Multiaxial Fatigue

Introduction

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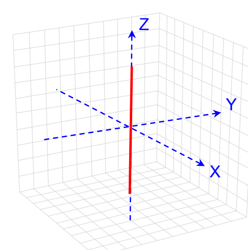
When is Multiaxial Fatigue Important ?

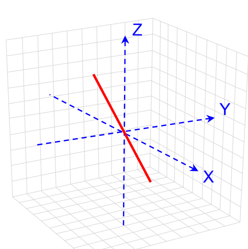
- Complex state of stress
- Complex out of phase loading

Uniaxial Stress



one principal stress
one direction

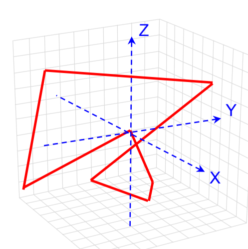
Proportional Biaxial



principal stresses vary proportionally
but do not rotate

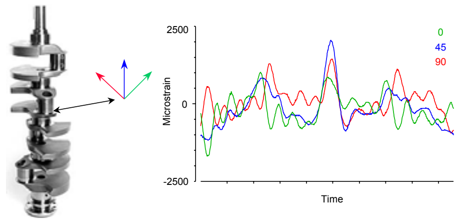
$\sigma_1 = \alpha\sigma_2 = \beta\sigma_3$

Nonproportional Multiaxial

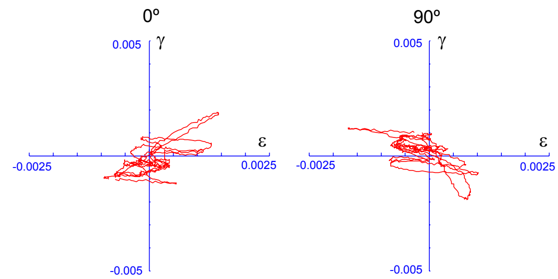


Principal stresses may vary nonproportionally and/or change direction

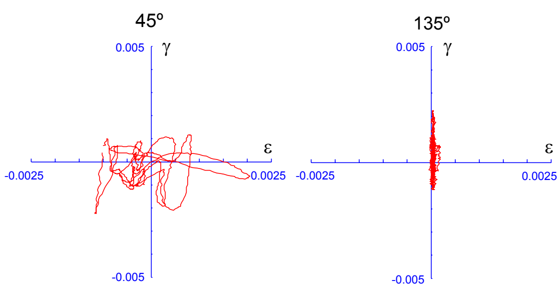
Crankshaft



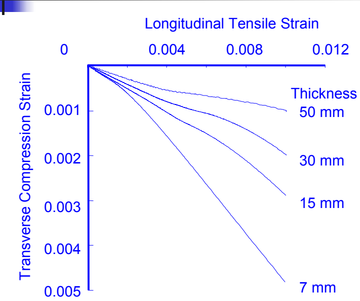
Shear and Normal Strains

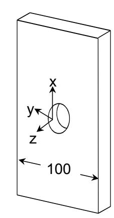


Shear and Normal Strains

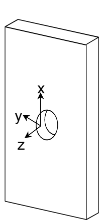


3D stresses





Notch Stresses



t	ε_x	ε_z	σ_x	σ_z
7	0.01	-0.005	63.5	0
15	0.01	-0.003	70.6	14.1
30	0.01	-0.002	73.0	21.8
50	0.01	-0.001	75.1	29.3

Multiaxial Fatigue

State of Stress

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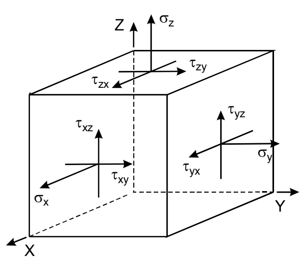
Outline

- **State of Stress**
- Stress-Strain Relationships
- Fatigue Mechanisms
- Multiaxial Testing
- Stress Based Models
- Strain Based Models
- Fracture Mechanics Models
- Nonproportional Loading
- Stress Concentrations

State of Stress

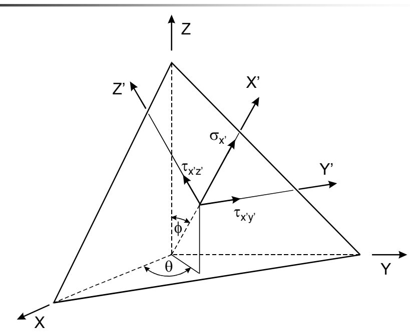
- Stress components
- Common states of stress
- Shear stresses

Stress Components

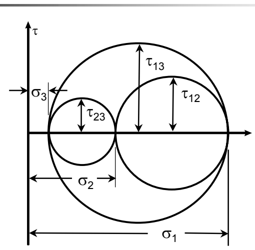


Six stresses and six strains

Stresses Acting on a Plane

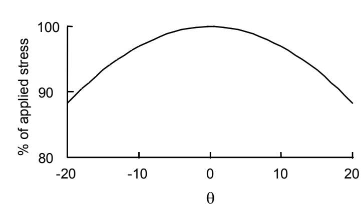


Principal Stresses



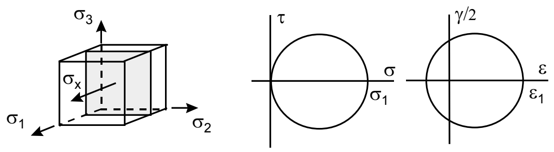
$$\sigma^3 - \sigma^2(\sigma_X + \sigma_Y + \sigma_Z) + \sigma(\sigma_X\sigma_Y + \sigma_Y\sigma_Z\sigma_X\sigma_Z - \tau^2_{XY} - \tau^2_{YZ} - \tau^2_{XZ}) - (\sigma_X\sigma_Y\sigma_Z + 2\tau_{XY}\tau_{YZ}\tau_{XZ} - \sigma_X\tau^2_{YZ} - \sigma_Y\tau^2_{ZX} - \sigma_Z\tau^2_{XY}) = 0$$

Stress and Strain Distributions



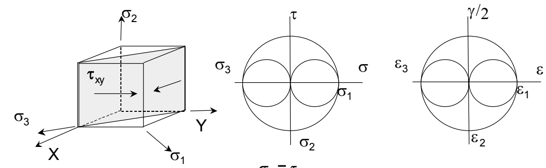
Stresses are nearly the same over a 10° range of angles

Tension



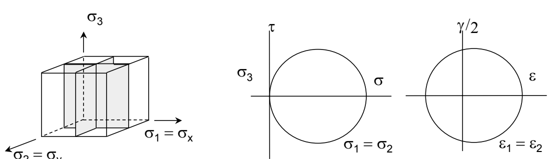
$\sigma_2 = \sigma_3 = 0$ $\quad$ $\varepsilon_2 = \varepsilon_3 = -\nu\varepsilon_1$

Torsion



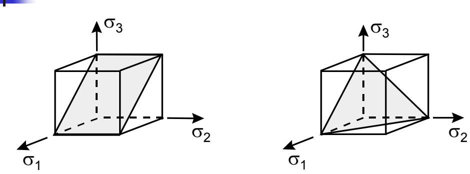
$\sigma_1 = \tau_{xy}$

Biaxial Tension



$$\varepsilon_3 = -\frac{2\nu}{1-\nu}\varepsilon$$

Shear Stresses

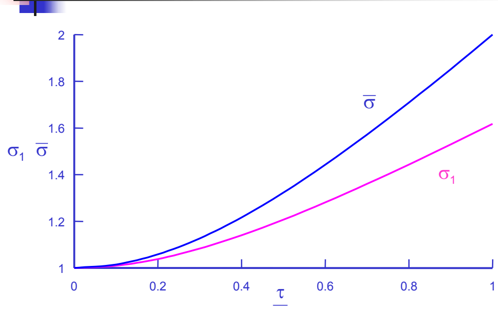


Maximum shear stress $\quad$ Octahedral shear stress

$$\tau_{13} = \frac{|\sigma_1 - \sigma_3|}{2} \qquad \tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_3)^2+(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2}$$

Mises: $\bar{\sigma} = \frac{3}{\sqrt{2}}\tau_{oct}$ $\qquad$ $\tau_{oct} = \frac{3}{2\sqrt{2}}\tau_{13} = 0.94\,\tau_{13}$

Shear Stress Influence



State of Stress Summary

- Stresses acting on a plane
- Principal stress
- Maximum shear stress
- Octahedral shear stress

Multiaxial Fatigue

Stress Strain Relationships

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Outline

- State of Stress
- **Stress-Strain Relationships**
- Fatigue Mechanisms
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Jenkin 1922

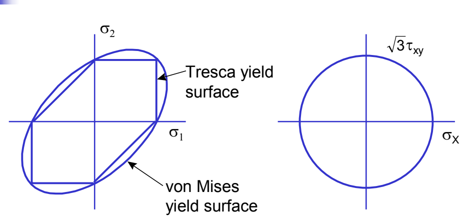
"About six months ago I wrote a paper, knowing that I should be very busy in the autumn and made a model to illustrate a point in it. But as I played with the model to learn how to use it, it grew too strong for me and took command and for the last six months I have been its obedient slave --- for the model explained the whole of my subject *Fatigue*."

"Fatigue in Metals," *The Engineer*, Dec. 8, 1922

Elastic Stress Strain Relationships

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\ \nu & 1-\nu & \nu & 0 & 0 & 0\\ \nu & \nu & 1-\nu & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{bmatrix}$$

Yield Surfaces



Equations

Tresca

$$F_1 = \sigma_1 - \sigma_3 - \sigma_{ys} = 0$$
$$F_2 = \sigma_1 - \sigma_2 - \sigma_{ys} = 0$$

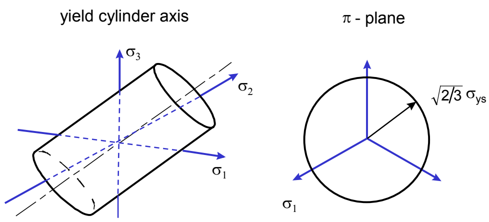
or

$$F_3 = \sigma_2 - \sigma_3 - \sigma_{ys} = 0$$

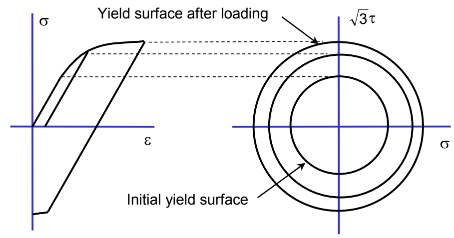
Mises

$$F = \sigma_x^2 + \sigma_y^2 - \sigma_x\,\sigma_y + 3\tau_{xy}^2 - \sigma_{ys}^2 = 0$$

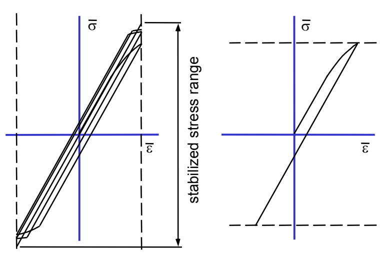
Mises Yield Surfaces



Isotropic Hardening

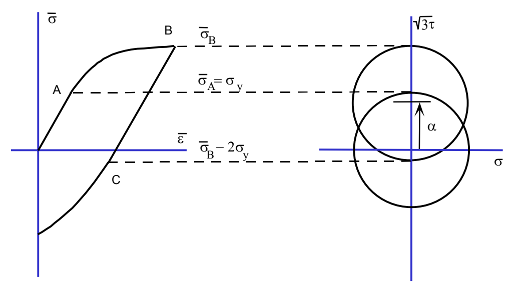


Cyclic Loading

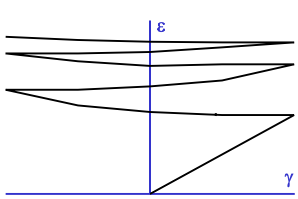


Strain Control   Stress Control

Kinematic Hardening

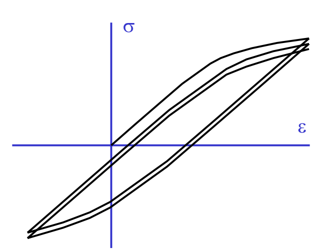


Ratcheting



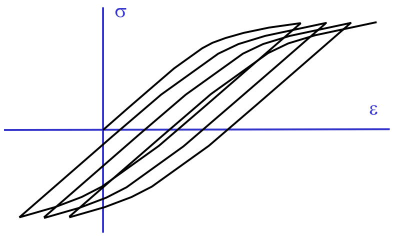
Cyclic torsion with a mean tension stress

Cyclic Mean Stress Relaxation



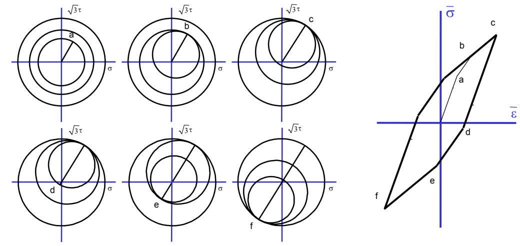
Cyclic strain with mean stress

Cyclic Creep

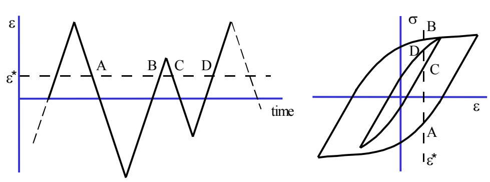


Cyclic stress with a mean stress

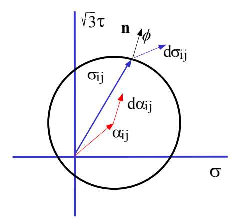
Plasticity Models



Why is modeling needed?



Flow and Hardening Rules



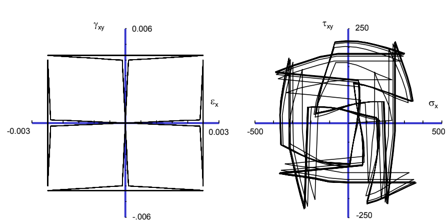
Flow rule

$$d\varepsilon_{ij}^{p} = d\lambda \frac{\partial F}{\partial \sigma_{ij}}$$

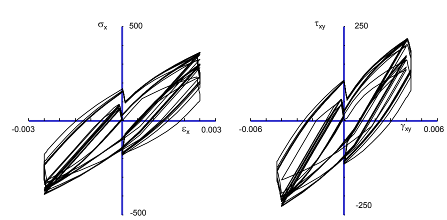
Hardening rule

$$F = f(\sigma_{ij}) + g(\bar{\varepsilon}^{p}) = 0$$

Multiaxial Example



Multiaxial Example



Summary

- Isotropic Hardening
- Kinematic Hardening
- Cyclic creep or ratcheting
- Mean stress relaxation
- Nonproportional hardening

Multiaxial Fatigue

Fatigue Mechanisms

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Outline

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Size Scale for Studying Fatigue

Atoms Dislocations Crystals Specimens Structures

10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2}

Understand the physics on this scale

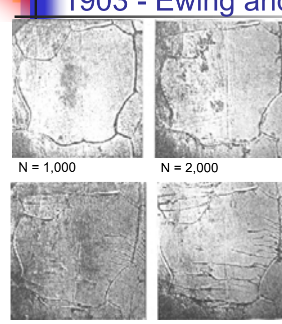
Model the physics on this scale

Use the models on this scale

The Fatigue Process

- Crack nucleation
- Small crack growth in an elastic-plastic stress field
- Macroscopic crack growth in a nominally elastic stress field
- Final fracture

1903 - Ewing and Humfrey



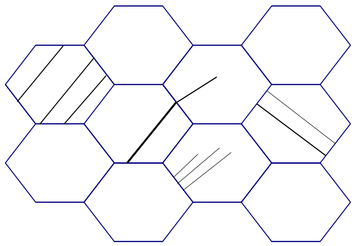
N = 1,000 N = 2,000

N = 10,000 N = 40,000 N_f = 170,000

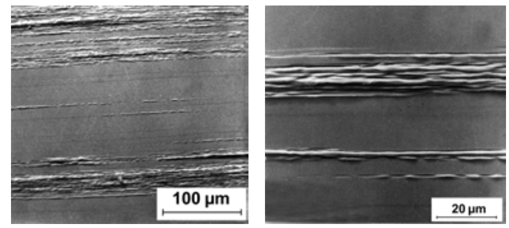
Cyclic deformation leads to the development of slip bands and fatigue cracks

Ewing, J.A. and Humfrey, J.C. "The fracture of metals under repeated alterations of stress", *Philosophical Transactions of the Royal Society*, Vol. A200, 1903, 241-250

Crack Nucleation

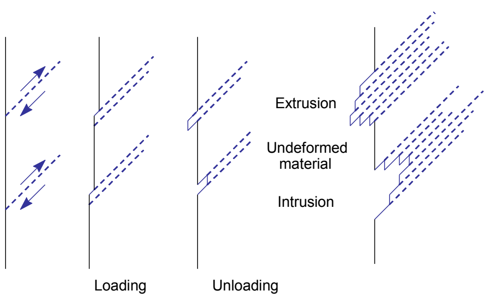


Slip Band in Copper

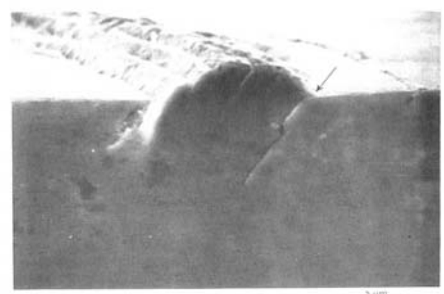


Polak, J. Cyclic Plasticity and Low Cycle Fatigue Life of Metals, Elsevier, 1991

Slip Band Formation

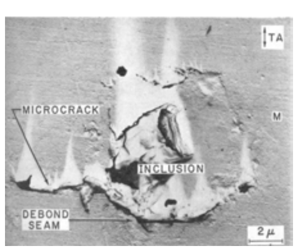


Slip Bands



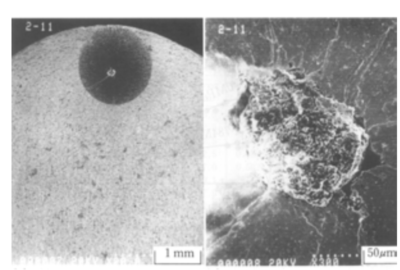
Ma, B-T and Laird C. "Overview of fatigue behavior in copper sinle crystals –II Population, size, distribution and growth Kinetics of stage I cracks for tests at constant strain amplitude", Acta Metallurgica, Vol 37, 1989, 337-348

Crack Initiation at Inclusions



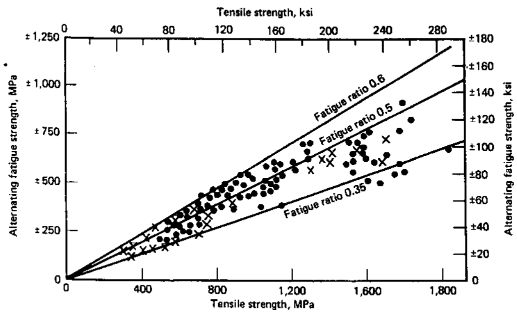
Langford and Kusenberger, "Initiation of Fatigue Cracks in 4340 Steel", *Metallurgical Transactions*, Vol 4, 1977, 553-559

Subsurface Crack Initiation



Y. Murakami, Metal Fatigue: *Effects of Small Defects and Nonmetallic Inclusions*, 2002

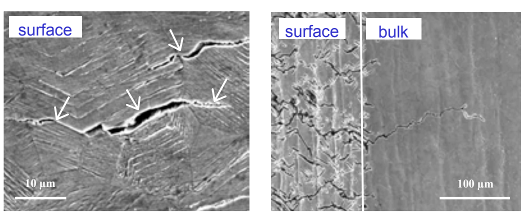
Fatigue Limit and Strength Correlation



Crack Nucleation Summary

- Highly localized plastic deformation
- Surface phenomena
- Stochastic process

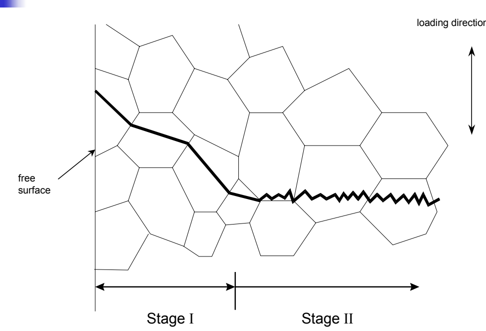
Surface Damage



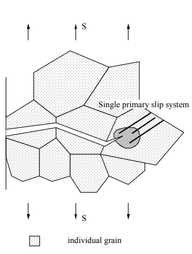
20-25 austenitic steel in symmetrical push-pull fatigue (20°C, $\Delta\varepsilon_p/2 = \pm 0.4\%$) : short cracks on the surface and in the bulk

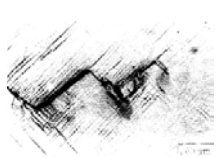
From Jacques Stolarz, Ecole Nationale Superieure des Mines
Presented at LCF 5 in Berlin, 2003

Stage I and Stage II



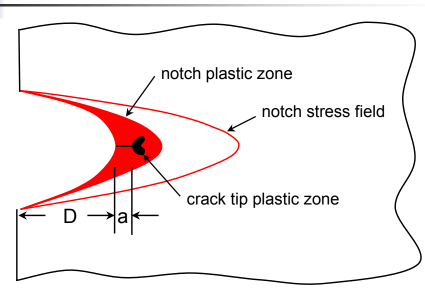
Stage I Crack Growth





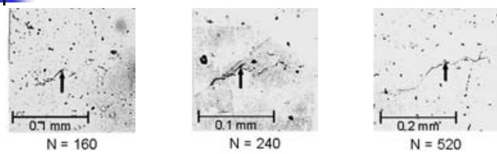
Stage I crack is strongly affected by slip characteristics, microstructure dimensions, stress level, extent of near tip plasticity

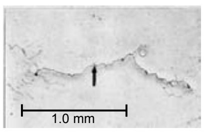
Small Cracks at Notches



Crack growth controlled by the notch plastic strains

Small Crack Growth

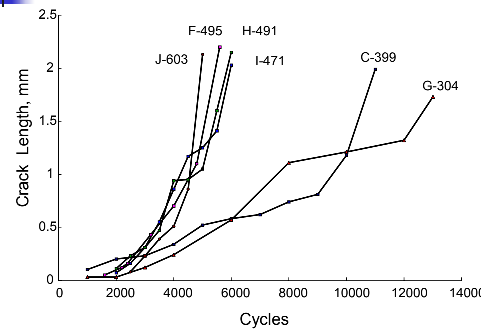




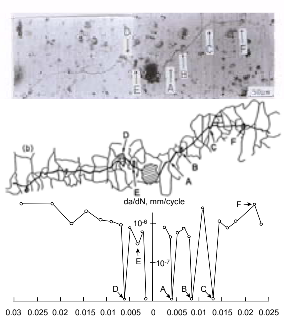
N = 900

Inconel 718
$\Delta\varepsilon = 0.02$
$N_f = 936$

Crack Length Observations

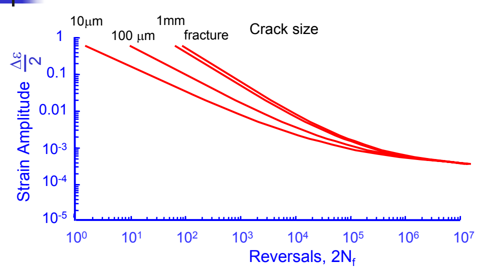


Crack - Microstructure Interactions



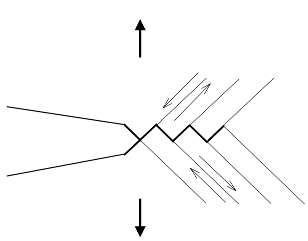
Akiniwa, Y., Tanaka, K., and Matsui, E.,"Statistical Characteristics of Propagation of Small Fatigue Cracks in Smooth Specimens of Aluminum Alloy 2024-T3, *Materials Science and Engineering*, Vol. A104, 1988, 105-115

Strain-Life Data



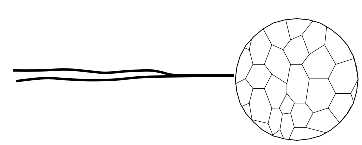
Most of the life is spent in microcrack growth in the plastic strain dominated region

Stage II Crack Growth



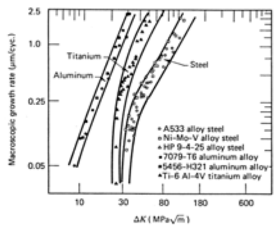
Locally, the crack grows in shear
Macroscopically it grows in tension

Long Crack Growth



Plastic zone size is much larger than the material microstructure so that the microstructure does not play such an important role.

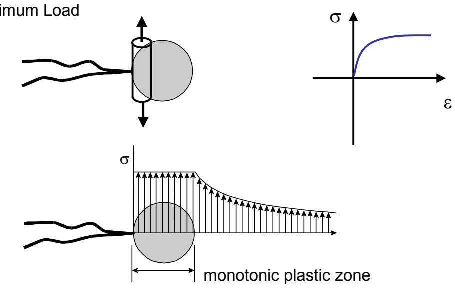
Crack Growth Rates of Metals



Material strength does not play a major role in fatigue crack growth

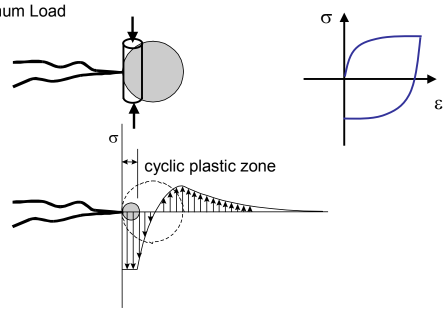
Stresses Around a Crack

Maximum Load

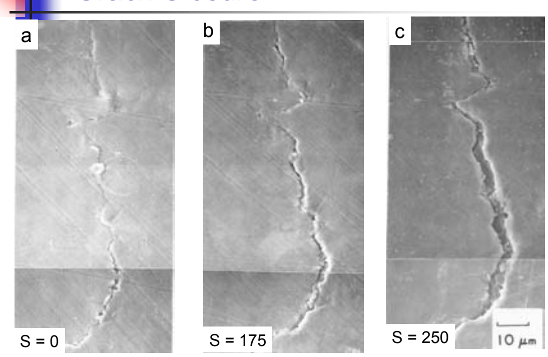


Stresses Around a Crack (continued)

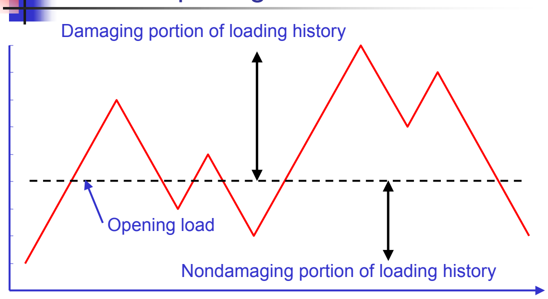
Minimum Load



Crack Closure

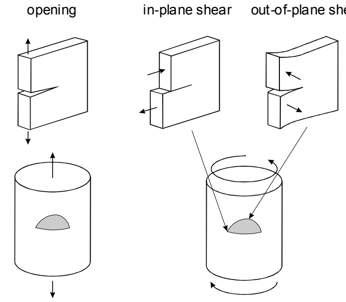


Crack Opening Load

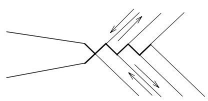


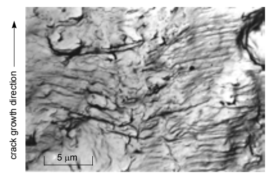
Mode I, Mode II, and Mode III

Mode I	Mode II	Mode III
opening	in-plane shear	out-of-plane shear

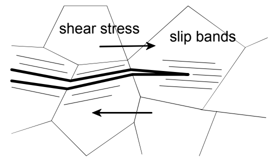


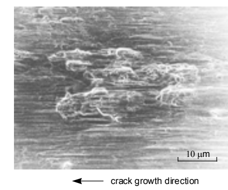
Mode I Growth



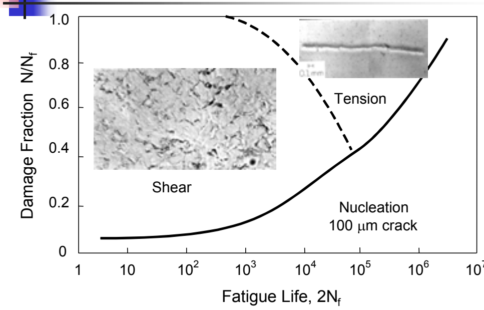


Mode II Growth

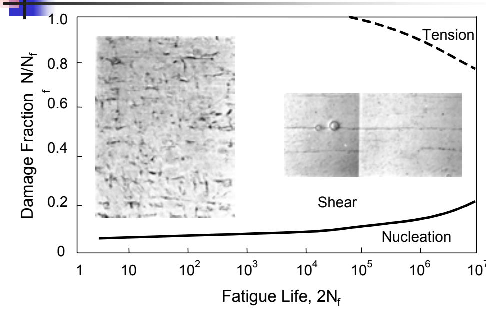




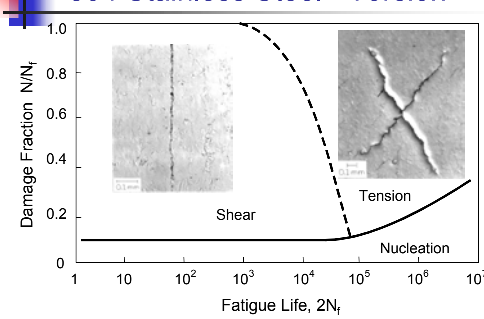
1045 Steel - Tension



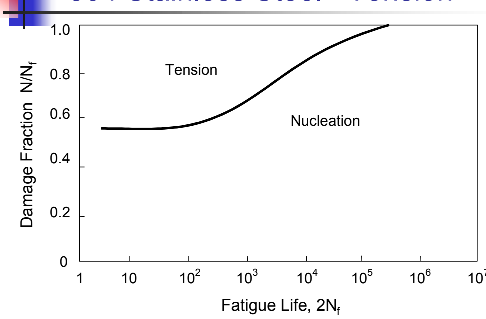
1045 Steel - Torsion



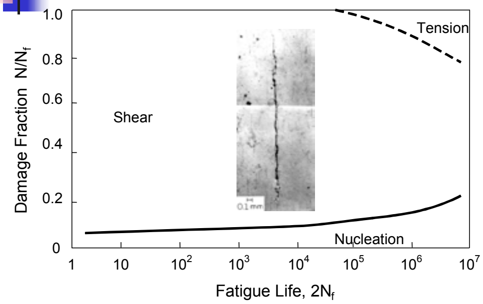
304 Stainless Steel - Torsion



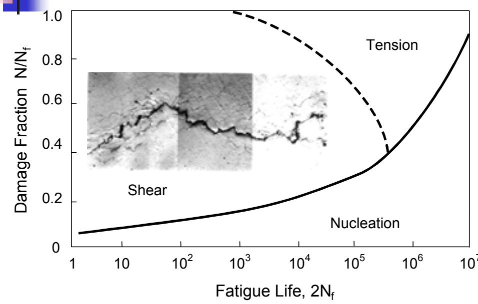
304 Stainless Steel - Tension



Inconel 718 - Torsion



Inconel 718 - Tension



Multiaxial Fatigue

Stress Based Models

Professor Darrell F. Socie
University of Illinois at Urbana-Champaign

Outline

- State of Stress
- Stress-Strain Relationships
- Fatigue Mechanisms
- Multiaxial Testing
- **Stress Based Models**
- Strain Based Models
- Fracture Mechanics Models
- Nonproportional Loading
- Stress Concentrations

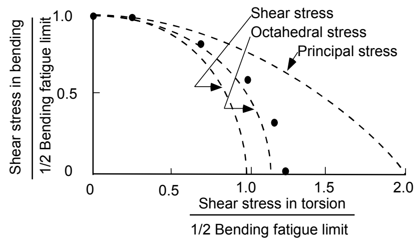
Fatigue Mechanisms Summary

- Fatigue cracks nucleate in shear
- Fatigue cracks grow in either shear or tension depending on material and state of stress

Stress Based Models

- Sines
- Findley
- Dang Van

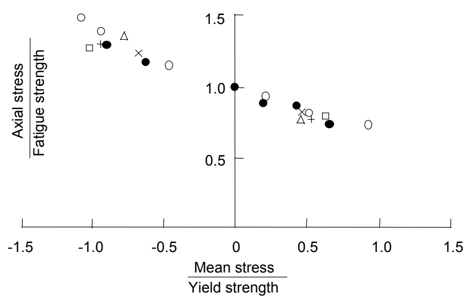
Bending Torsion Correlation



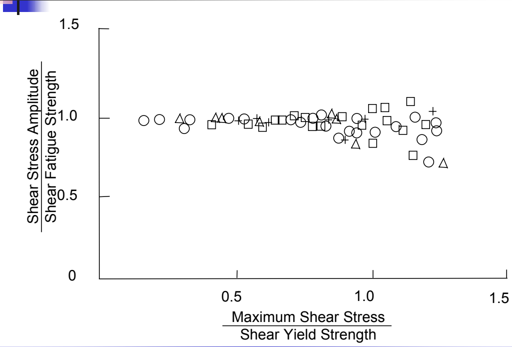
Test Results

- Cyclic tension with static tension
- Cyclic torsion with static torsion
- Cyclic tension with static torsion
- Cyclic torsion with static tension

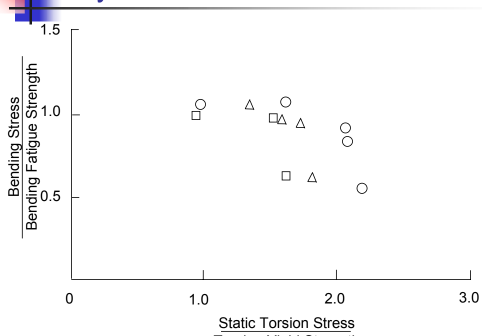
Cyclic Tension with Static Tension



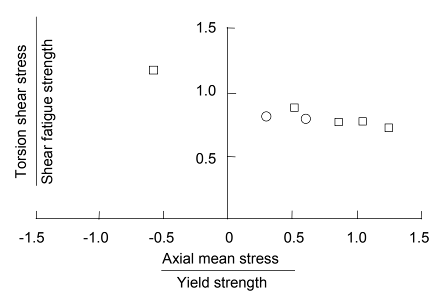
Cyclic Torsion with Static Torsion



Cyclic Tension with Static Torsion



Cyclic Torsion with Static Tension



Conclusions

- Tension mean stress affects both tension and torsion
- Torsion mean stress does not affect tension or torsion

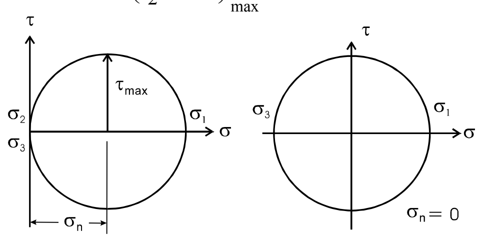
Sines

$$\frac{\Delta\tau_{oct}}{2} + \alpha(3\sigma_h) = \beta$$

$$\frac{1}{6}\sqrt{(\Delta\sigma_x - \Delta\sigma_y)^2 + (\Delta\sigma_x - \Delta\sigma_z)^2 + (\Delta\sigma_y - \Delta\sigma_z)^2 + 6(\Delta\tau_{xy}^2 + \Delta\tau_{xz}^2 + \Delta\tau_{yz}^2)} +$$
$$\alpha(\sigma_x^{mean} + \sigma_y^{mean} + \sigma_z^{mean}) = \beta$$

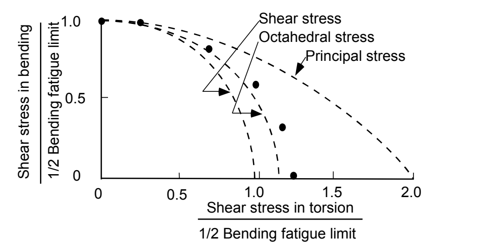
Findley

$$\left(\frac{\Delta\tau}{2} + k\sigma_n\right)_{max} = f$$



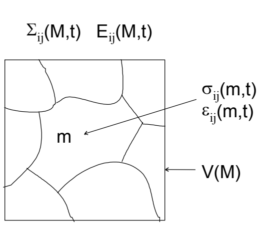
tension torsion

Bending Torsion Correlation

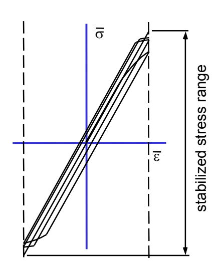


Dang Van

$$\tau(t) + a\sigma_h(t) = b$$

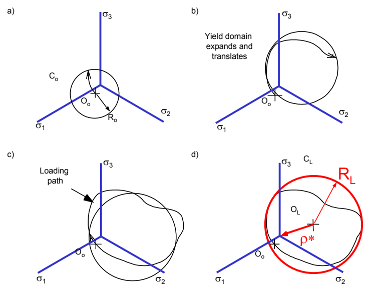


Isotropic Hardening



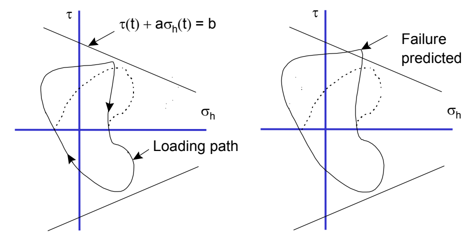
Failure occurs when the stress range is not elastic

Multiaxial Kinematic and Isotropic



ρ* stabilized residual stress

Dang Van (continued)



Stress Based Models Summary

Sines: $\frac{\Delta\tau_{oct}}{2} + \alpha(3\sigma_h) = \beta$

Findley: $\left(\frac{\Delta\tau}{2} + k\sigma_n\right)_{max} = f$

Dang Van: $\tau(t) + a\sigma_h(t) = b$

Multiaxial Fatigue

Strain Based Models

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Outline

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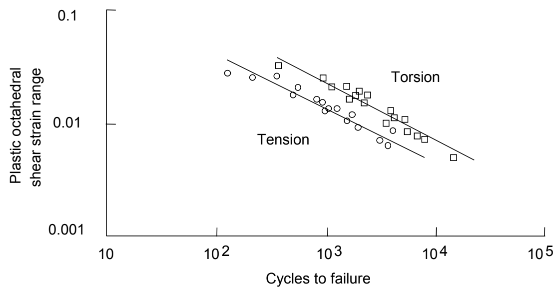
- Fracture Mechanics Models
- Nonproportional Loading
- Stress Concentrations

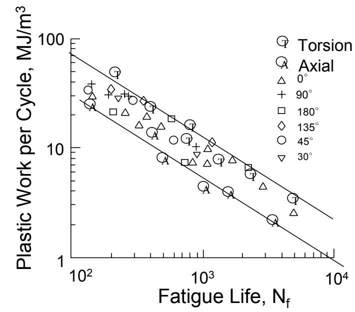
Strain Based Models

- Plastic Work
- Brown and Miller
- Fatemi and Socie
- Smith Watson and Topper
- Liu

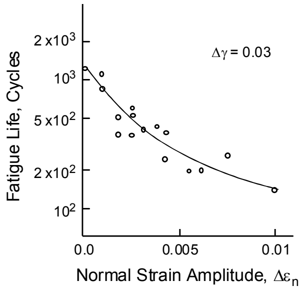
Octahedral Shear Strain



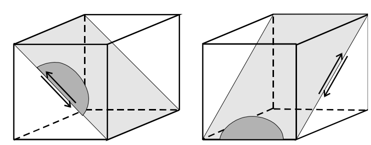
Plastic Work



Brown and Miller



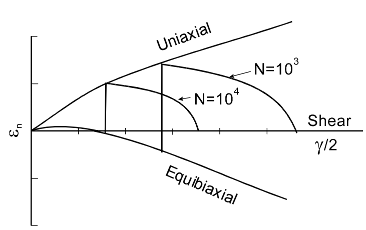
Case A and B



Case A — Growth along the surface

Case B — Growth into the surface

Brown and Miller (continued)

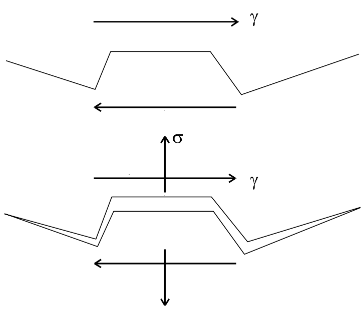


Brown and Miller (continued)

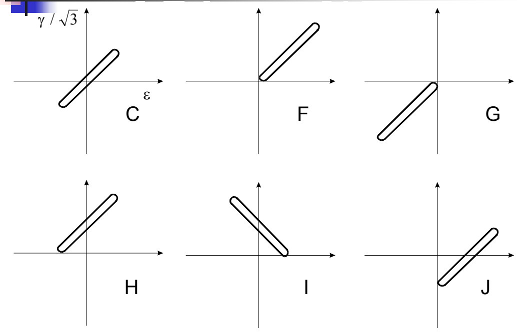
$$\Delta\hat{\gamma} = \left(\Delta\gamma_{max}^{\alpha} + S\Delta\varepsilon_n^{\alpha}\right)^{\frac{1}{\alpha}}$$

$$\frac{\Delta\gamma_{max}}{2} + S\Delta\varepsilon_n = A\frac{\sigma_f' - 2\sigma_{n,mean}}{E}(2N_f)^b + B\varepsilon_f'(2N_f)^c$$

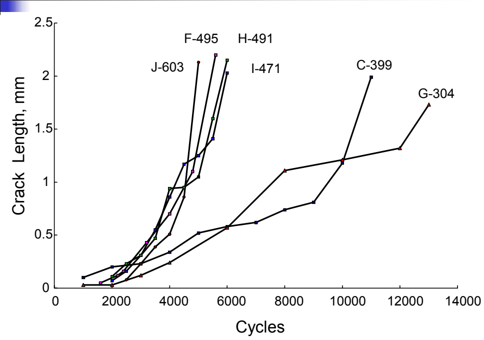
Fatemi and Socie



Loading Histories



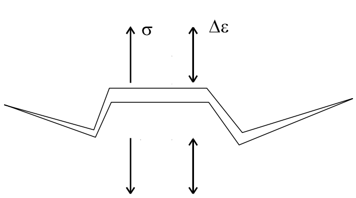
Crack Length Observations



Fatemi and Socie

$$\frac{\Delta\gamma}{2}\left(1+k\frac{\sigma_{n,max}}{\sigma_y}\right)=\frac{\tau_f'}{G}(2N_f)^{bo}+\gamma_f'(2N_f)^{co}$$

Smith Watson Topper



SWT

$$\sigma_n\frac{\Delta\varepsilon_1}{2}=\frac{\sigma_f'^{\,2}}{E}(2N_f)^{2b}+\sigma_f'\varepsilon_f'(2N_f)^{b+c}$$

Liu

Virtual strain energy for both mode I and mode II cracking

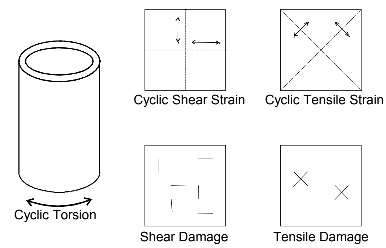
$$\Delta W_I = (\Delta\sigma_n\, \Delta\varepsilon_n)_{max} + (\Delta\tau\, \Delta\gamma)$$

$$\Delta W_I = 4\sigma'_f \varepsilon'_f (2N_f)^{b+c} + \frac{4\sigma'^2_f}{E}(2N_f)^{2b}$$

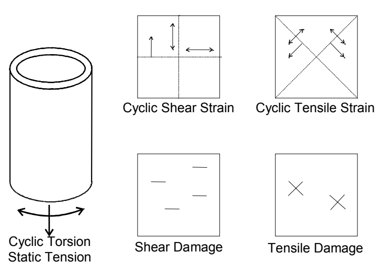
$$\Delta W_{II} = (\Delta\sigma_n\, \Delta\varepsilon_n) + (\Delta\tau\, \Delta\gamma)_{max}$$

$$\Delta W_{II} = 4\tau'_f \gamma'_f (2N_f)^{bo+co} + \frac{4\tau'^2_f}{G}(2N_f)^{2bo}$$

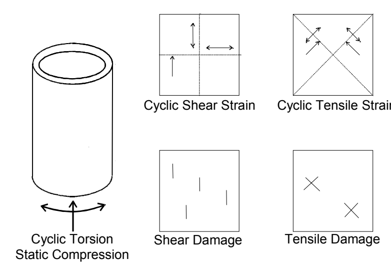
Cyclic Torsion



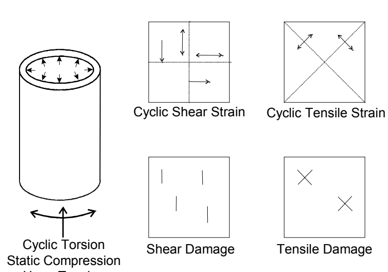
Cyclic Torsion with Static Tension



Cyclic Torsion with Compression



Cyclic Torsion with Tension and Compression



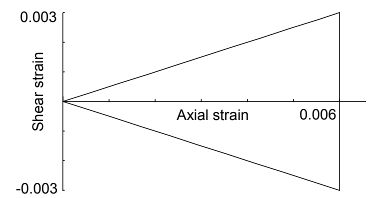
Test Results

Load Case	$\Delta\gamma/2$	σ_{hoop} MPa	σ_{axial} MPa	N_f
Torsion	0.0054	0	0	45,200
with tension	0.0054	0	450	10,300
with compression	0.0054	0	-500	50,000
with tension and compression	0.0054	450	-500	11,200

Conclusions

- All critical plane models correctly predict these results
- Hydrostatic stress models can not predict these results

Loading History



Model Comparison

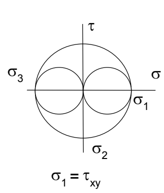
Summary of calculated fatigue lives

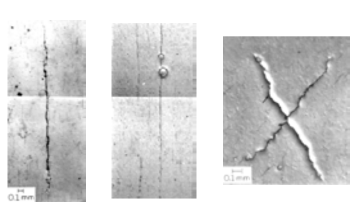
Model	Equation	Life
Epsilon	6.5	14,060
Garud	6.7	5,210
Ellyin	6.17	4,450
Brown-Miller	6.22	3,980
SWT	6.24	9,930
Liu I	6.41	4,280
Liu II	6.42	5,420
Chu	6.37	3,040
Gamma		26,775
Fatemi-Socie	6.23	10,350
Glinka	6.39	33,220

Strain Based Models Summary

- Two separate models are needed, one for tensile growth and one for shear growth
- Cyclic plasticity governs stress and strain ranges
- Mean stress effects are a result of crack closure on the critical plane

Separate Tensile and Shear Models





Inconel 1045 steel stainless steel

Cyclic Plasticity

$\Delta\varepsilon$

$\Delta\gamma$

$\Delta\varepsilon^p$

$\Delta\gamma^p$

$\Delta\varepsilon\Delta\sigma$

$\Delta\gamma\Delta\tau$

$\Delta\varepsilon^p\Delta\sigma$

$\Delta\gamma^p\Delta\tau$

Mean Stresses

$$\Delta\varepsilon_{eq} = \frac{\sigma_f' - \sigma_{mean}}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

$$\frac{\Delta\gamma_{max}}{2} + S\Delta\varepsilon_n = (1.3+0.7S)\frac{\sigma_f' - 2\sigma_n}{E}(2N_f)^b + (1.5+0.5S)\varepsilon_f'(2N_f)^c$$

$$\frac{\Delta\gamma}{2}\left(1 + k\frac{\sigma_{n,max}}{\sigma_y}\right) = \frac{\tau_f'}{G}(2N_f)^{bo} + \gamma_f'(2N_f)^{co}$$

$$\sigma_n\frac{\Delta\varepsilon_1}{2} = \frac{\sigma_f'^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}$$

$$\Delta W_t = \left[(\Delta\sigma_n\Delta\varepsilon_n)_{max} + (\Delta\tau\Delta\gamma)\right]\left(\frac{2}{1-R}\right)$$

Multiaxial Fatigue

Fracture Mechanics Models

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University of Illinois at Urbana-Champaign

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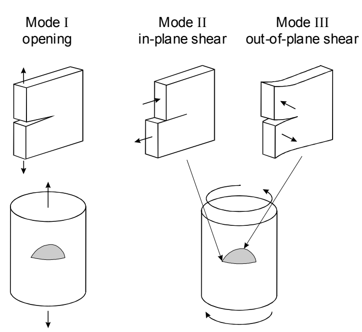
- **Fracture Mechanics Models**
- Nonproportional Loading
- Stress Concentrations

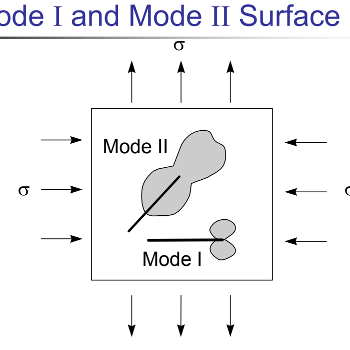
Fracture Mechanics Models

- Mode I growth
- Torsion
- Mode II growth
- Mode III growth

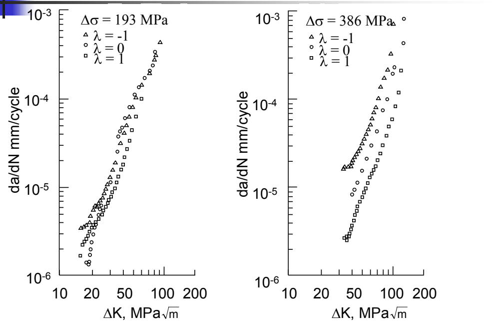
Mode I, Mode II, and Mode III



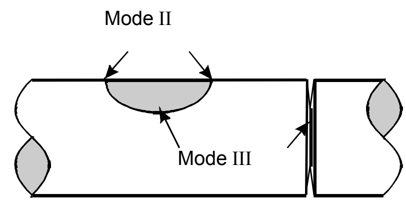
Mode I and Mode II Surface Cracks



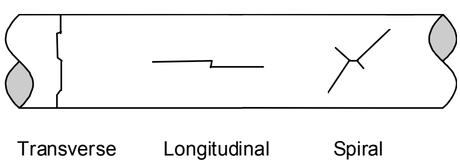
Biaxial Mode I Growth



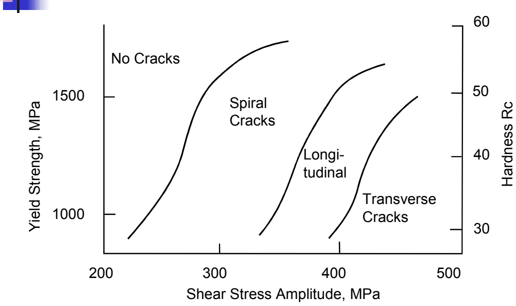
Surface Cracks in Torsion



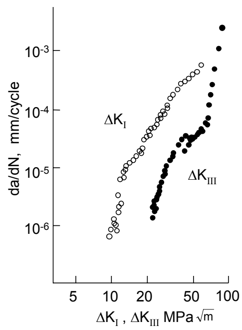
Failure Modes in Torsion



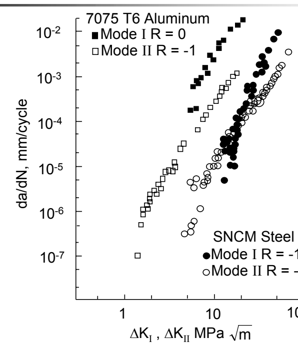
Fracture Mechanism Map



Mode I and Mode III Growth



Mode I and Mode II Growth



Fracture Mechanics Models

$$\frac{da}{dN} = C(\Delta K_{eq})^m$$

$$\Delta K_{eq} = \left[\Delta K_I^4 + 8\Delta K_{II}^4 + 8\Delta K_{III}^4/(1-\nu)\right]^{0.25}$$

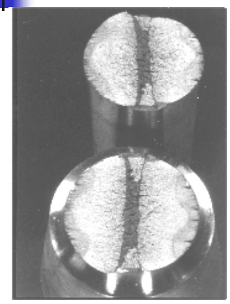
$$\Delta K_{eq} = \left[\Delta K_I^2 + \Delta K_{II}^2 + (1+\nu)\Delta K_{III}^2\right]^{0.5}$$

$$\Delta K_{eq} = \left[\Delta K_I^2 + \Delta K_I\,\Delta K_{II} + \Delta K_{II}^2\right]^{0.5}$$

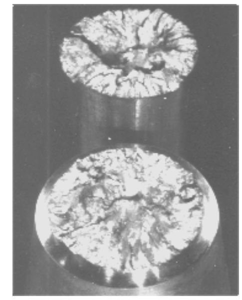
$$\Delta K_{eq}(\varepsilon) = \left[(F_{II}\frac{E}{2(1+\nu)}\Delta\gamma)^2 + (F_I E \Delta\varepsilon)^2\right]^{0.5}\sqrt{\pi a}$$

$$\Delta K_{eq}(\varepsilon) = FG\Delta\gamma\left(1 + k\frac{\sigma_{n,max}}{\sigma_{ys}}\right)\sqrt{\pi a}$$

Fracture Surfaces

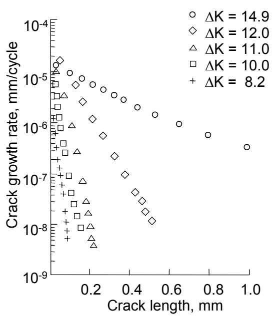


Bending



Torsion

Mode III Growth



Fracture Mechanics Models Summary

- Multiaxial loading has little effect in Mode I
- Crack closure makes Mode II and Mode III calculations difficult

Multiaxial Fatigue

Nonproportional Loading

Professor Darrell F. Socie
University of Illinois at Urbana-Champaign

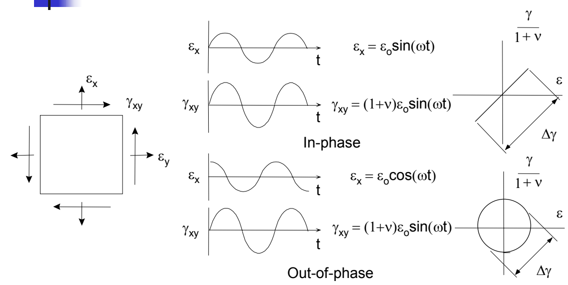
Outline

- State of Stress
- Stress-Strain Relationships
- Fatigue Mechanisms
- Multiaxial Testing
- Stress Based Models
- Strain Based Models
- Fracture Mechanics Models
- **Nonproportional Loading**
- Stress Concentrations

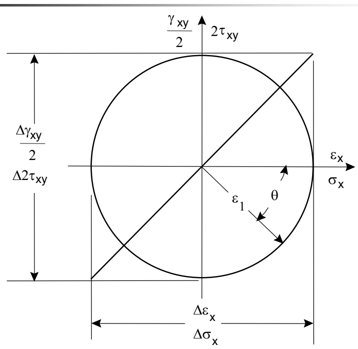
Nonproportional Loading

- In and Out-of-phase loading
- Nonproportional cyclic hardening
- Variable amplitude

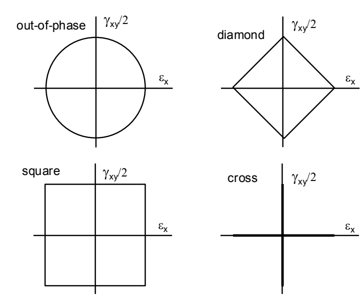
In and Out-of-Phase Loading



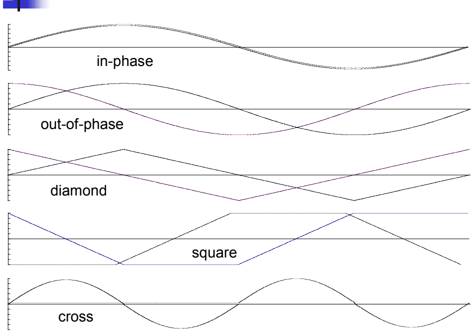
In-Phase and Out-of-Phase



Loading Histories

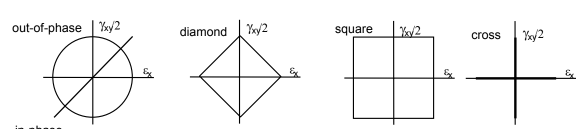


Loading Histories

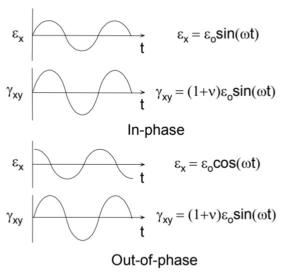


Findley Model Results

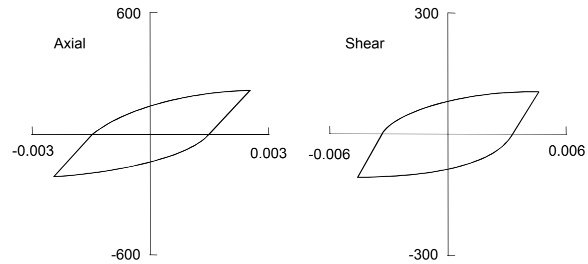
	$\Delta\tau/2$ MPa	$\sigma_{n,max}$ MPa	$\Delta\tau/2 + 0.3\ \sigma_{n,max}$	N/N_{ip}
in-phase	353	250	428	1.0
90° out-of-phase	250	500	400	2.0
diamond	250	500	400	2.0
square	353	603	534	0.11
cross - tension cycle	250	250	325	16
cross - torsion cycle	250	0	250	216



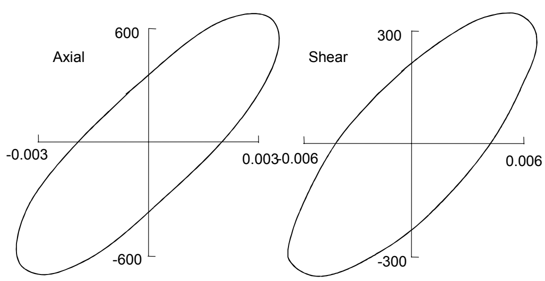
Nonproportional Hardening



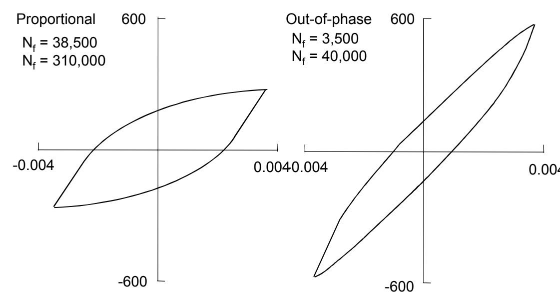
In-Phase



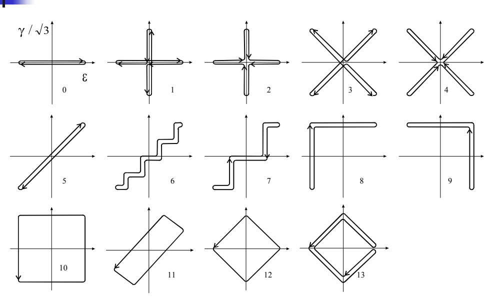
90° Out-of-Phase



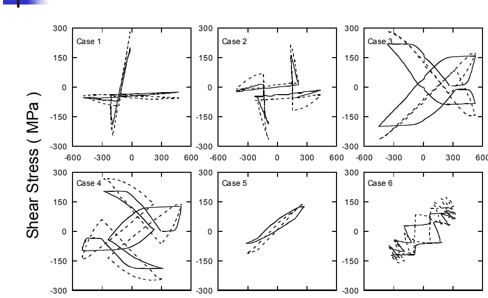
Critical Plane



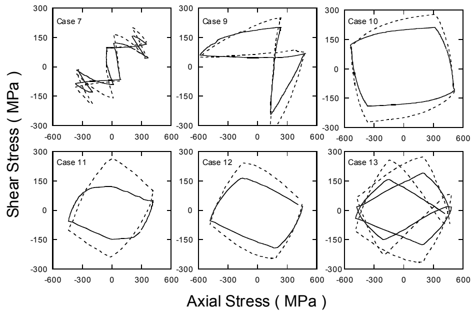
Loading Histories



Stress-Strain Response

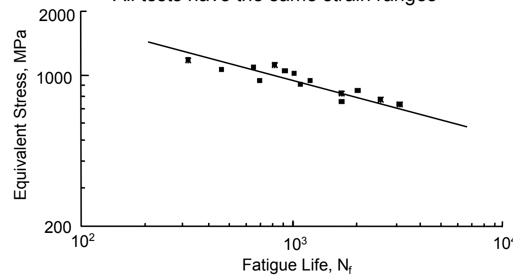


Stress-Strain Response (continued)



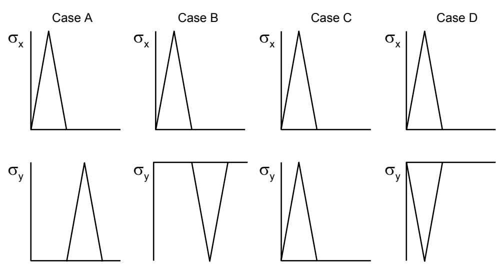
Maximum Stress

All tests have the same strain ranges

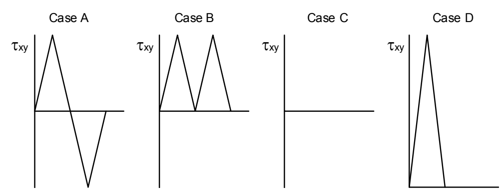


Nonproportional hardening results in lower fatigue lives

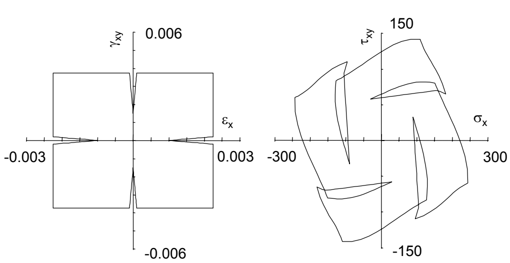
Nonproportional Example



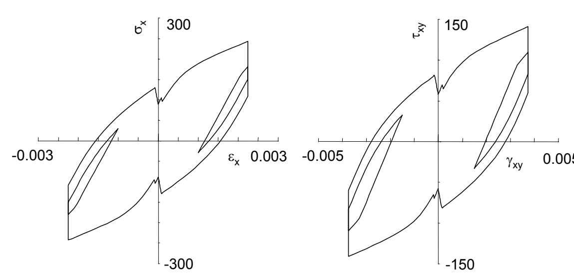
Shear Stresses



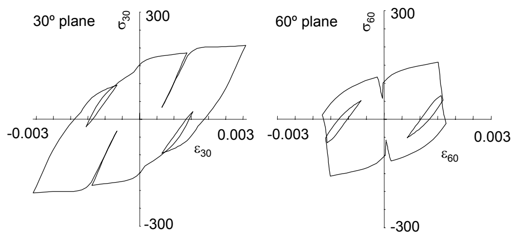
Simple Variable Amplitude History



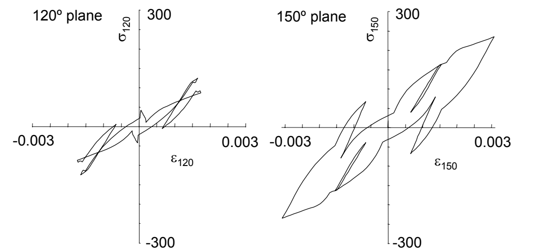
Stress-Strain on 0° Plane



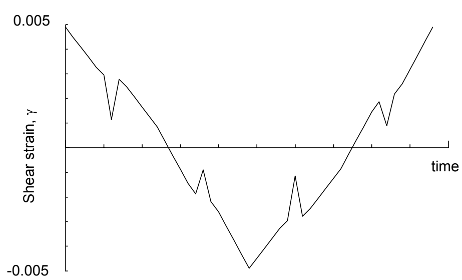
Stress-Strain on 30° and 60° Planes



Stress-Strain on 120° and 150° Planes



Shear Strain History on Critical Plane

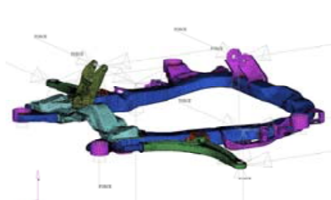


Fatigue Calculations

Load or strain history → Cyclic plasticity model → Stress and strain tensor → Search for critical plane

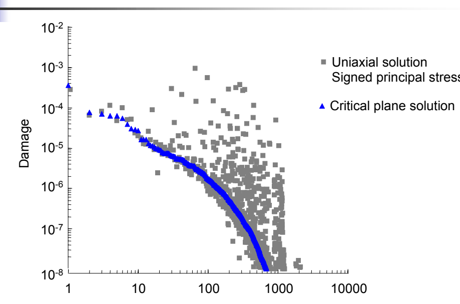
An Example

- Analysis model
  - Single event
  - 16 input channels
  - 2240 elements



From Khosrovaneh, Pattu and Schnaidt "Discussion of Fatigue Analysis Techniques for Automotive Applications" Presented at SAE 2004.

Biaxial and Uniaxial Solution



Nonproportional Loading Summary

- Nonproportional cyclic hardening increases stress levels
- Critical plane models are used to assess fatigue damage

Multiaxial Fatigue

Stress Concentrations

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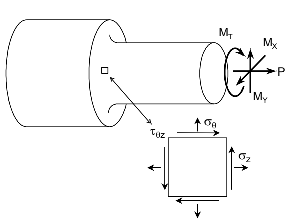
- **Stress Concentrations**

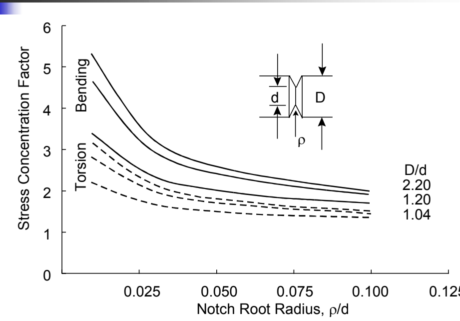
Notches

- Stress and strain concentrations
- Nonproportional loading and stressing
- Fatigue notch factors
- Cracks at notches

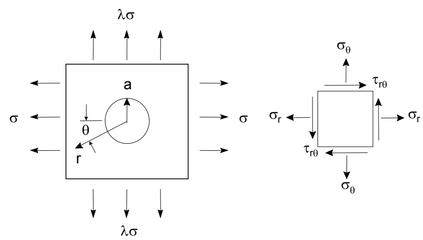
Notched Shaft Loading



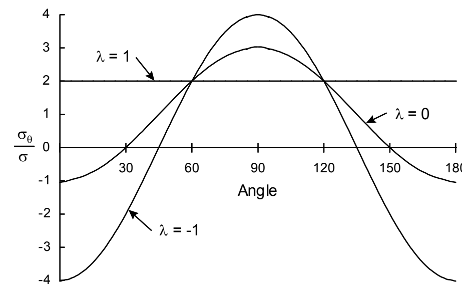
Stress Concentration Factors



Hole in a Plate

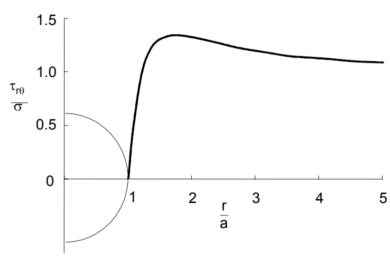


Stresses at the Hole

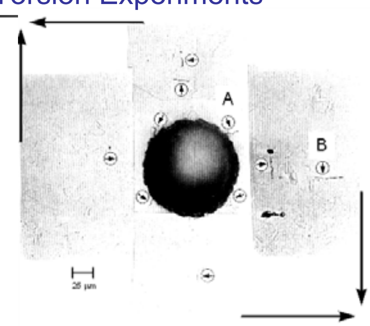


Stress concentration factor depends on type of loading

Shear Stresses during Torsion



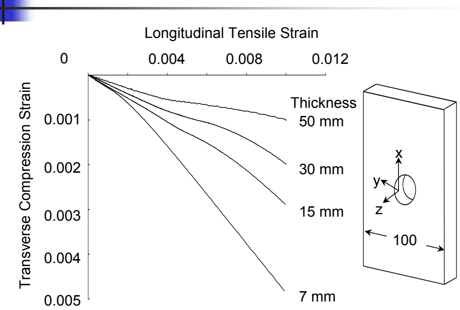
Torsion Experiments



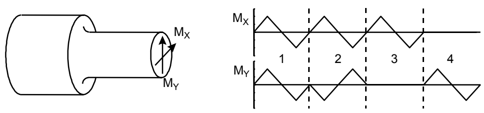
Multiaxial Loading

- Uniaxial loading that produces multiaxial stresses at notches
- Multiaxial loading that produces uniaxial stresses at notches
- Multiaxial loading that produces multiaxial stresses at notches

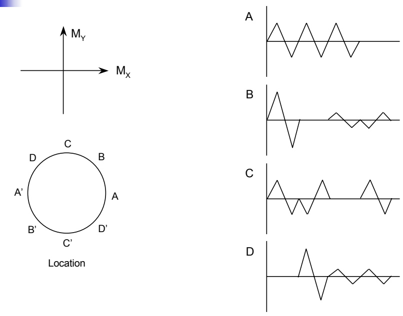
Thickness Effects



Applied Bending Moments



Bending Moments on the Shaft



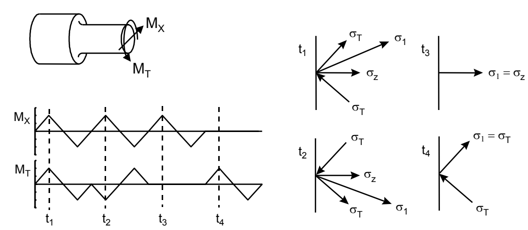
Bending Moments

ΔM	A	B	C	D
2.82		1		1
2.00	3		2	
1.41		2		1
1.00			2	
0.71				2

$$\Delta\overline{M} = \sqrt[5]{\sum \Delta M^5}$$

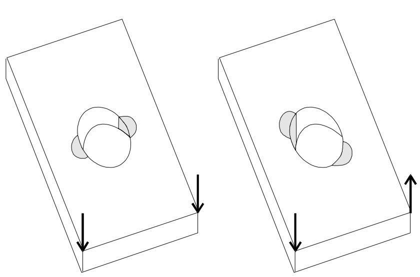
	A	B	C	D
$\Delta\overline{M}$	2.49	2.85	2.31	2.84

Torsion Loading



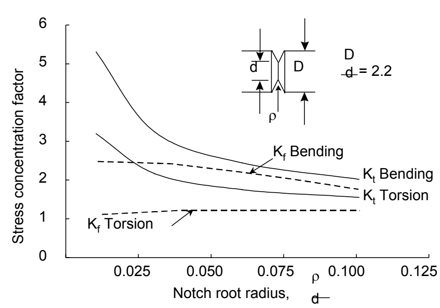
Out-of-phase shear loading is needed to produce nonproportional stressing

Plate and Shell Structures

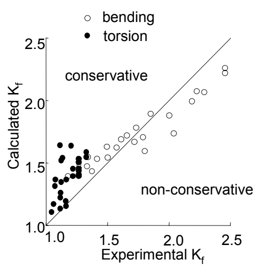


$K_t = 3$ $K_t = 4$

Fatigue Notch Factors



Fatigue Notch Factors (continued)



Peterson's Equation

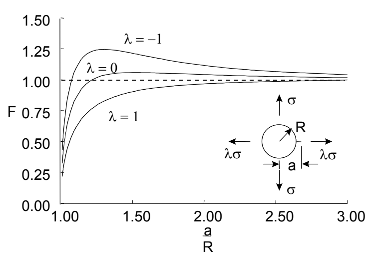
$$K_f = 1 + \frac{K_T - 1}{1 + \frac{a}{r}}$$

Fracture Surfaces in Torsion

Circumferencial Notch

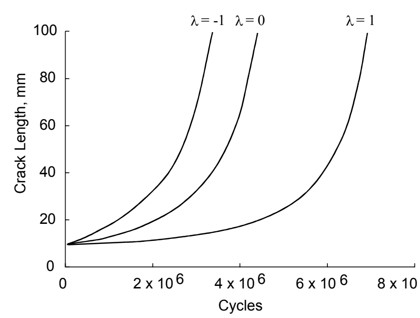
Shoulder Fillet

Stress Intensity Factors



$$\frac{da}{dN} = C\left(\Delta K_{eq}\right)^m \qquad K_I = F\,\Delta\sigma\sqrt{\pi a}$$

Crack Growth From a Hole



Notches Summary

- Uniaxial loading can produce multiaxial stresses at notches
- Multiaxial loading can produce uniaxial stresses at notches
- Multiaxial stresses are not very important in thin plate and shell structures
- Multiaxial stresses are not very important in crack growth

Multiaxial Fatigue