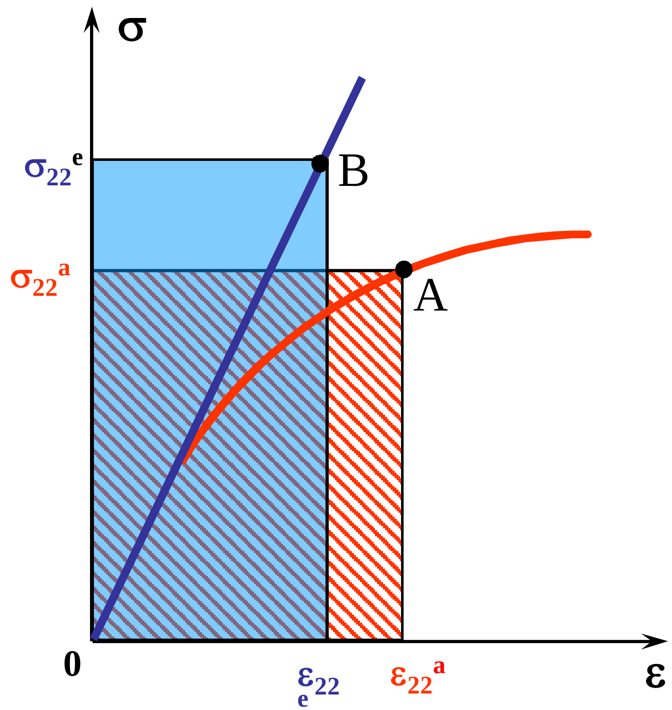
$$\begin{cases} \dfrac{(\sigma_n K_t)^2}{E} = \sigma_{22}^{e}\varepsilon_{22}^{e} = \sigma_{22}^{a}\varepsilon_{22}^{a} \\ \varepsilon_{22}^{a} = \dfrac{\sigma_{22}^{a}}{E} + f\left(\sigma_{22}^{a}\right) \end{cases}$$

The ESED Method



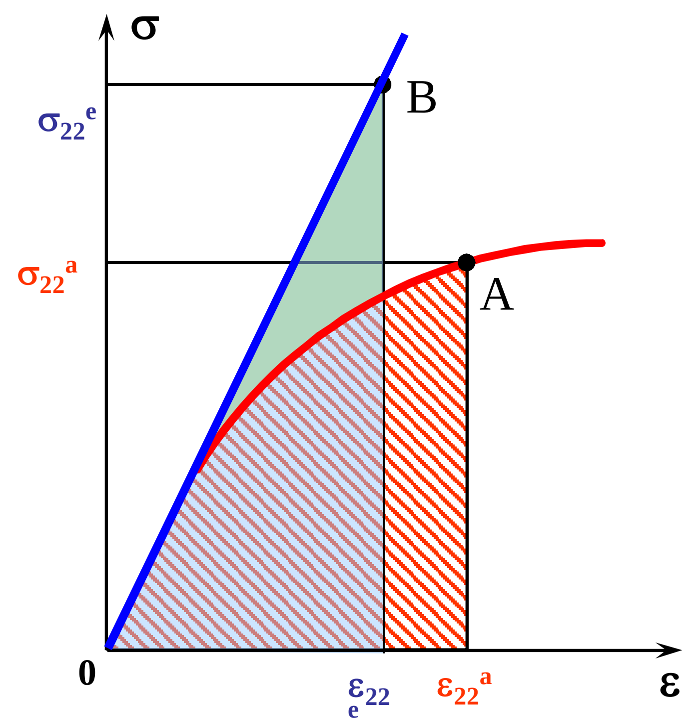
$$\begin{cases} \dfrac{(\sigma_n K_t)^2}{2E} = \dfrac{\sigma_{22}^{e}\varepsilon_{22}^{e}}{2} = \displaystyle\int_0^{\varepsilon_{22}^{a}} \sigma_{22}^{a}\, d\varepsilon_{22}^{a} \\ \varepsilon_{22}^{a} = \dfrac{\sigma_{22}^{a}}{E} + f\left(\sigma_{22}^{a}\right) \end{cases}$$

Neuber's Rule and the Ramberg-Osgood curve



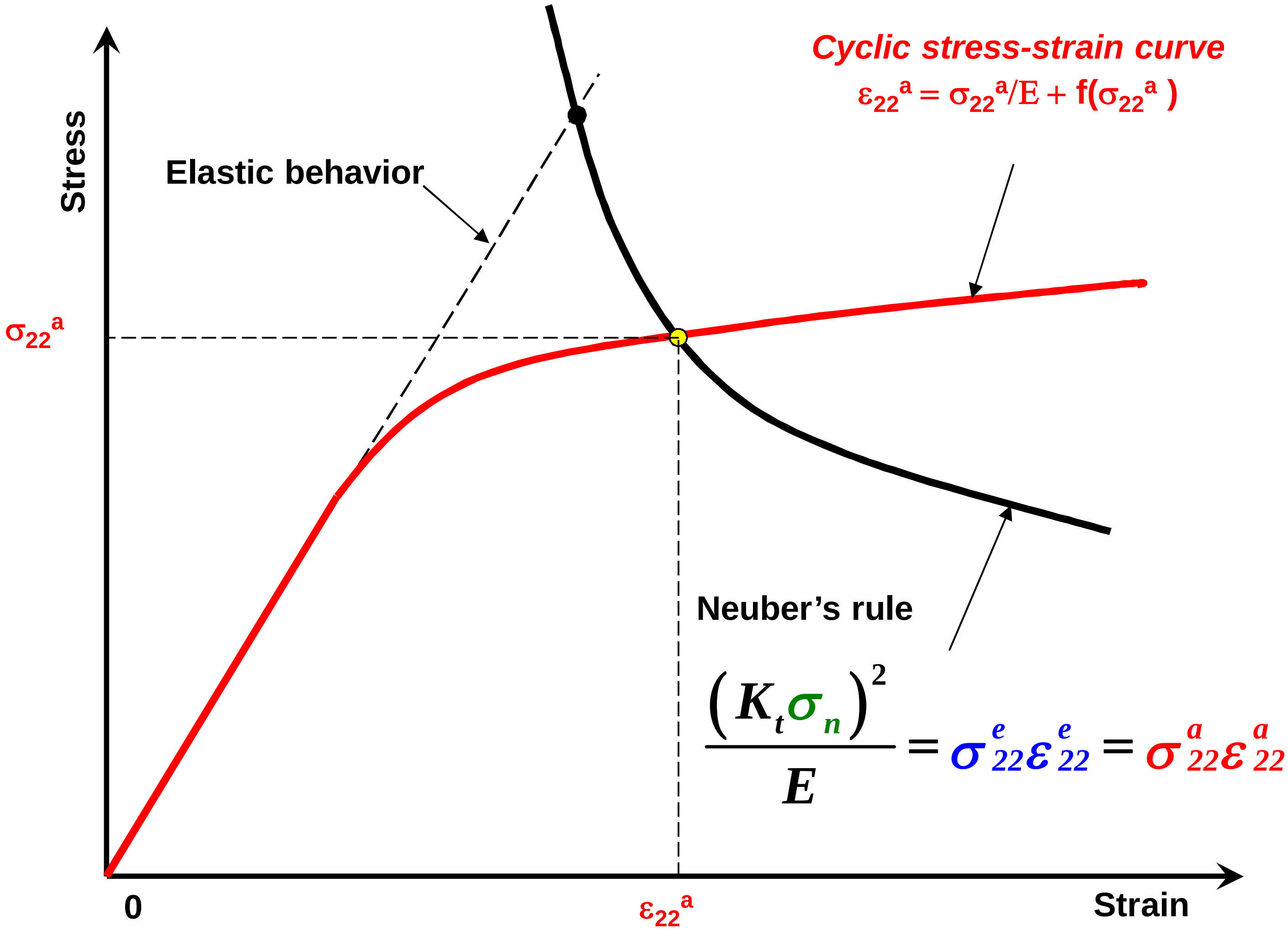
$$\begin{cases} \dfrac{(\sigma_n K_t)^2}{E} = \sigma_{22}^e \varepsilon_{22}^e = \sigma_{22}^a \varepsilon_{22}^a \\ \varepsilon_{22}^a = \dfrac{\sigma_{22}^a}{E} + \left(\dfrac{\sigma_{22}^a}{K}\right)^{\frac{1}{n}} \end{cases}$$

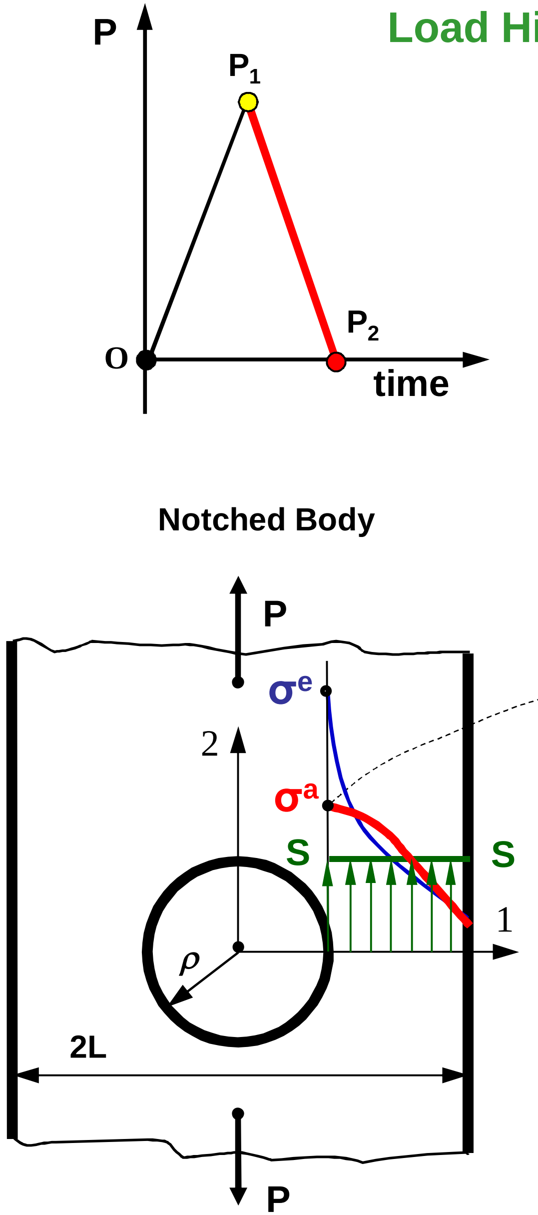
The ESED method and the Ramberg-Osgood curve

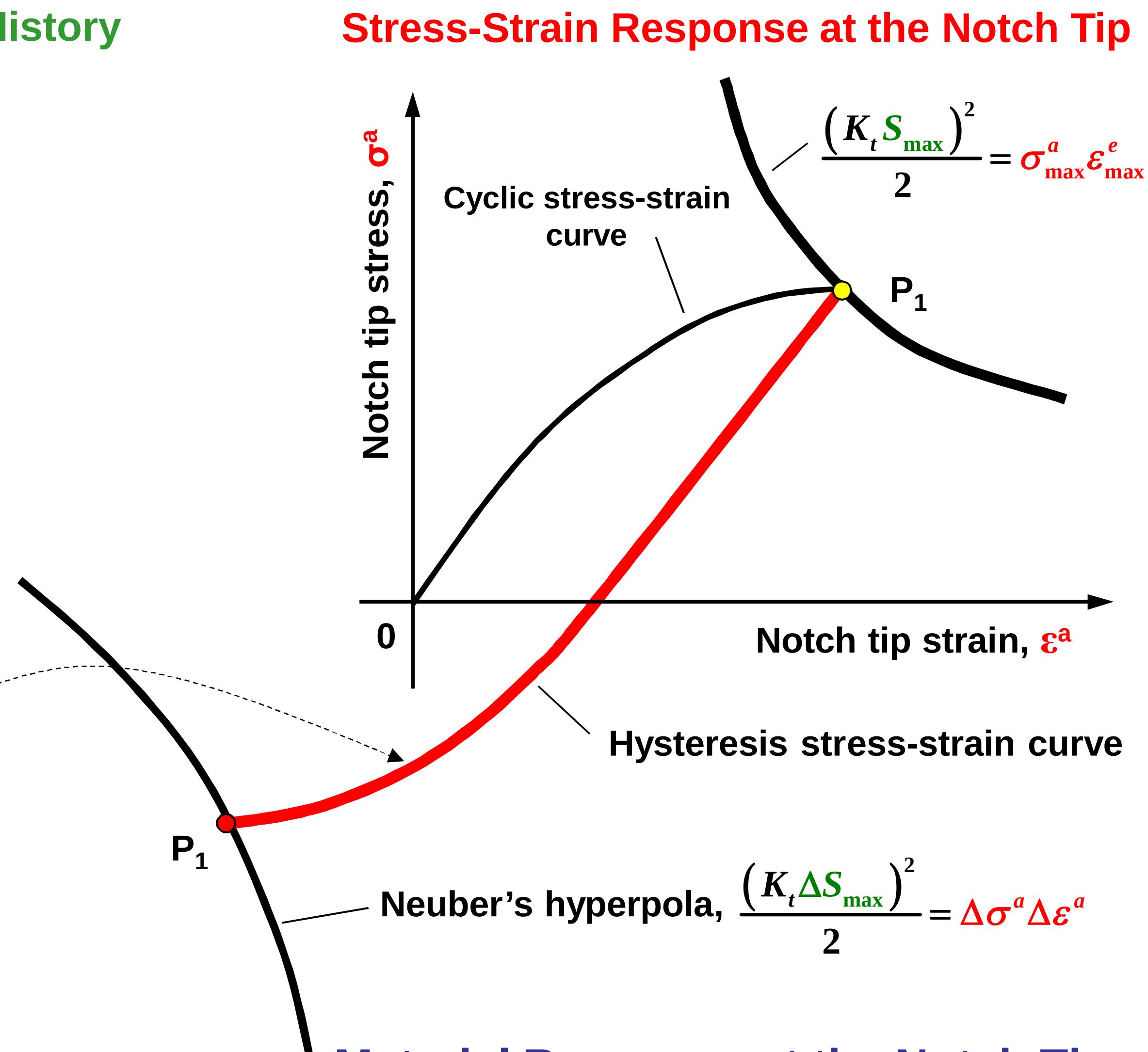


$$\begin{cases} \dfrac{(\sigma_n K_t)^2}{2E} = \dfrac{\sigma_{22}^e \varepsilon_{22}^e}{2} = \dfrac{(\sigma_{22}^a)^2}{2E} + \dfrac{\sigma_{22}^a}{n+1}\left(\dfrac{\sigma_{22}^a}{K}\right)^{\frac{1}{n}} \\ \varepsilon_{22}^a = \dfrac{\sigma_{22}^a}{E} + \left(\dfrac{\sigma_{22}^a}{K}\right)^{\frac{1}{n}} \end{cases}$$

Graphical solution to the Neuber rule and the equation of the Stress-Strain curve

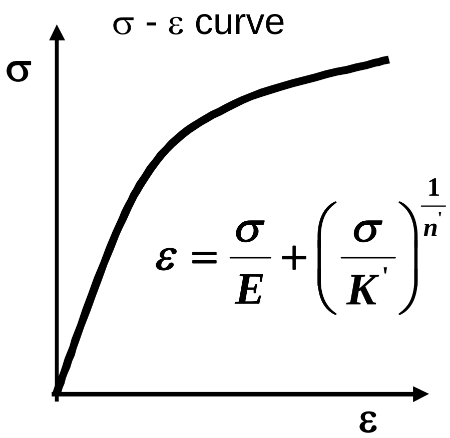




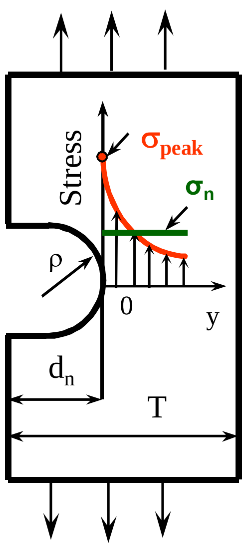


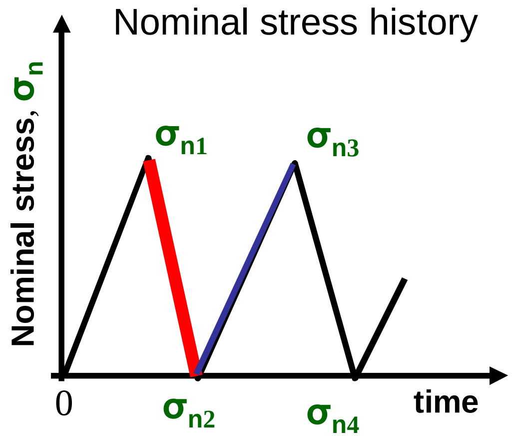
Material Response at the Notch Tip Due to Loading and Unloading Reversals of Load P

Simulation of Stress-Strain Response at the Notch Tip (Neuber's Rule) Induced by Cyclic Loading

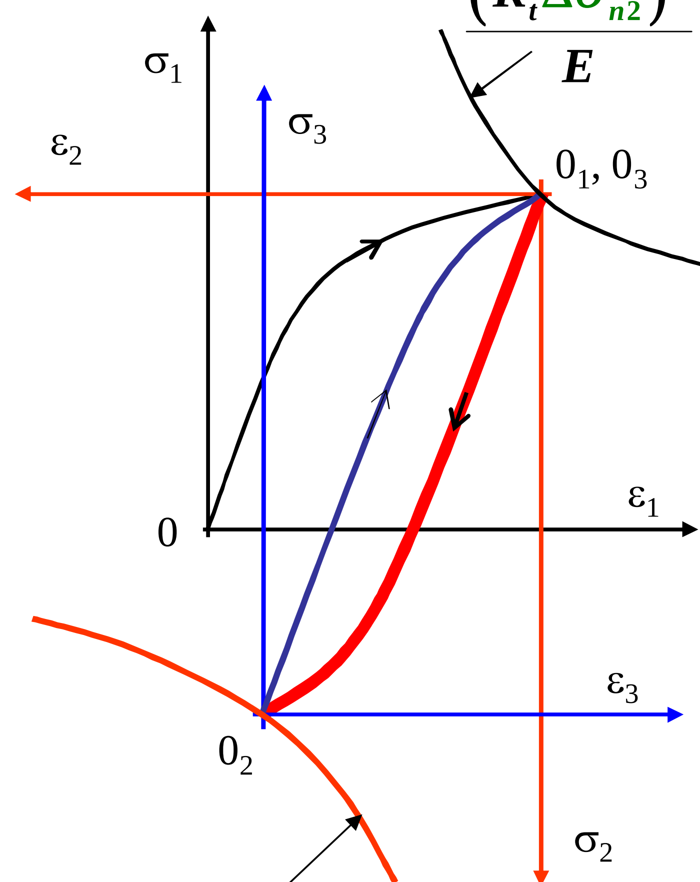


$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}}$$





$$\frac{\left(K_t \Delta\sigma_{n2}\right)^2}{E} = \sigma_1 \cdot \varepsilon_1$$

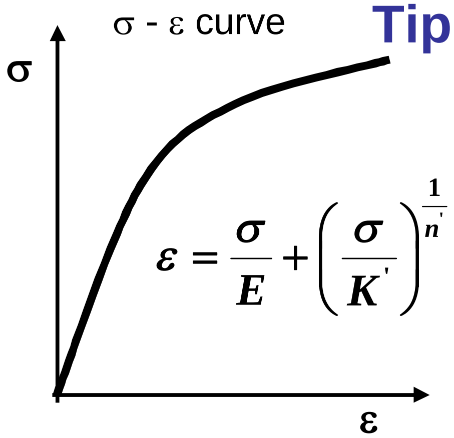


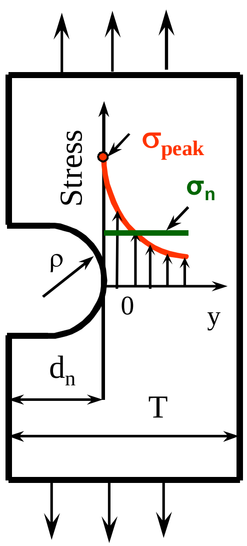
$$\frac{\left(K_t \cdot \Delta\sigma_{n2}\right)^2}{E} = \Delta\sigma_2 \cdot \Delta\varepsilon_2$$

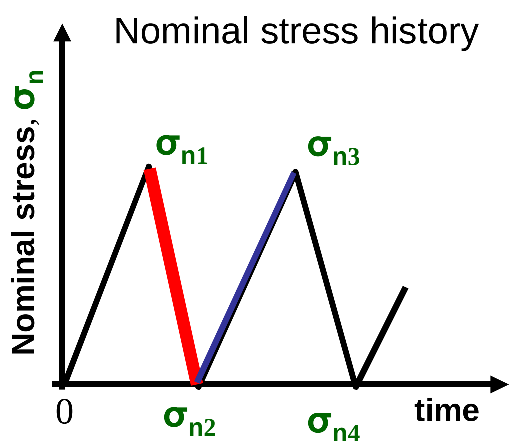
$$\sigma_1 \cdot \varepsilon_1 = \frac{\left(\sigma_1\right)^2}{E} + \sigma_1\left(\frac{\sigma_1}{K'}\right)^{\frac{1}{n'}}$$

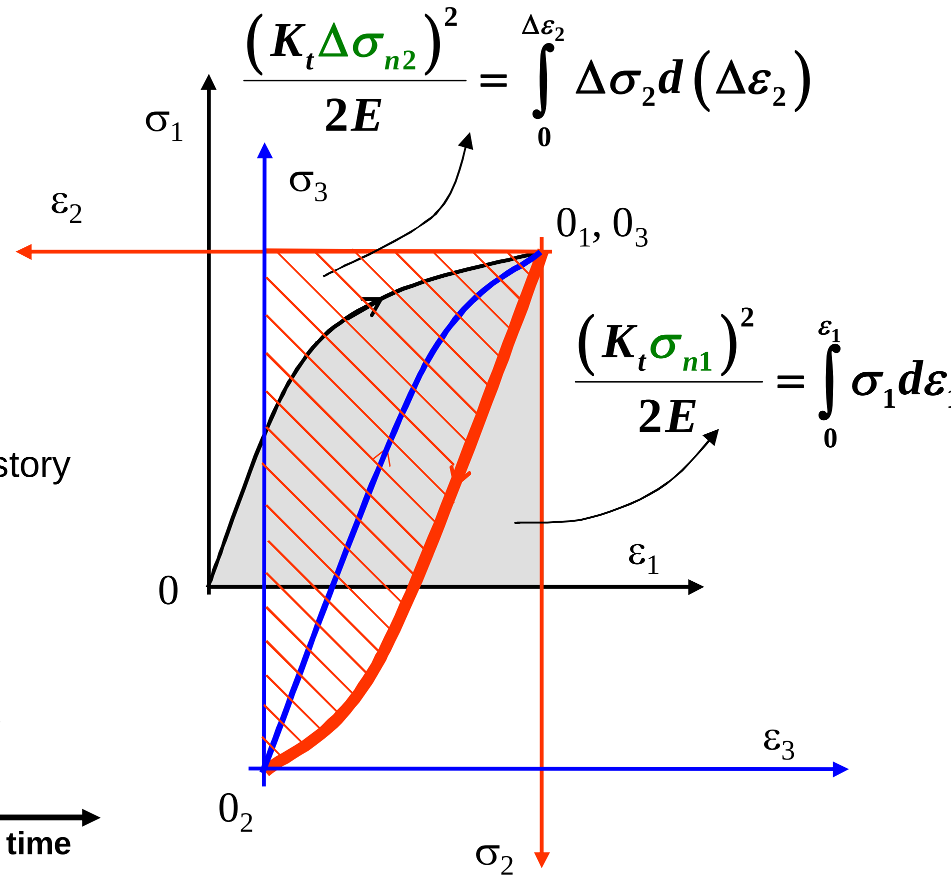
$$\Delta\sigma_2 \cdot \Delta\varepsilon_2 = \frac{\left(\Delta\sigma_2\right)^2}{E} + 2\cdot\Delta\sigma_2\left(\frac{\Delta\sigma_2}{2K'}\right)^{\frac{1}{n'}}$$

Simulation of Stress-Strain Response at the Notch Tip (ESED Method) Induced by Cyclic Loading







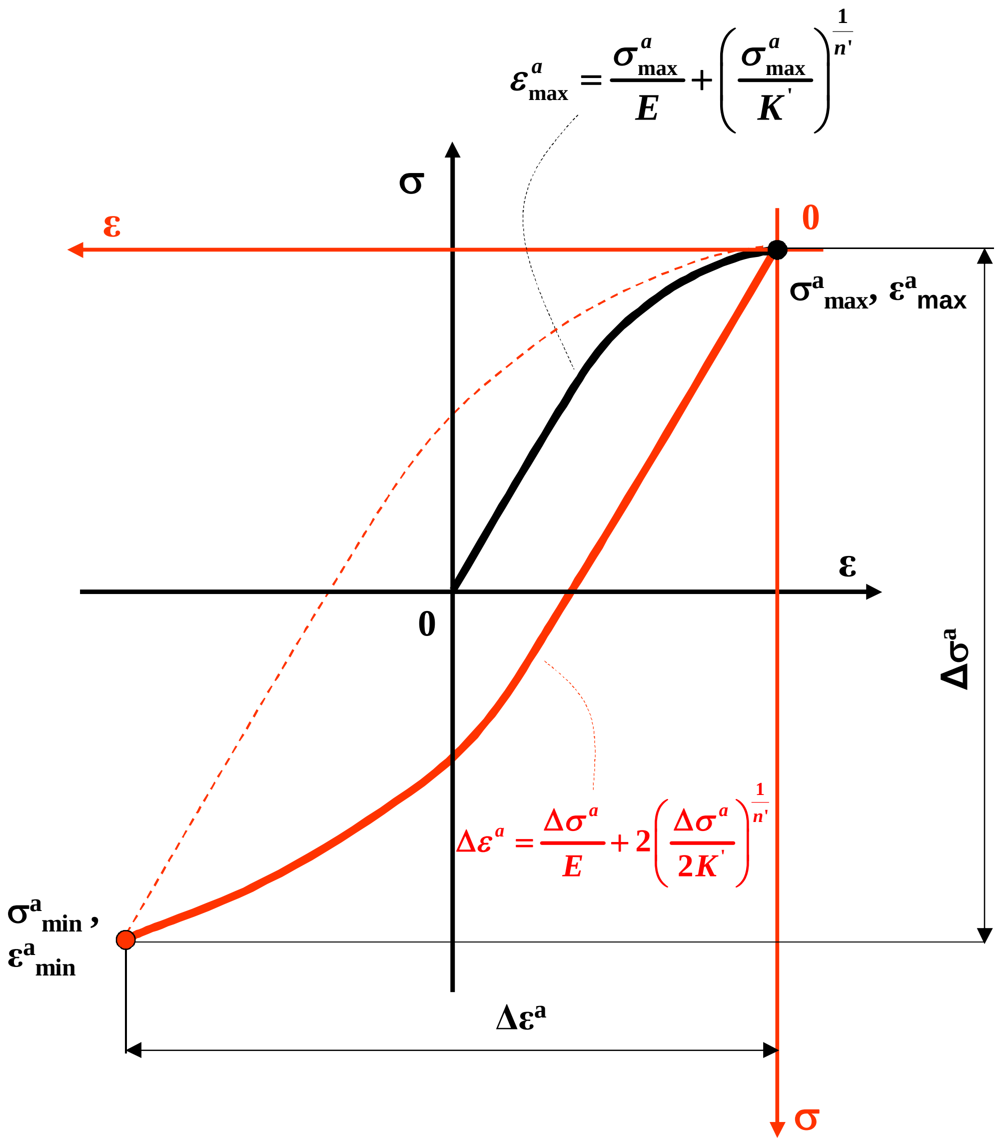


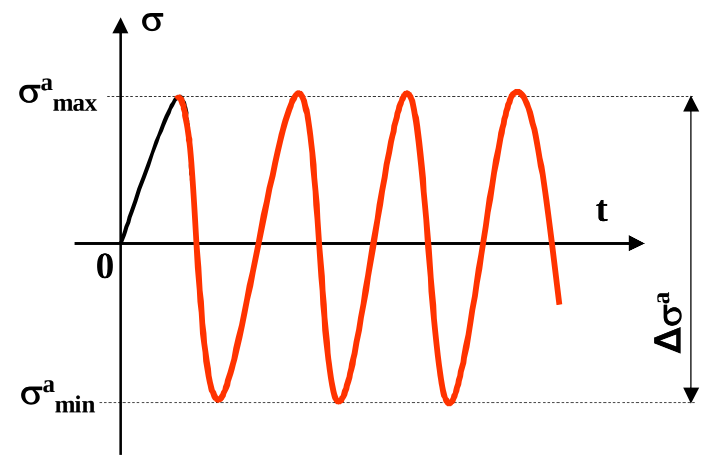
$$\int_0^{\varepsilon_1} \sigma_1 d\varepsilon_1 = \frac{\sigma_1^2}{2E} + \frac{\sigma_1}{n'+1}\left(\frac{\sigma_1}{K'}\right)^{\frac{1}{n'}}$$

$$\int_0^{\Delta\varepsilon_1} \Delta\sigma_2 d(\Delta\varepsilon_2) = \frac{(\Delta\sigma_2)^2}{2E} + \frac{2\cdot\Delta\sigma_2}{n'+1}\left(\frac{\Delta\sigma_2}{2K'}\right)^{\frac{1}{n'}}$$

Cyclic loading and cyclic stress-strain response

smooth component, non-linear elastic-plastic stress-strain curve





$$\varepsilon^a_{max} = \frac{\sigma^a_{max}}{E} + \left(\frac{\sigma^a_{max}}{K'}\right)^{\frac{1}{n'}}$$

$$\Delta\varepsilon^a = \frac{\Delta\sigma^a}{E} + 2\left(\frac{\Delta\sigma^a}{2K'}\right)^{\frac{1}{n'}}$$

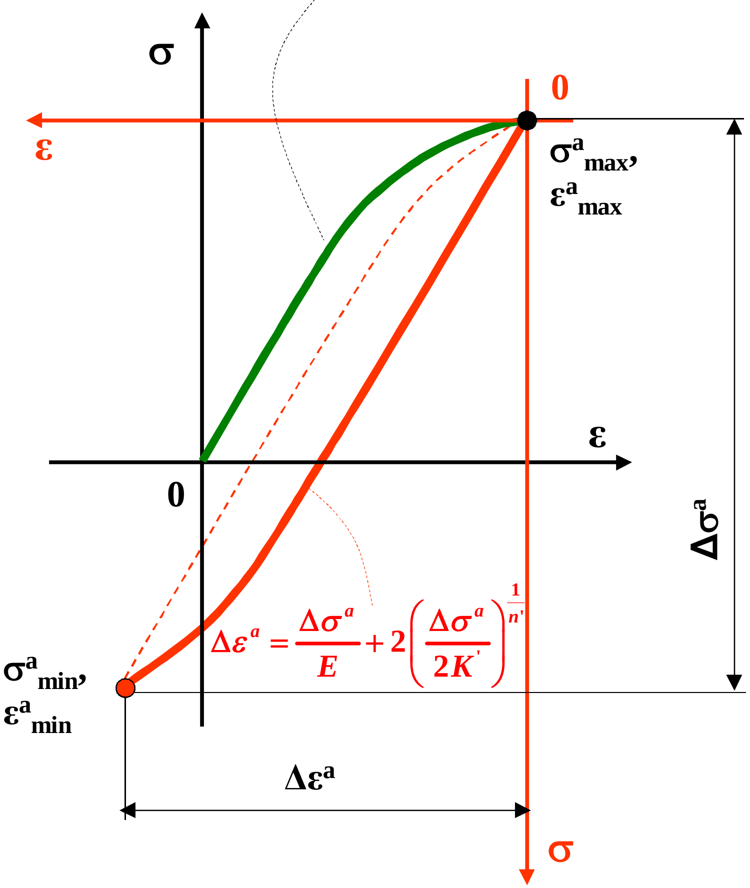
$$\varepsilon^a_{min} = \varepsilon^a_{max} - \Delta\varepsilon^a$$

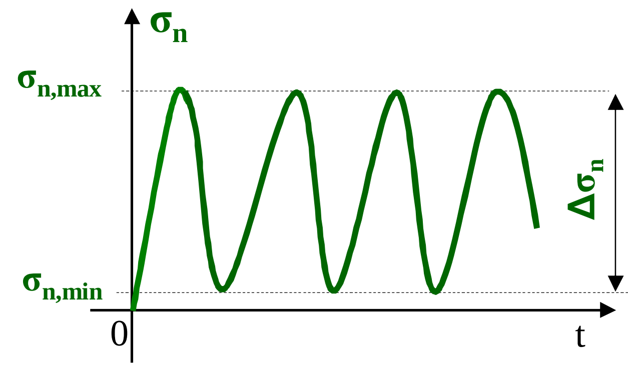
$$\sigma^a_{min} = \sigma^a_{max} - \Delta\sigma^a$$

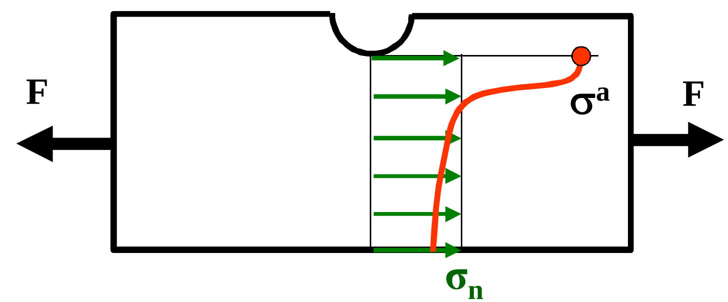
Cyclic loading and cyclic stress-strain response

notched component, non-linear elastic-plastic stress-strain curve

$$\varepsilon^a_{\max} = \frac{\sigma^a_{\max}}{E} + \left(\frac{\sigma^a_{\max}}{K'}\right)^{\frac{1}{n'}}$$



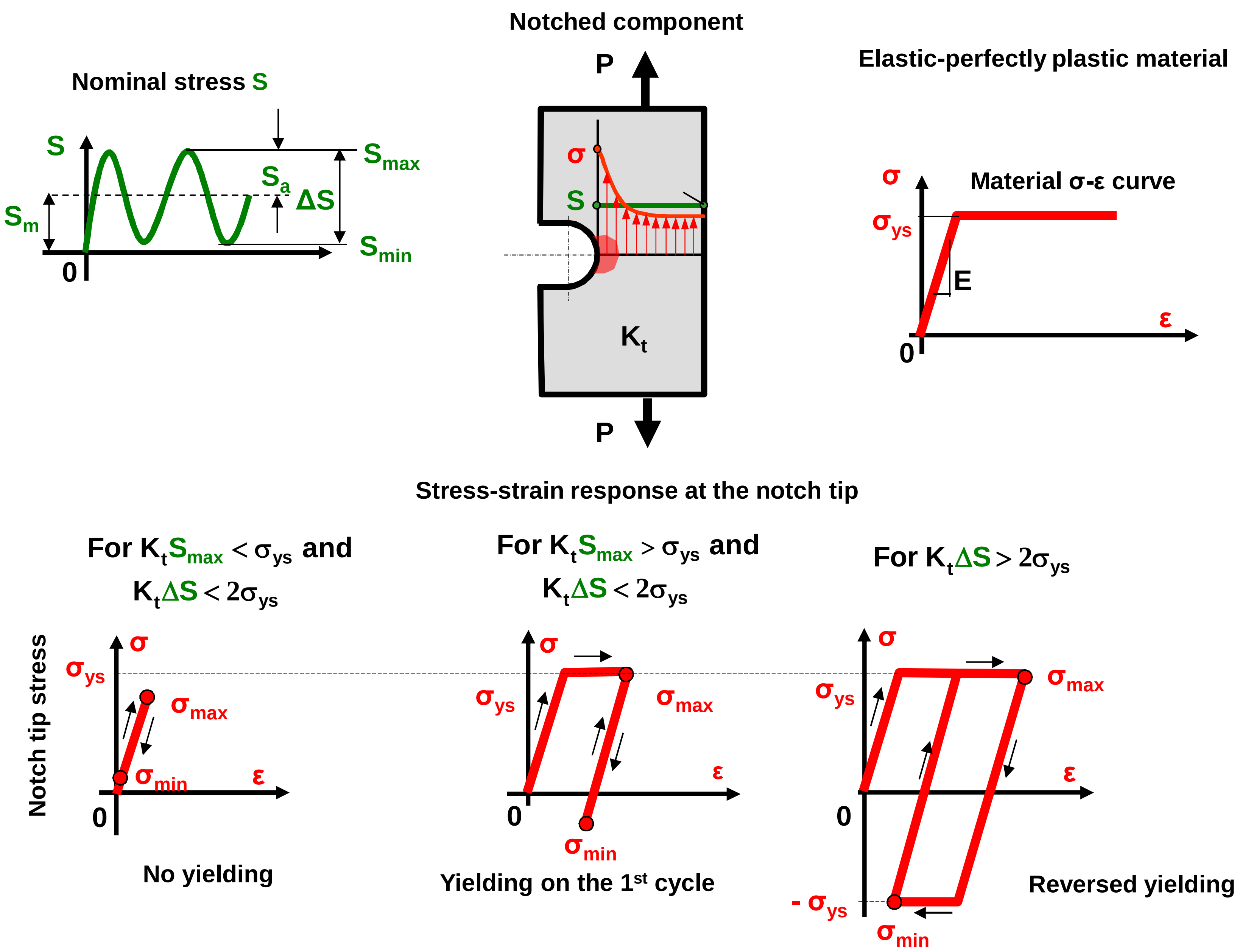




$$\begin{cases} \dfrac{\left(K_t \sigma_{n,\max}\right)^2}{E} = \sigma^a_{\max}\varepsilon^a_{\max} \\[2ex] \varepsilon^a_{\max} = \dfrac{\sigma^a_{\max}}{E} + \left(\dfrac{\sigma^a_{\max}}{K'}\right)^{\frac{1}{n'}} \end{cases}$$

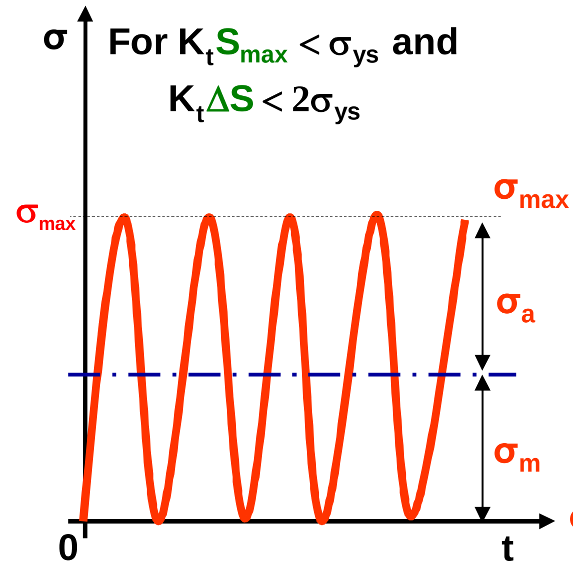
$$\begin{cases} \dfrac{\left(K_t \Delta\sigma_n\right)^2}{E} = \Delta\sigma^a \cdot \Delta\varepsilon^a \\[2ex] \Delta\varepsilon^a = \dfrac{\Delta\sigma^a}{E} + 2\left(\dfrac{\Delta\sigma^a}{2K'}\right)^{\frac{1}{n'}} \end{cases}$$

$$\varepsilon^a_{\min} = \varepsilon^a_{\max} - \Delta\varepsilon^a; \qquad \sigma^a_{\min} = \sigma^a_{\max} - \Delta\sigma^a$$



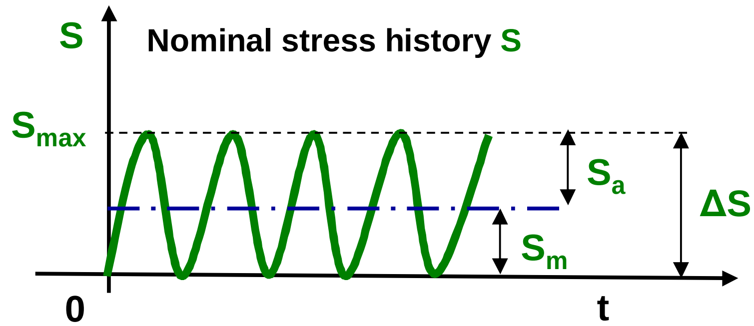
Notched component
Nominal stress S
Elastic-perfectly plastic material
P
S
S_{max}
S_a
ΔS
S_m
S_{min}
0
σ
S
K_t
P
σ
Material σ-ε curve
σ_{ys}
E
ε
0
Stress-strain response at the notch tip
For $K_t S_{max} < \sigma_{ys}$ and $K_t \Delta S < 2\sigma_{ys}$
For $K_t S_{max} > \sigma_{ys}$ and $K_t \Delta S < 2\sigma_{ys}$
For $K_t \Delta S > 2\sigma_{ys}$
Notch tip stress
σ
σ_{ys}
σ_{max}
σ_{min}
ε
0
No yielding
σ
σ_{ys}
σ_{max}
ε
0
σ_{min}
Yielding on the 1st cycle
σ
σ_{ys}
σ_{max}
ε
0
$-\sigma_{ys}$
σ_{min}
Reversed yielding

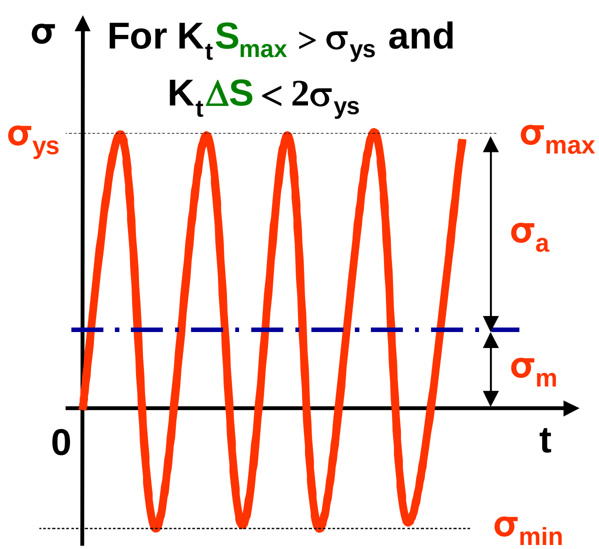
The ‘erratic’ relationship between the nominal mean stress S_m and the local *(at the notch tip)* mean stress σ_m



$$\sigma_{max} = K_t S_{max}$$
$$\sigma_a = K_t S_a$$
$$\sigma_m = K_t S_m$$
$$\sigma_{min} = K_t S_{max} - 2K_t S_a$$

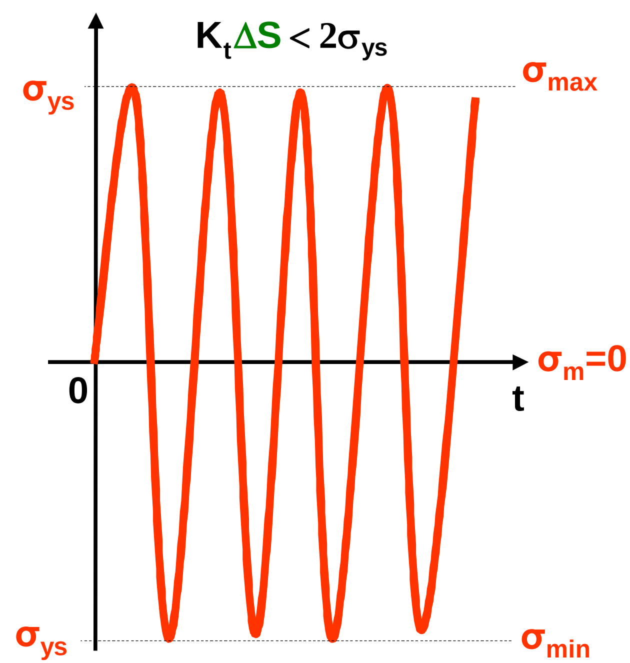
σ_{min}





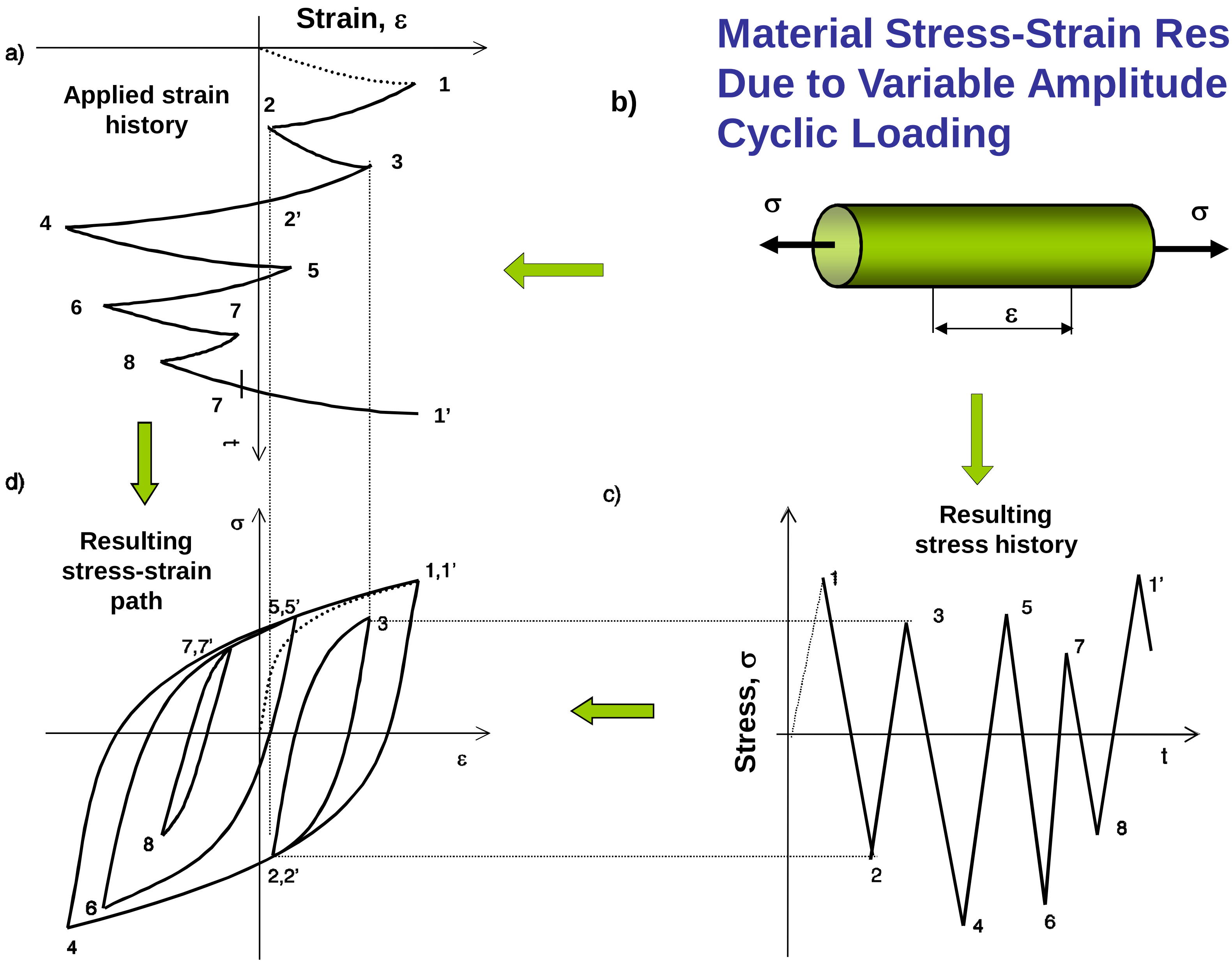
$$\sigma_{max} = \sigma_{ys}$$
$$\sigma_a = K_t S_a$$
$$\sigma_m = \sigma_{ys} - K_t S_a$$
$$\sigma_{min} = \sigma_{ys} - 2K_t S_a$$

Local notch tip stress histories σ

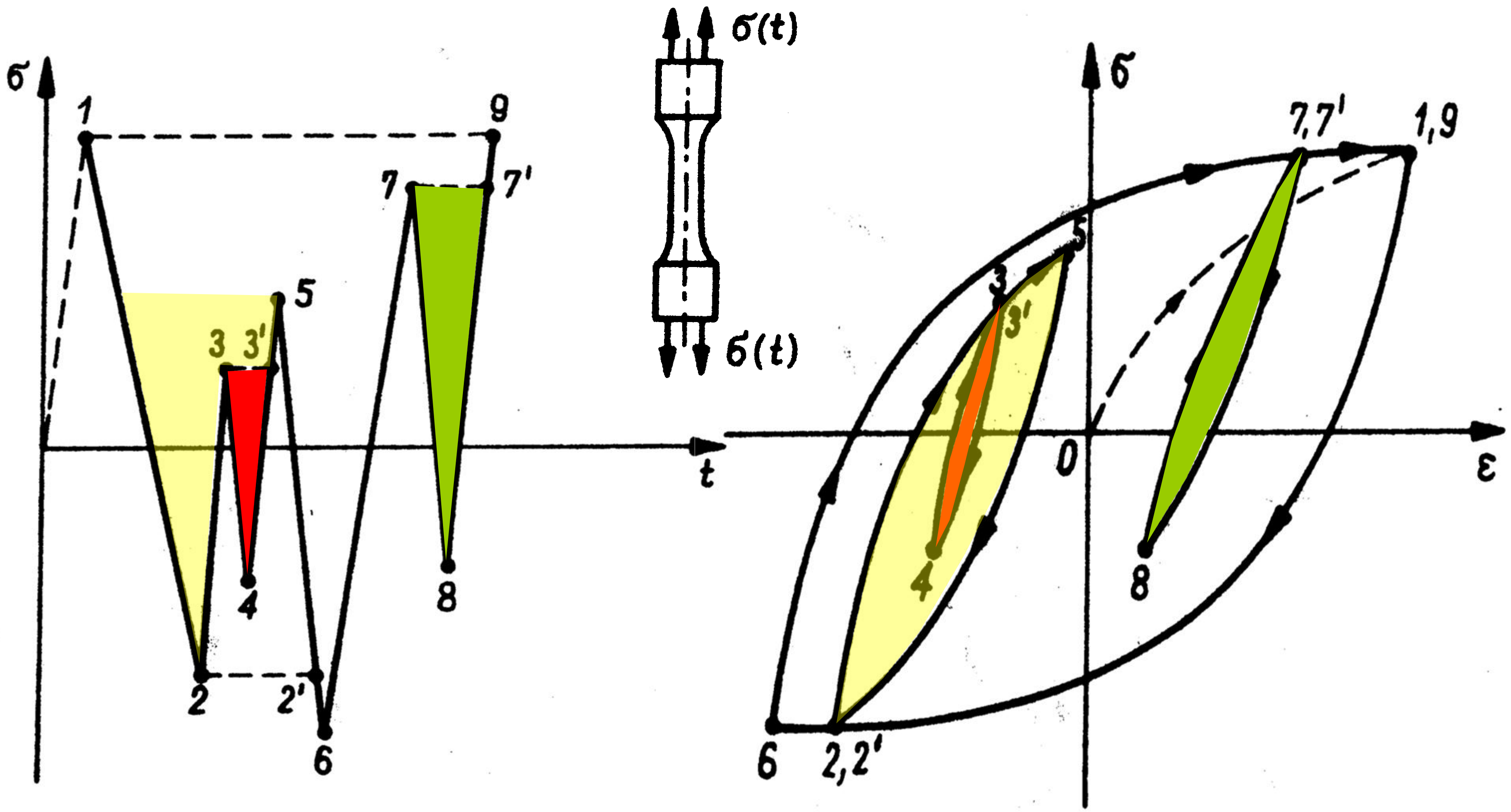


$$\sigma_{max} = \sigma_{ys}$$
$$\sigma_a = \sigma_{ys}$$
$$\sigma_m = 0$$
$$\sigma_{min} = -\sigma_{ys}$$

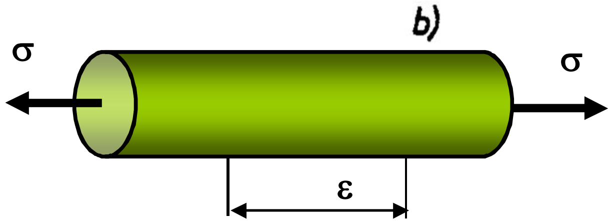
Material Stress-Strain Response Due to Variable Amplitude Cyclic Loading

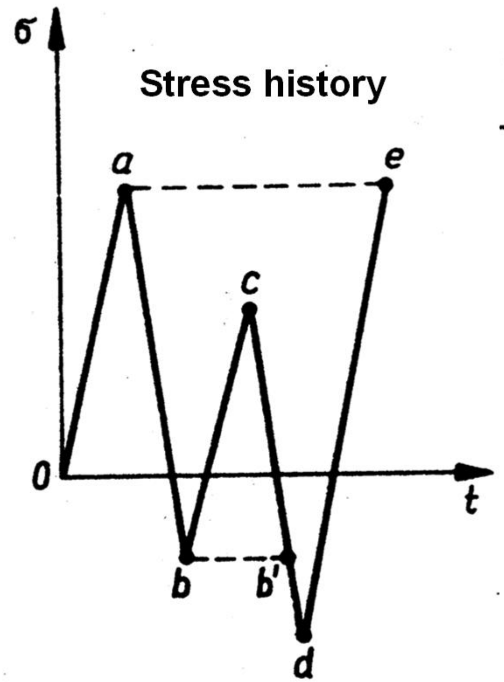


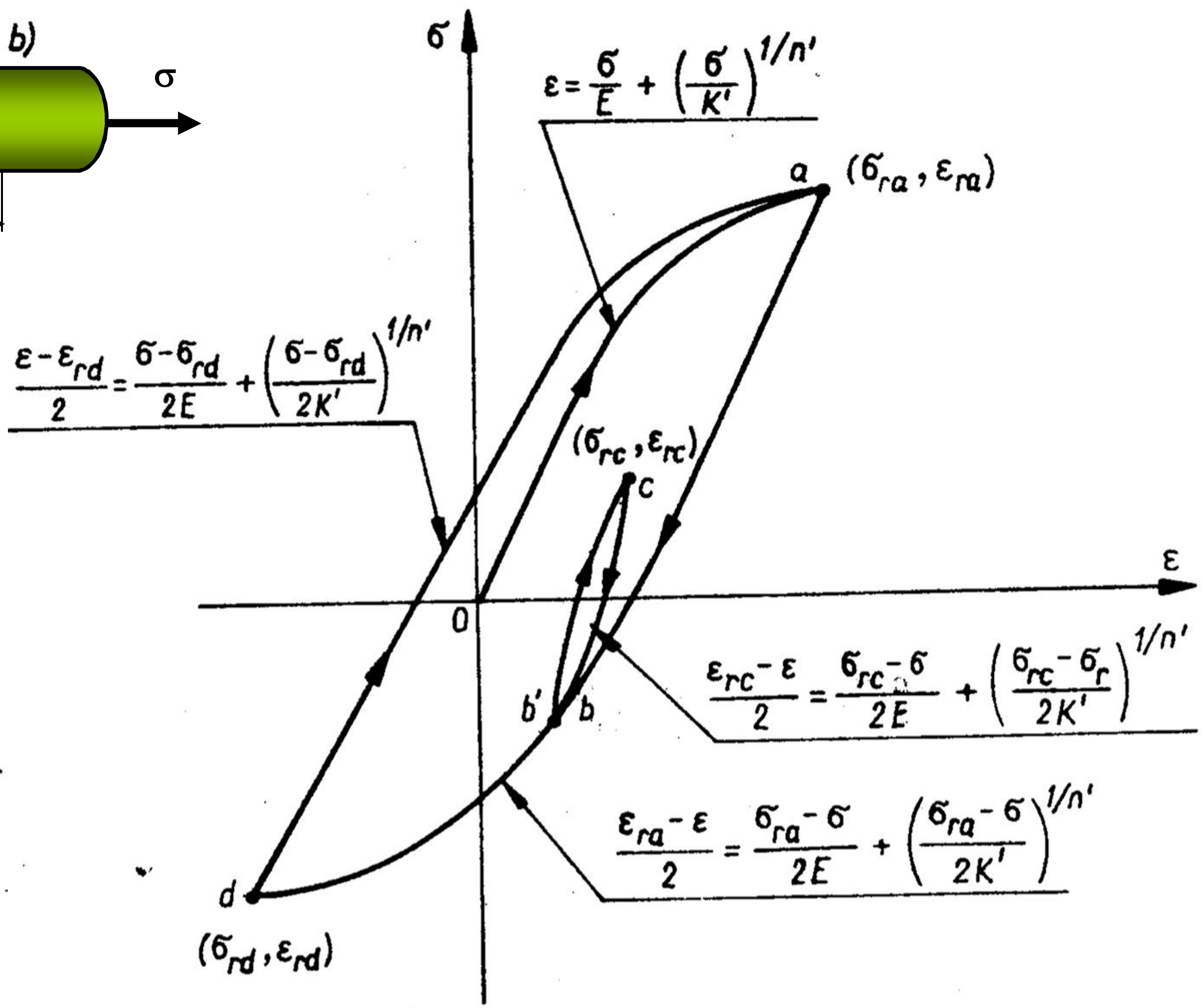
Strain-Stress Hysteresis Loops vs. "Rainflow Counted" Cycles



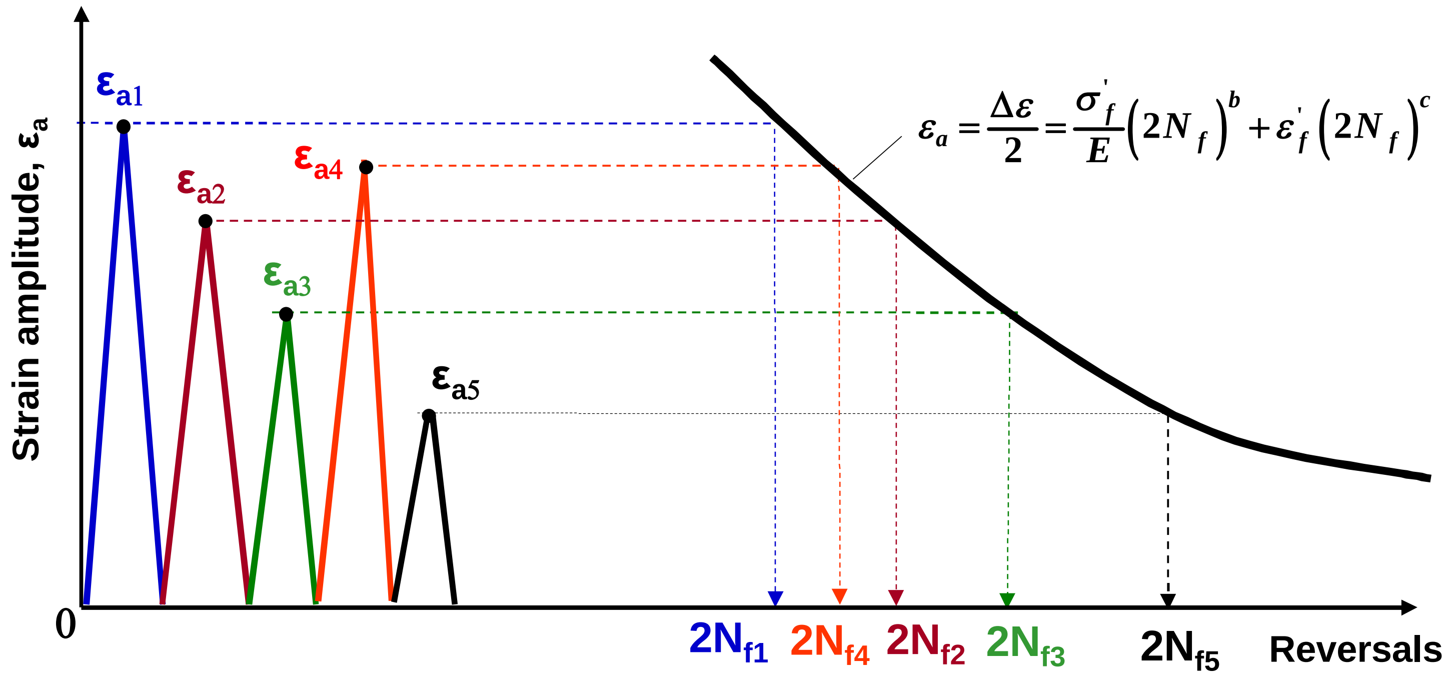
Mathematical Description of Material Stress-Strain Response Induced by a Variable Amplitude Stress or Strain History







The linear hypothesis of Fatigue Damage accumulation (the Miner rule)



$$\varepsilon_a = \frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$

$$\varepsilon_{a,i} = \frac{\Delta\varepsilon_i}{2} = \frac{\sigma'_f}{E}\left(2N_{fi}\right)^b + \varepsilon'_f\left(2N_{fi}\right)^c \Rightarrow N_{fi}$$

$$D_1 = \frac{1}{N_{f1}}; \quad D_2 = \frac{1}{N_{f2}}; \quad D_3 = \frac{1}{N_{f3}};$$

$$D_4 = \frac{1}{N_{f4}}; \quad D_5 = \frac{1}{N_{f5}};$$

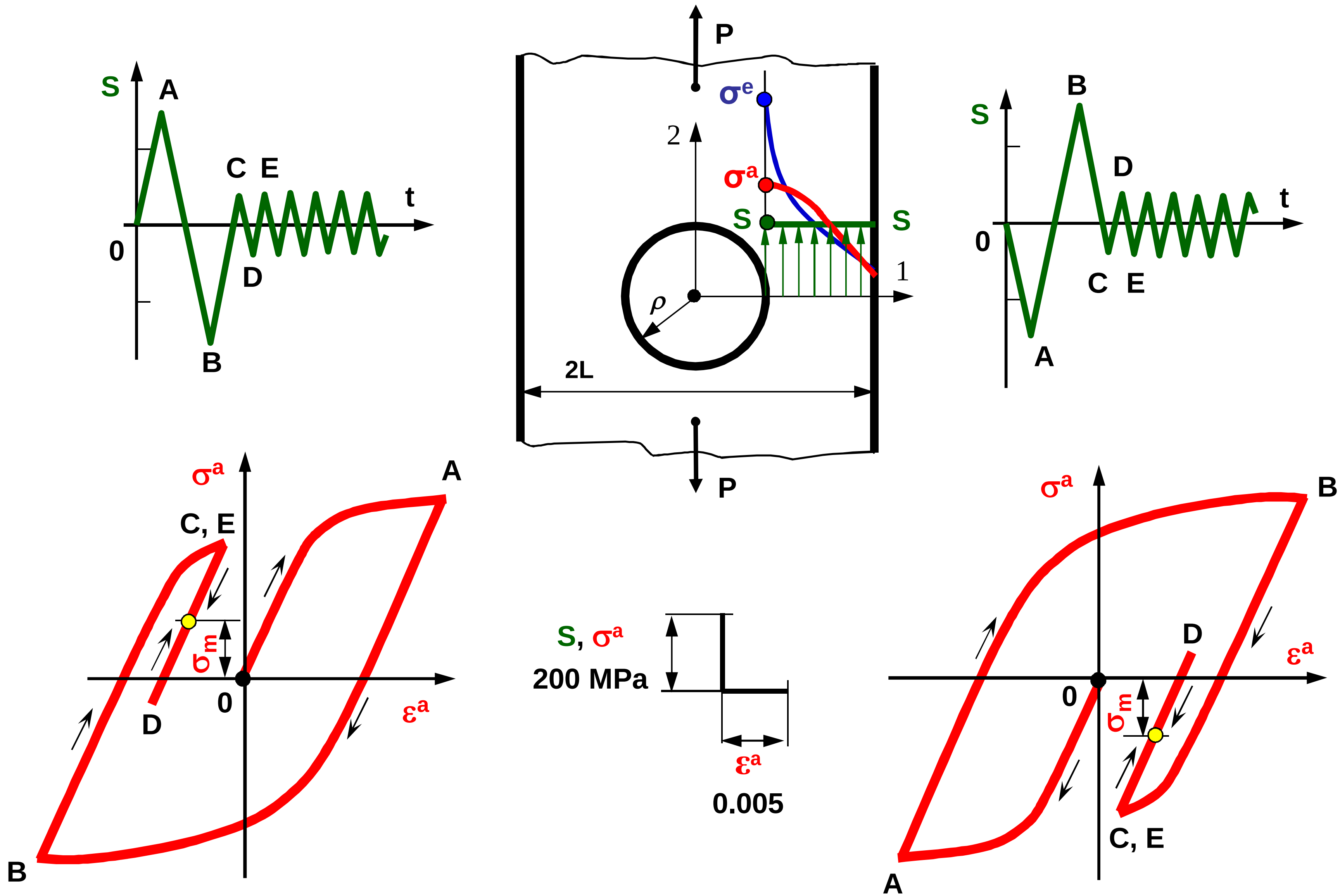
$$D = \sum_{i=1}^{5} D_i = D_1 + D_2 + D_3 + D_4 + D_5$$

$$= \frac{1}{N_{f1}} + \frac{1}{N_{f2}} + \frac{1}{N_{f3}} + \frac{1}{N_{f4}} + \frac{1}{N_{f5}};$$

$$\text{if} \quad D \geq 1 \rightarrow \text{Failure!!}$$

$$L_R = \frac{1}{D} = \frac{1}{1/N_{f1} + 1/N_{f2} + 1/N_{f3} + 1/N_{f4} + 1/N_{f5}}$$

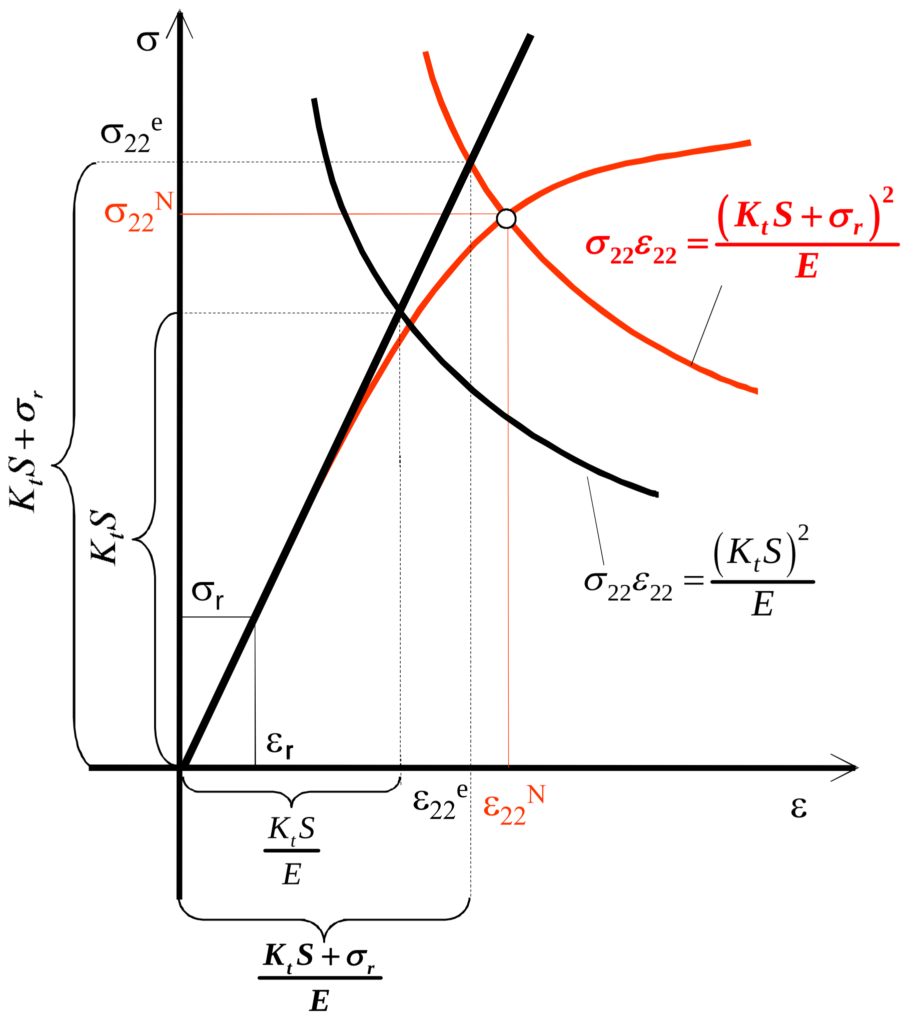
The Loading Sequence Effect

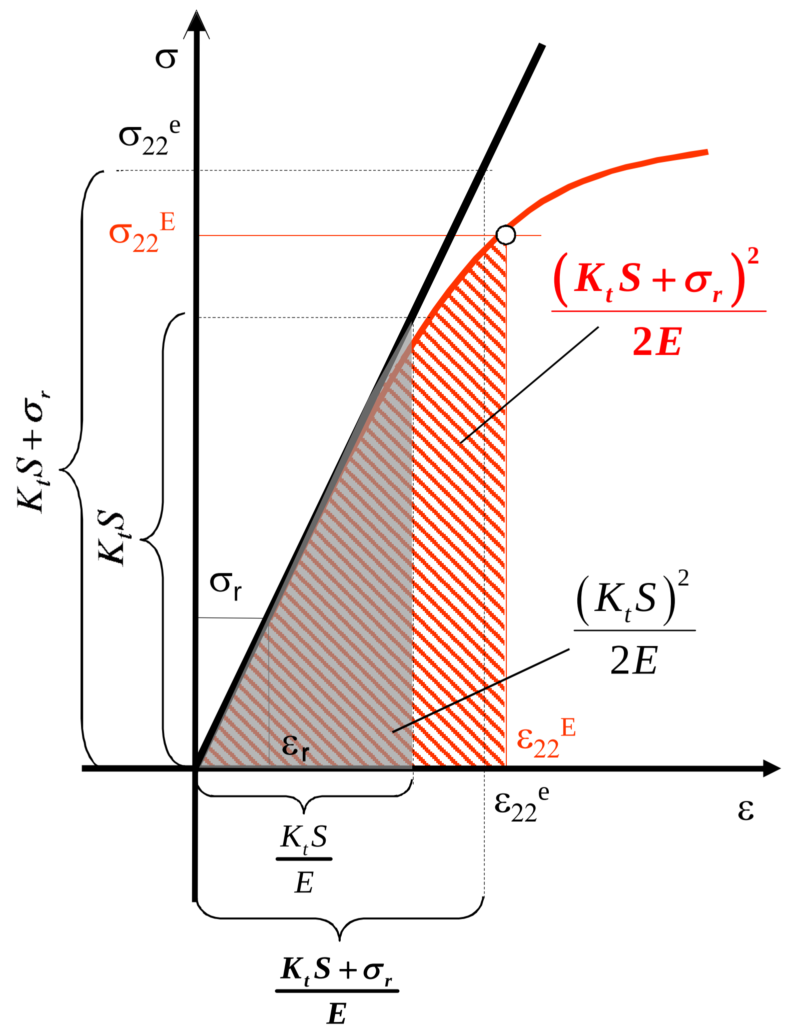


Modeling the residual stress effect

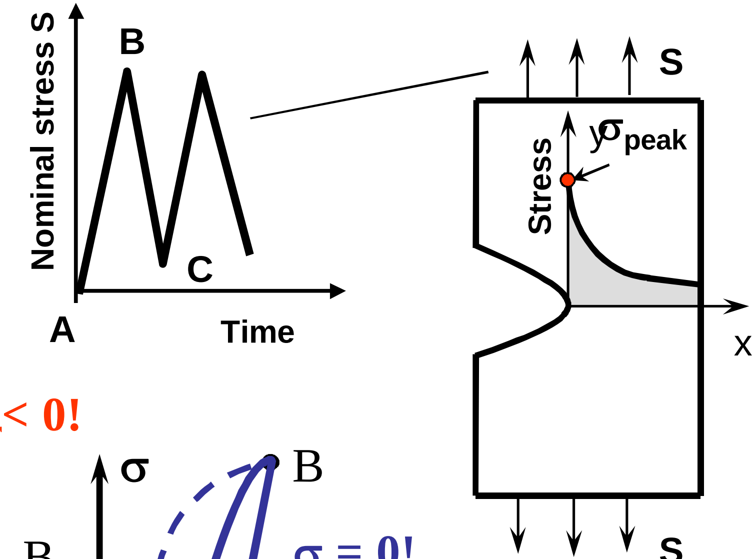
$$\frac{\left(K_t S+\sigma_r\right)^2}{E}=\sigma_{22}^{N}\varepsilon_{22}^{N}$$ - Neuber's rule

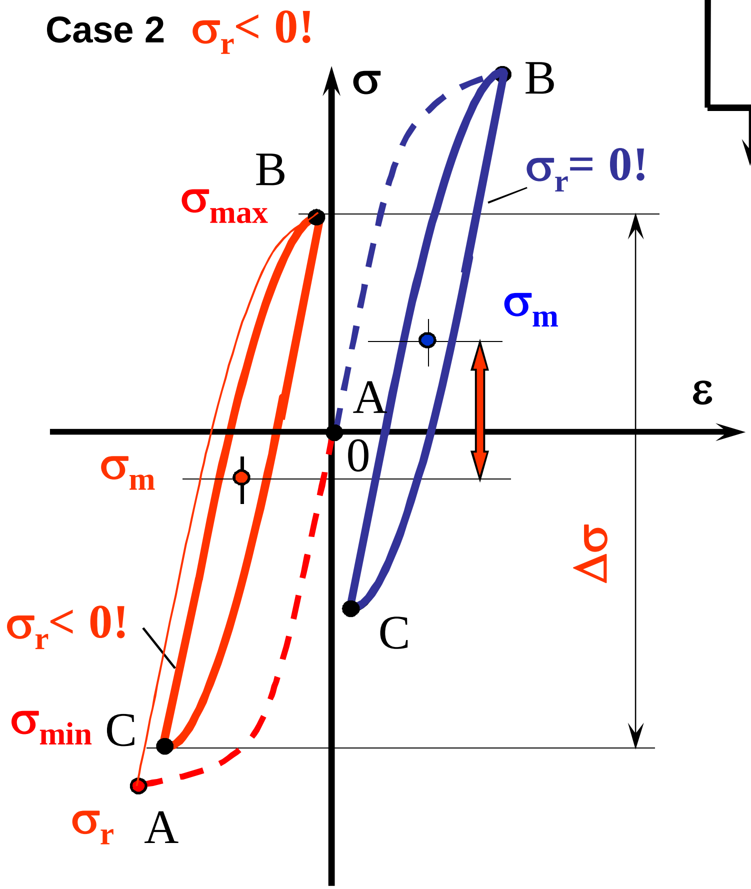
$$\frac{\left(K_t S+\sigma_r\right)^2}{2E}=\int_0^{\varepsilon_{22}^{E}}\sigma_{22}^{E}\,d\varepsilon_{22}^{E}$$ - ESED method

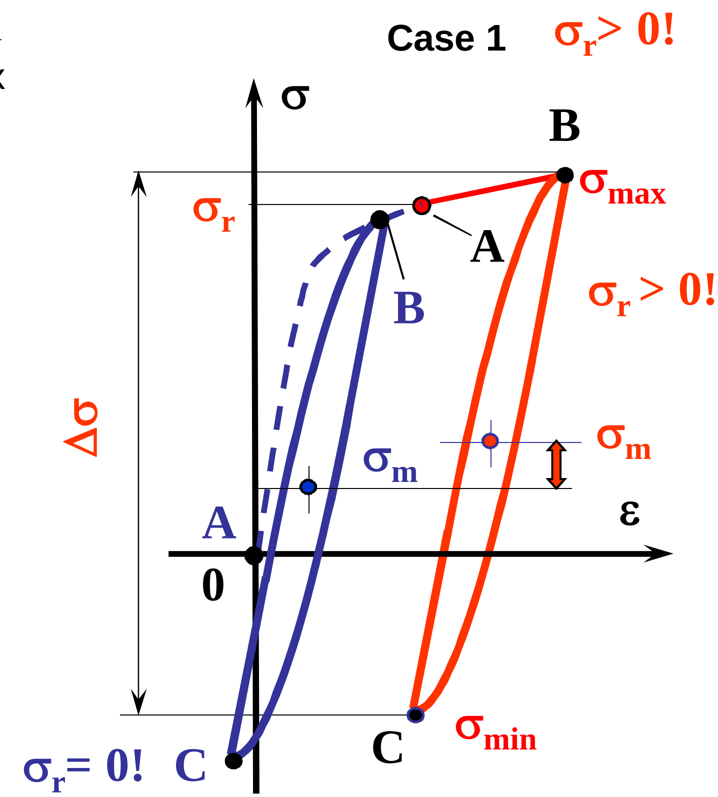




Residual Stress Effect on the Stress-Strain Response at the Notch Tip







Summary of the Local Strain-Life (ε-N) Approach

Advantages:

- The method takes into account the actual stress-strain response of the material due to cyclic loading.
- Plastic strain, and the mechanism that leads to crack initiation, is accurately modeled.
- This method can model the effect of the residual mean stresses resulting from the sequence effect in load histories and the manufacturing residual stresses. This allows for more accurate damage accumulation under variable amplitude cyclic loading.
- The ε-N method can be more easily extrapolated to situations involving complicated geometries.
- This method can be used in high temperature applications where fatigue-creep interaction is critical.
- In situations where it is important, this method can incorporate transient material behavior.
- This method can be used for both low cycle (high strains) and high cycle fatigue (low strains)
- There is only one essential empirical element in the method, i.e. the correction for the mean stress effect.